Basic Education

KwaZulu-Natal Department of Basic Education
REPUBLIC OF SOUTH AFRICA

MATHEMATICS

COMMON TEST

MARCH 2016

NATIONAL SENIOR CERTIFICATE

GRADE 12

Marks: 100

Time: 2 hours

N.B. This question paper consists of 7 pages and an information sheet.

INSTRUCTIONS AND INFORMATION

Read the following instructions carefully before answering the questions.

1. This question paper consists of 8 questions.

2. Answer **ALL** questions.

3. Clearly show **ALL** calculations, diagrams, graphs, et cetera that you have used in determining your answers.

4. Answers only will not necessarily be awarded full marks.

5. An approved scientific calculator (non-programmable and non-graphical) may be used, unless stated otherwise.

6. If necessary, answers should be rounded off to TWO decimal places, unless stated otherwise.

7. Diagrams are NOT necessarily drawn to scale.

8. Number the answers correctly according to the numbering system used in this question paper. Write neatly and legibly.

QUESTION 1

1.1 Given: $f(x)=\frac{-5}{x}$, determine $f'(x)$ from first principles. (5)

1.2 Determine:-

1.2.1 $\frac{dy}{dx}$ if $y=(x^2+3)\sqrt{x}$ (4)

1.2.2 $f'(x)$ if $f(x)=\frac{x^3-8}{2-x}$ (4)

[13]

QUESTION 2

Given the equation of the curve $h(x)=x^3+3x^2+15x$

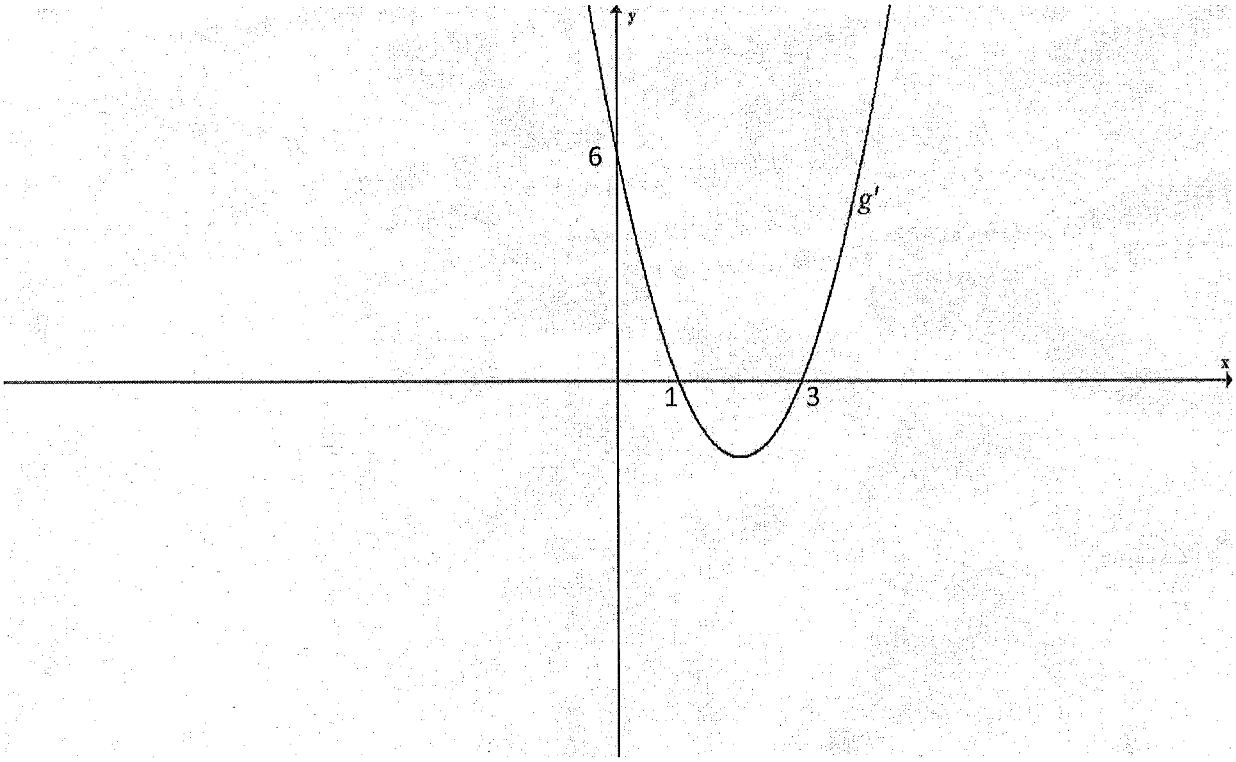
2.1 Determine the equation of the tangent to h at $x=-3$. (5)

2.2 Show that the function h is increasing for all real value(s) of x. (5)

[10]

QUESTION 3

The sketch below represents $g'(x) = ax^2 + bx + c$ with $g'(1) = g'(3) = 0$ and $g'(0) = 6$.



3.1 Write down the x-coordinates of the stationary points of g. (2)

3.2 Determine the value of x, for which the gradient of the curve is 6, except $x = 0$. (3)

3.3 For which value(s) of x is the graph of g strictly increasing? (3)

3.4 Calculate the x – coordinate of the point of inflection of g. (1)

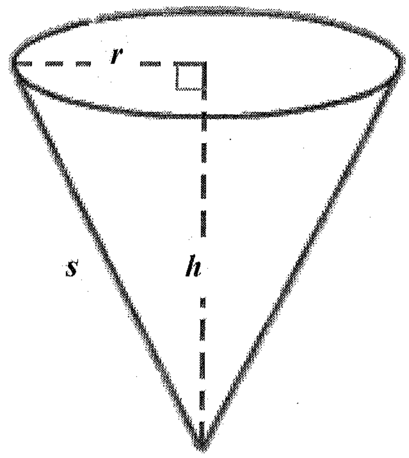
3.5 If it is further given that $g(-1) = \frac{4}{3}$ and g is a cubic function.
$g(x) = ax^3 + bx^2 + cx + d$, calculate the value of the y – intercept of g. (7)

3.6 Determine the value(s) of x for which g is concave up. (2)

[18]

QUESTION 4

A right circular cone with perpendicular height (h cm) , radius (r cm) has a slant height(s) of 12 cm.



$V = \frac{1}{3}\pi r^2 h$

$V = \frac{4}{3}\pi r^3$

$A = \pi r^2 + \pi rs$

$A = 4\pi r^2$

4.1 Express the radius of the cone in terms h. (2)

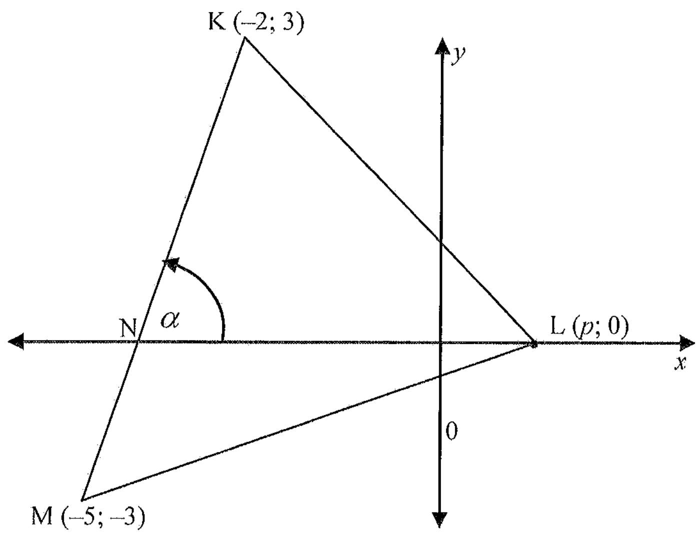
4.2 Show that the Volume of the cone is given by $V = 48\pi h - \frac{1}{3}\pi h^3$. (2)

4.3 Calculate the height of the cone for which the volume is at a maximum. (5)

[9]

QUESTION 5

In the sketch alongside, points K(–2:3), L(p;0) and M(–5;–3) are vertices of Δ KLM in a Cartesian plane. KM cuts the x-axis at N and $K\hat{N}L = \alpha$.



5.1.1 Determine the equation of the line KM. (3)

5.1.2 Hence, calculate the co-ordinates of N (1)

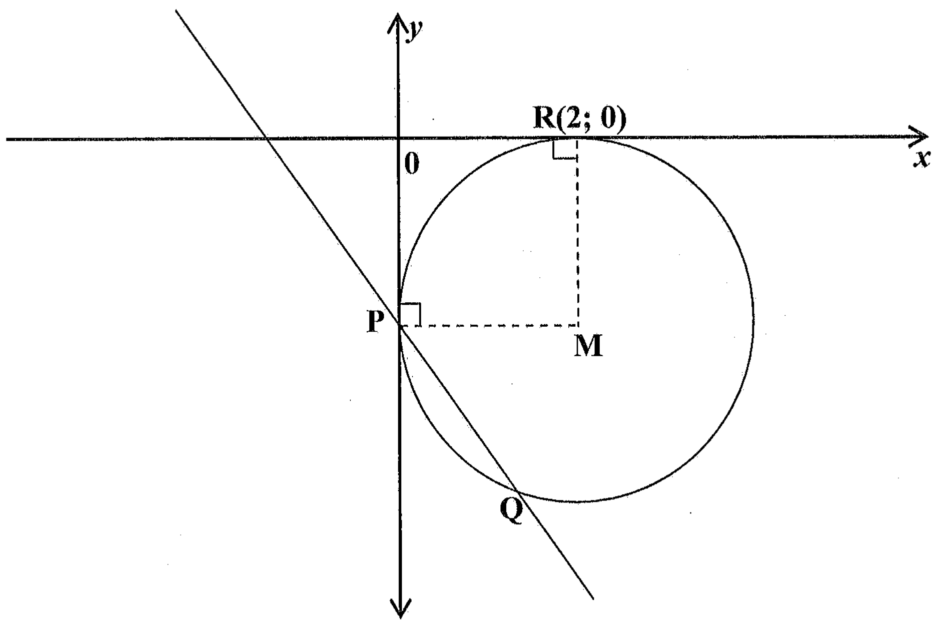
5.2 Calculate the value of p if KM = LM (4)

5.3 Determine α, correct to ONE decimal digit. (2)

[10]

QUESTION 6

In the sketch below, the circle with centre M touches, the y-axis at P and the x-axis at R(2:0). The straight line defined by the equation $y = -x - 2$ cuts the circle at point Q and passes through point P.



6.1	Write down the co-ordinates of P.	(1)
6.2	Write down the co-ordinates of M, the centre of the circle.	(1)
6.3	Show that the equation of the circle with centre M is : $x^2 + y^2 - 4x + 4y + 4 = 0$.	(3)
6.4	The straight line with equation y $= -x + c$ is a tangent to the circle with centre M. Calculate the numerical values of c.	(5)
		[10]

QUESTION 7

7.1 If $4 \tan \alpha - 3 = 0$ and $90^\circ \leq \alpha \leq 360^\circ$, determine without the use of a calculator the value of $\cos^2 \alpha - \sin \alpha$. (5)

7.2 Simplify, without using a calculator:

7.2.1 $\dfrac{\sin 61^\circ \, . \, \sin(90^\circ - \theta)}{\cos 29^\circ \, . \, \sin(180^\circ - \theta)}$ (4)

7.2.2 $\sin 15^\circ \cos 15^\circ$ (3)

7.3 Prove the following identity:

$$\frac{\sin A - \cos A}{\sin A + \cos A} + \frac{\sin A + \cos A}{\sin A - \cos A} = \frac{-2}{\cos 2A}$$ (6)

[18]

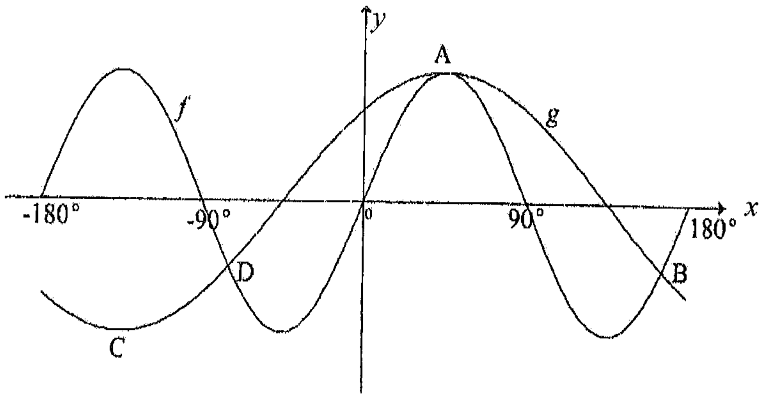
QUESTION 8

8.1 Determine the general solution of $\cos 2x + \cos x - 2 = 0$ (5)

8.2 The sketch below, shows the graphs of :

$f = \{(x\,;y\,/\,y = \sin px\}$ and
$g = \{(x\,;y)\,/\,y = \cos(x+q)\,;x \in [-180°\,;180°]$

A $(45°\,;1)$ and B $\left(165°\,;-\frac{1}{2}\right)$ are two points of intersection of f and g.



8.2.1 Determine the value(s) of p and q. (4)

8.2.2 What is the period of g? (1)

8.2.3 Write down the co-ordinates of C, the turning point of the curve g (1)

8.2.4 Write down the co-ordinates of D, a point of intersection of f and g. (1)
[12]

TOTAL: 100

INFORMATION SHEET: MATHEMATICS
INLIGTINGSBLAD: WISKUNDE

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$A = P(1+ni)$ $\quad A = P(1-ni)$ $\quad A = P(1-i)^n$ $\quad A = P(1+i)^n$

$\sum_{i=1}^{n} 1 = n$ $\quad \sum_{i=1}^{n} i = \frac{n(n+1)}{2}$ $\quad T_n = a + (n-1)d$ $\quad S_n = \frac{n}{2}(2a + (n-1)d)$

$T_n = ar^{n-1}$ $\quad S_n = \frac{a(r^n - 1)}{r-1}$; $\quad r \neq 1$ $\quad S_\infty = \frac{a}{1-r}$; $-1 < r < 1$

$F = \frac{x[(1+i)^n - 1]}{i}$ $\quad P = \frac{x[1-(1+i)^{-n}]}{i}$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $\quad M\left(\frac{x_1 + x_2}{2}; \frac{y_1 + y_2}{2}\right)$

$y = mx + c$ $\quad y - y_1 = m(x - x_1)$ $\quad m = \frac{y_2 - y_1}{x_2 - x_1}$ $\quad m = \tan\theta$

$(x-a)^2 + (y-b)^2 = r^2$

In ΔABC: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$ $\quad a^2 = b^2 + c^2 - 2bc.\cos A$ $\quad area\,\Delta ABC = \frac{1}{2}ab.\sin C$

$\sin(\alpha + \beta) = \sin\alpha.\cos\beta + \cos\alpha.\sin\beta$ $\quad \sin(\alpha - \beta) = \sin\alpha.\cos\beta - \cos\alpha.\sin\beta$

$\cos(\alpha + \beta) = \cos\alpha.\cos\beta - \sin\alpha.\sin\beta$ $\quad \cos(\alpha - \beta) = \cos\alpha.\cos\beta + \sin\alpha.\sin\beta$

$$\cos 2\alpha = \begin{cases} \cos^2\alpha - \sin^2\alpha \\ 1 - 2\sin^2\alpha \\ 2\cos^2\alpha - 1 \end{cases}$$

$\sin 2\alpha = 2\sin\alpha.\cos\alpha$

$(x\,;y) \to (x\cos\theta + y\sin\theta\,; y\cos\theta - x\sin\theta)$ $\quad (x\,;y) \to (x\cos\theta - y\sin\theta\,; y\cos\theta + x\sin\theta)$

$\bar{x} = \frac{\sum fx}{n}$ $\quad \sigma^2 = \frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n}$

$P(A) = \frac{n(A)}{n(S)}$ $\quad P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$

$\hat{y} = a + bx$ $\quad b = \frac{\sum(x - \bar{x})(y - \bar{y})}{\sum(x - \bar{x})^2}$

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MEMORANDUM

NATIONAL SENIOR CERTIFICATE

GRADE 12

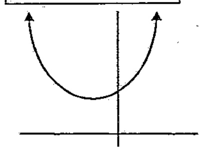
MARKS: 100

This memorandum consists of 11 pages.

QUESTION 1

1.1	$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$ $= \lim_{h \to 0} \frac{\frac{-5}{x+h} - \frac{-5}{x}}{h}$ $= \lim_{h \to 0} \frac{\frac{-5x+5x+5h}{x(x+h)}}{h}$ $= \lim_{h \to 0} \frac{5h}{x(x+h)} \times \frac{1}{h}$ $= \frac{5}{x^2}$	Penalise 1 mark for incorrect notation	✓Formula ✓Substitution ✓Simplifying ✓Simplifying ✓Answer (5)
1.2.1	$y = x^{\frac{5}{2}} + 3x^{\frac{1}{2}}$ $\frac{dy}{dx} = \frac{5}{2}x^{\frac{3}{2}} + \frac{3}{2}x^{-\frac{1}{2}}$ **OR** $y = (x^2+3)x^{\frac{1}{2}}$ $\frac{dy}{dx} = (x^2+3)\frac{1}{2}x^{-\frac{1}{2}} + x^{\frac{1}{2}}.2x$ $= \frac{1}{2}x^{\frac{3}{2}} + \frac{3}{2}x^{-\frac{1}{2}} + 2x^{\frac{3}{2}}$ $= \frac{5}{2}x^{\frac{3}{2}} + \frac{3}{2}x^{-\frac{1}{2}}$	Product Rule	✓✓each term ✓✓each answer (4) ✓✓$(x^2+3)\frac{1}{2}x^{-\frac{1}{2}}$ ✓✓$x^{\frac{1}{2}}.2x$ (4)
1.2.2	$f(x) = \frac{(x-2)(x^2+2x+4)}{-(x-2)}$ $= -x^2 - 2x - 4$ $f'(x) = -2x - 2$	If second bracket is linear – break down	✓Factorising numerator ✓ $-x^2-2x-4$ ✓✓Answers (4) [13]

QUESTION 2

2.1	$h'(x)=3x^2+6x+15$ $h'(-3)=3(-3)^2+6(-3)+15=24$ $h(-3)=(-3)^3+3(-3)^2+15(-3)=-45$ $y-y_1=m(x-x_1)$ $y+45=24(x+3)$ $y=24x+27$ *OR* $h'(x)=3x^2+6x+15$ $h'(-3)=3(-3)^2+6(-3)+15=24$ $h(-3)=(-3)^3+3(-3)^2+15(-3)=-45$ $y=mx+c$ $-45=24(-3)+c$ $c=27$ $y=24x+27$	✓derivative ✓gradient value ✓y - value ✓substitution of gradient = 24 and point (–3;–45) ✓answer (5) **OR** ✓derivative ✓gradient value ✓y - value ✓substitution of gradient = 24 and point (–3;–45) ✓answer (5)
2.2	$h'(x)=3x^2+6x+15$ $=3x^2+6x+3+12$ $=3(x^2+2x+1)+12$ $=3(x+1)^2+12$ $3(x+1)^2\geq 0$ for all $x\in R$ $\Rightarrow 3(x+1)^2+12\geq 12$ $\therefore h'(x)\geq 12 \Rightarrow h'(x)>0$ h is increasing for all values of x Graph Method full marks if concluded h'(x) > 0  **OR** $h'(x)=3x^2+6x+15$ $=3[x^2+2x+5]$ $=3[x^2+2x+1-1+5]$ $=3[(x+1)^2+4]$ $=3(x+1)^2+12$ $3(x+1)^2\geq 0$ for all $x\in R$ $\Rightarrow 3(x+1)^2+12\geq 12$ $\therefore h'(x)\geq 12 \Rightarrow h'(x)>0$ h is increasing for all values of x	✓writing 15 as 3+12 ✓factorising ✓writing as perfect square ✓$3(x+1)^2\geq 0$ ✓proving $h'(x)>0$ (5) ✓removing 3 as a common factor ✓completing the square ✓Multiplying by 3 ✓$3(x+1)^2\geq 0$ ✓proving $h'(x)>0$ (5) **[10]**

QUESTION 3

3.1	$x=1$ or $x=3$	✓✓answers (2)
3.2	Midpointof x values of TP's $x=\frac{1+3}{2}=2$ Answer only full marks Using symmetry $x=4$	✓$x=2$ ✓Using symmetry ✓answer (3)
3.3	$x<1$ or $x>3$	✓$x<1$ ✓$x>3$ ✓ or (3)
3.4	$x=2$	✓answer (1)

3.5	$y=a(x-x_1)(x-x_2)$ $6=a(0-1)(0-3)$ $6=3a$ $2=a$ $y=2(x-1)(x-3)$ $y=2x^2-8x+6$ $g'(x)=3ax^2+2bx+c$ $3a=2 \quad 2b=-8 \quad c=6$ $a=\frac{2}{3} \quad b=-4$ $g(x)=\frac{2}{3}x^3-4x^2+6x+d$ $\frac{4}{3}=\frac{2}{3}(-1)^3-4(-1)^2+6(-1)+d$ $\frac{4}{3}=-\frac{32}{3}+d$ $12=d$ - y-intercept. *OR* $y=a(x-x_1)(x-x_2)$ $6=a(0-1)(0-3)$ $6=3a$ $2=a$ $y=2(x-1)(x-3)$ $y=2x^2-8x+6$ $g(x)=\frac{2}{3}x^3-4x^2+6x+d$ $\frac{4}{3}=\frac{2}{3}(-1)^3-4(-1)^2+6(-1)+d$ $\frac{4}{3}=-\frac{32}{3}+d$ $12=d$ - y-intercept.	✓substituting into formula ✓ a value ✓ equation of derivative ✓ a value and b value ✓$g(x)$ ✓substituting point ✓ y intercept (7) ✓substituting into formula ✓ a value ✓ equation of derivative ✓✓anti derivative ✓substituting point ✓ y intercept (7)
3.6	$g''(x)=4x-8$ $4x-8>0$ $x>2$ Answer only – full marks	✓ $4x-8>0$ ✓answer (2)
		[18]

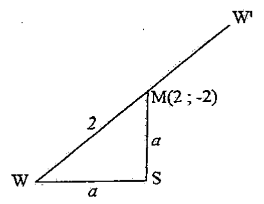
QUESTION 4

4.1	$r^2+h^2=144$ $r^2=144-h^2$ $r=\sqrt{144-h^2}$	✓ $r^2+h^2=144$ ✓answer (2)
4.2	$V=\frac{1}{3}\pi r^2 h$ $=\frac{1}{3}\pi h(144-h^2)$ $=48\pi h-\frac{1}{3}\pi h^3$	✓Formula ✓Subst. into formula (2)
4.3	$V=48\pi h-\frac{1}{3}\pi h^3$ $V'=48\pi-\pi h^2=0$ $48-h^2=0$ $h=\sqrt{48}=4\sqrt{3}$ $h=6{,}93cm$ No penalty for rounding off	✓✓For derivative and equating to 0 ✓for simplifying ✓for h being the subject ✓answer (5)
		[9]

QUESTION 5

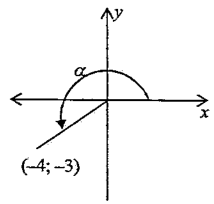
5.1.1	$m_{KM} = \frac{-3-3}{-5-(-2)} = \frac{-6}{-3} = 2$ $y = 2x + c$ $(-2; 3)$: $3 = 2(-2) + c$ $7 = c$ $y = 2x + 7$	✓ gradient ✓ substitution (3) ✓equation of the line
5.1.2	$N(-\frac{7}{2};0)$	✓Co-ordinates of N (1)
5.2	$KM^2 = LM^2$ $(-5-(-2))^2 + (-3-3)^2 = (-5-p)^2 + (-3-0)^2$ $(-5+2)^2 + (-6)^2 = (-5-p)^2 + (3)^2$ $9 + 36 = (-5-p)^2 + 9$ $45 = (-5-p)^2 + 9$ $(-5-p)^2 = 45 - 9$ $25 + 10p + p^2 = 36$ $p^2 + 10p + 25 - 36 = 0$ $p^2 + 10p - 11 = 0$ $(p + 11)(p - 1) = 1$ $p = -11$ or $p = 1$ $\therefore p = 1$	✓ substitution ✓simplifying the square ✓simplifying ✓$p = 1$ (4)
5.3	$m_{KM} = 2 = \tan\alpha$ $\tan^{-1}\alpha = 2$ $\alpha = 63{,}4°$	✓ $\tan^{-1}\alpha = 2$ ✓ answer (2)
		[10]

QUESTION 6

6.1	P(0; –2)	✓ correct co-ordinates (1)
6.2	M (2 ; –2)	✓ correct co-ordinates (1)
6.3	Centre M(2;–2) and radius 2 units $(x-2)^2 + (y+2)^2 = (2)^2$ $x^2 - 4x + 4 + y^2 + 4y + 4 = 4$ $x^2 + y^2 - 4x + 4y + 8 = 4$ $x^2 + y^2 - 4x + 4y + 4 = 0$	✓ substitution of centre ✓ substitution of radius ✓ $x^2 + y^2 - 4x + 4y + 8 = 4$ (3)
6.4	$y = -x + c$ substituting $y = -x + c$ into the equation of the circle $x^2 + (-x+c)^2 - 4x + 4(-x+c) + 4 = 0$ $x^2 + x^2 - 2xc + c^2 - 4x - 4x + 4c + 4 = 0$ $2x^2 - 2xc + c^2 - 8x + 4c + 4 = 0$ $2x^2 - (2c+8)x + (c^2 + 4c + 4) = 0$ Since the line is a tangent to the circle, the above equation must have equal root ($\Delta = 0$) $\Delta = b^2 - 4ac = 0$ $0 = (2c+8)^2 - 4(2)(c^2 + 4c + 4)$ $0 = 4c^2 + 32c + 64 - 8c^2 - 32c - 32$ $0 = -4c^2 + 32$ $-\frac{4c^2}{4} = \frac{-32}{4}$ $c^2 = 8$ $c = \pm\sqrt{8}$ $= \pm 2\sqrt{2}$ *or* $\pm 2{,}82$ **OR** Let points of contact of the tangent with the circle be W and W'  	✓Subst $y = -x + c$ ✓writing the equation in a standard form ✓ using $\Delta = 0$ ✓ subst ✓values of c (5)

	m of tangent $= -1$ m of radius $= 1$ MS= WS $= a$ $a^2 + a^2 = 2^2$ $2a^2 = 4$ $a^2 = 2$ $a = \pm\sqrt{2}$ $\therefore W(2-\sqrt{2};-2-\sqrt{2})$ and $W'(2+\sqrt{2};-2+\sqrt{2})$ For tangent at W: $y = -x + c$ $-2-\sqrt{2} = -(2-\sqrt{2}) + c$ $c = -2\sqrt{2}$ For tangent at W': $y = -x + c$ $-2+\sqrt{2} = -(2+\sqrt{2}) + c$ $c = 2\sqrt{2}$	✓gradient of radius ✓theorem of pythagoras ✓both values of a ✓substitution into the equation of the tangent ✓both answers (5)

QUESTION 7

7.1	$4\tan\alpha - 3 = 0$ $4\tan\alpha = 3$ $\tan\alpha = \frac{3}{4}$ and $90° \le \alpha \le 360°$ implies that α lies in the third quadrant. $\therefore \tan\alpha = \frac{3}{4} = \frac{-3}{-4}$ $\therefore x = -3$ and $y = -3$ $r^2 = (-4)^2 + (-3)^2 = 25$ $\therefore r = 5$ No Sketch – but correct working – full marks $\cos^2\alpha - \sin\alpha$ $= \left(\frac{-4}{5}\right)^2 - \left(\frac{-3}{5}\right)$ $= \frac{16}{25} + \frac{3}{5}$ $= \frac{16+15}{25}$ $= \frac{31}{25}$ or $1\frac{6}{25}$	✓correct quadrant  ✓ $r = 5$ ✓subst ✓simplifying ✓ answer (5)
7.2.1	$\frac{\sin 61° . \sin(90° - \theta)}{\cos 29° . \sin(180° - \theta)}$ $= \frac{\sin(90° - 29°) . \sin(90° - \theta)}{\cos 29° . \sin(180° - \theta)}$ $= \frac{\cos 29° . \cos\theta}{\cos 29° . \sin\theta}$ $= \frac{1}{\tan\theta}$	✓ cos 29° in the numerator ✓reduction of cos θ ✓ reduction of sin θ ✓ $\frac{1}{\tan\theta}$ (4)
7.2.2	sin 15° . cos 15° $\frac{1}{2}(2\sin 15° . \cos 15°) = \frac{1}{2}\sin 2(15°)$ $\therefore \sin 15° . \cos 15° = \frac{1}{2}\sin 30°$ $= \frac{1}{2} . \frac{1}{2}$ $= \frac{1}{4}$	✓writing as double angle ✓ $\sin 30° = \frac{1}{2}$ ✓ answer (3)

7.3	$\frac{\sin A-\cos A}{\sin A+\cos A}+\frac{\sin A+\cos A}{\sin A-\operatorname{Cos} A}=\frac{-2}{\cos 2A}$ LHS $=\frac{\sin A-\cos A}{\sin A+\cos A}+\frac{\sin A+\cos A}{\sin A-\cos A}$ $=\frac{(\sin A-\cos A)(\sin A-\cos A)+(\sin A+\cos A)(\sin A+\cos A)}{(\sin A+\cos A)(\sin A-\cos A)}$ $=\frac{\sin^2 A-2\sin A\cos A+\cos^2 A+\sin^2 A+2\cos\sin A+\cos^2 A}{(\sin A+\cos A)(\sin A-\cos A)}$ $=\frac{2\sin^2 A+2\cos^2 A}{\sin^2 A-\cos^2 A}$ $=\frac{2(\sin^2 A+\cos^2 A)}{-(\cos^2 A-\sin^2 A)}$ $=\frac{-2(1)}{\cos 2A}$ $=\frac{-2}{\cos 2A}$ = RHS	✓ adding using LCD ✓ simplifying ✓ simplifying denominator ✓factorising numerator ✓ $\sin^2 A+\cos^2 A=1$ ✓ writing $1-2\cos^2 A$ as $-\cos 2A$ (6)
		[18]

QUESTION 8

8.1	$2\cos^2 x-1+\cos x-2=0$ $2\cos^2 x+\cos x-3=0$ $(2\cos x+3)(\cos x-1)=0$ $\cos x=\frac{-3}{2}$ or $\cos x=1$ N/A $x=0°+n.360°, n\in \mathbf{Z}$	✓ $2\cos^2 x-1$ ✓ factorising ✓ $\cos x=1$ ✓rejecting the solution ✓ $x=n.360°, n\in\mathbf{Z}$ (5)
8.2.1	Substituting A (45° ; 1) in *f*: $y=\sin px$ $1=\sin(p.45°)$ $p.45°=90°$ $\therefore p=2$ Answer only Full Marks Substituting A (45° ; 1) in *g*: $y=\cos(x+q)$ $1=\cos(45°+q)$ $\therefore 45°+q=0°$ $\therefore q=-45°$	✓ subst ✓ $p=2$ ✓ subst ✓ $q=-45°$ (4)
8.2.2	360°	✓ 360° (1)
8.2.3	$C(-135°;-1)$	✓ both co-ordinates (1)
8.2.4	$D\left(-75°;-\frac{1}{2}\right)$	✓ both co-ordinates (1)
		[12]

TOTAL MARKS: 100

TAXONOMY GRID

COMMON TEST – MARCH 2016

MATHEMATICS GRADE 12

QUESTION NUMBER	LEVEL	LEVEL 2	LEVEL 3	LEVEL 4
1.1		5		
1.2.1		4		
1.2.2		4		
2.1		5		
2.2				5
3.1	2			
3.2	3			
3.3			3	
3.4	1			
3.5			7	
3.6	2			
4.1	2			
4.2			2	
4.3			5	
5.1.1	3			
5.1.2	1			
5.2		4		
5.3	2			
6.1	1			
6.2	1			
6.3		4		
6.4				5
7.1			5	
7.2.1		4		
7.2.2				3
7.3			6	
8.1			5	
8.2.1		4		
8.2.2	1			
8.2.3	1			
8.2.4	1			
TOTAL	20	34	33	13