education

Department:
Education
PROVINCE OF KWAZULU-NATAL

NATIONAL SENIOR CERTIFICATE

GRADE 12

MATHEMATICS

COMMON TEST

MARCH 2019

MARKS: 100

TIME: 2 hours

This question paper consists of 9 pages and 1 information sheet.

INSTRUCTIONS AND INFORMATION

Read the following instructions carefully before answering the questions.

1. This question paper consists of 9 questions.

2. Answers **ALL** questions.

3. Clearly show **ALL** calculations, diagrams, graphs, etc cetera that you have used in determining yours answers.

4. Answers only will not necessarily be awarded full marks.

5. An approved scientific calculator (non-programmable and non-graphical) may be used, unless stated otherwise.

6. If necessary, answers should be rounded off to TWO decimal places, unless stated otherwise.

7. Diagrams are NOT necessarily drawn to scale.

8. Number the answers correctly according to the numbering system used in this question paper. Write neatly and legibly.

QUESTION 1

Given the quadratic sequence: 2 ; 5 ; 10 ; 17 ; . . .

1.1 Write down the next **two** terms of the quadratic sequence. (2)

1.2 Calculate the n^{th} term of the quadratic sequence. (4)

[6]

QUESTION 2

2.1 Given the combined constant and arithmetic sequences:

5 ; 4 ; 5 ; 7 ; 5 ; 10 ; ...

Determine the position of the term 1051 in the combined sequence. (3)

2.2 The series 3 + 8 + 13 + … consists of n terms. The sum of the last three terms is 699.

2.2.1 Determine the sum to n terms in terms of n. (2)

2.2.2 If the last three terms are excluded from the series, then determine in terms of n the sum of the remaining terms. (2)

2.2.3 Hence, or otherwise, determine the value of n. (3)

[10]

QUESTION 3

3.1 The sixth term of a geometric sequence is 80 more than the fifth term.

3.1.1 Show that $a = \dfrac{80}{r^5 - r^4}$. (2)

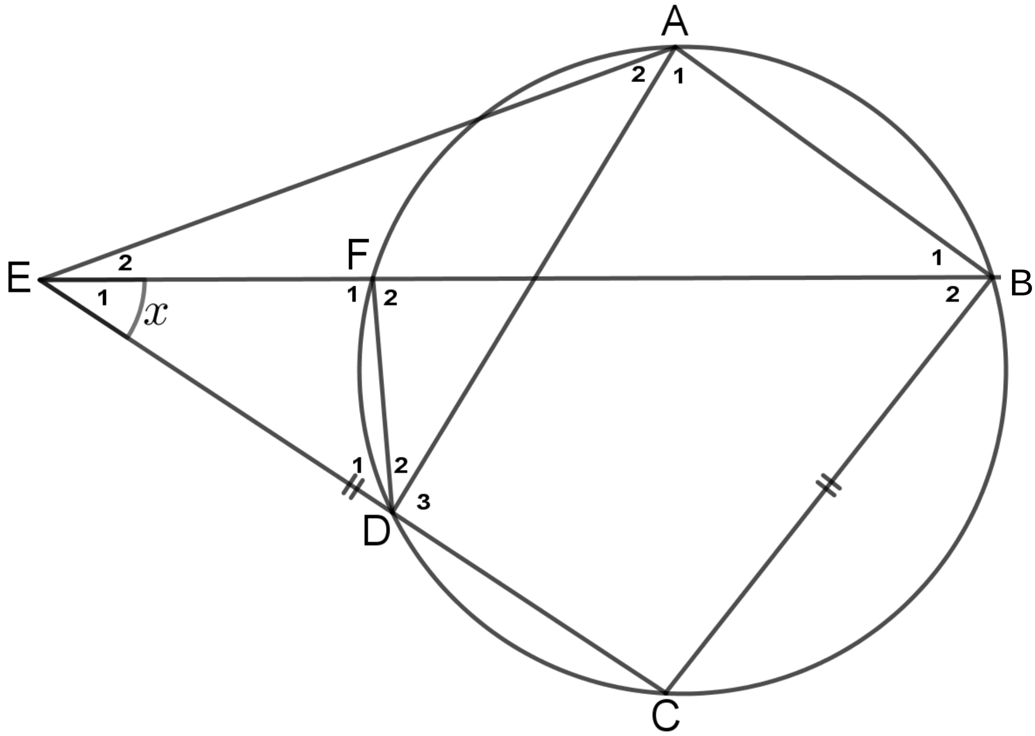
3.1.2 If it is further given that sum of the fifth and sixth terms is 240, determine the value of the common ratio. (5)

3.2 Write the geometric series 9 + 3 + 1 ; … to 130 terms in sigma notation. (2)

[9]

QUESTION 4

In the diagram below, BC = CE; $\hat{E}_1 = x$ and $\hat{D}_1 = \hat{D}_2$.



4.1 Name, with reasons, TWO other angles each equal to x and show that FD = FE. (4)

4.2 Prove that BF bisects $C\hat{B}A$. (4)

4.3 Hence, or otherwise, prove that $\hat{A}_1 = C\hat{B}A$. (4)

[12]

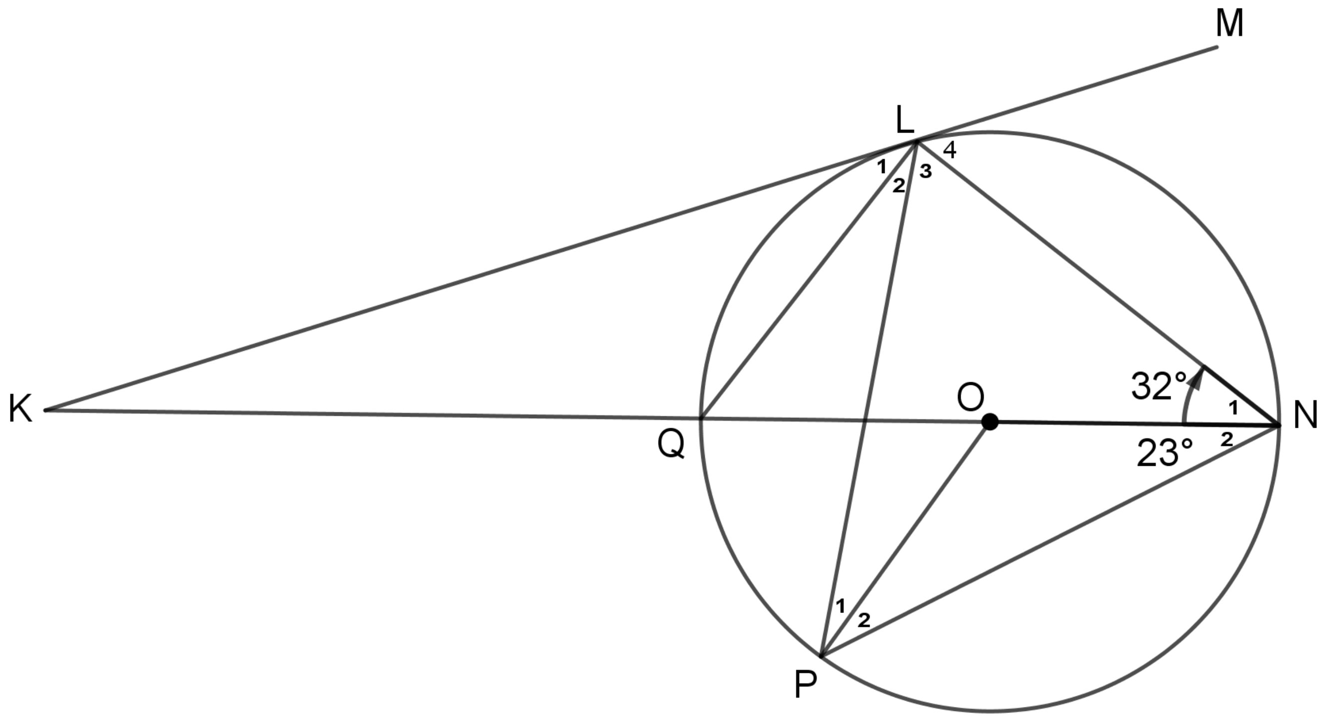
QUESTION 5

5.1 The angle at the point of contact between a tangent to a circle and a chord is --------------- . (1)

5.2 In the sketch below, circle centre O has a tangent KLM.

Diameter NQ produced meet the tangent in K.

$\hat{N}_1 = 32^o$ and $\hat{N}_2 = 23^0$.



Calculate, with reasons, the size of:

5.2.1 $\hat{P}_2$ (1)

5.2.2 $P\hat{O}Q$ (2)

5.2.3 $\hat{L}_2$ (2)

5.2.4 N$\hat{L}$Q (1)

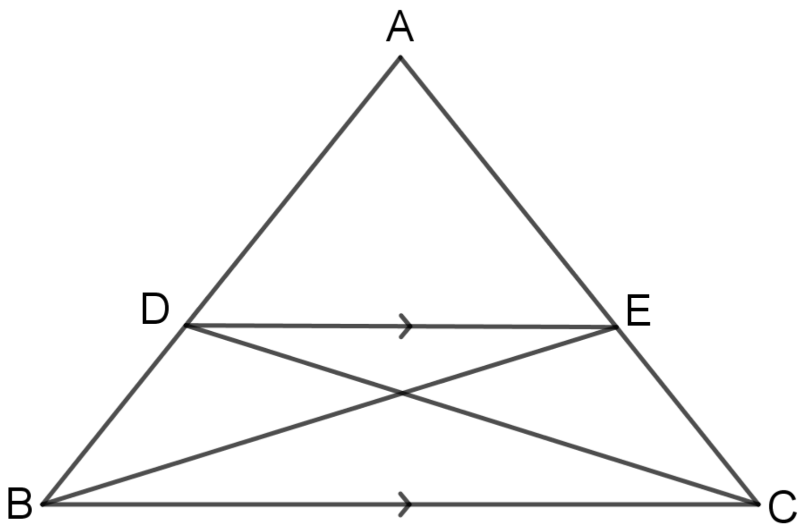
5.2.5 $\hat{L}_3$ (2)

5.2.6 $P\hat{L}K$ (2)

[11]

QUESTION 6

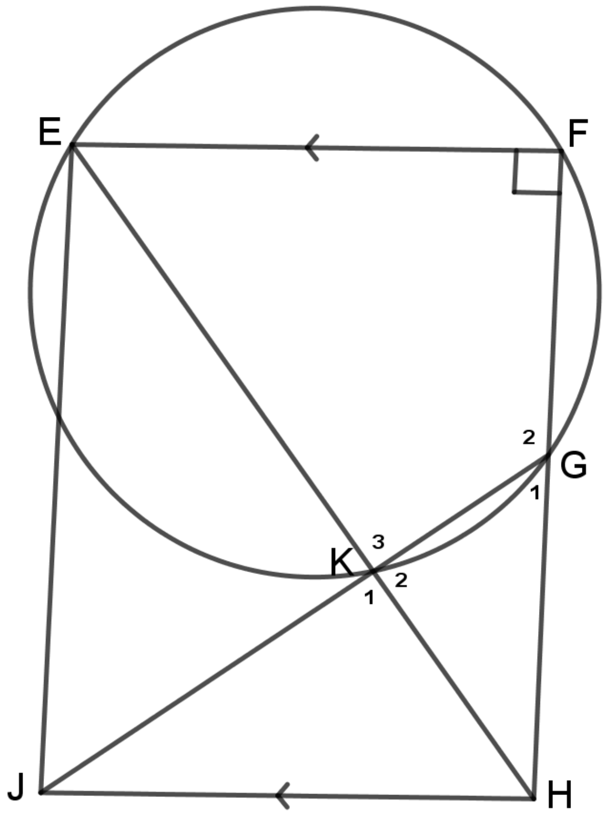
In the diagram below, ΔABC has $DE \parallel BC$. Prove the theorem that states $\frac{AD}{DB} = \frac{AE}{EC}$.



[7]

QUESTION 7

In the diagram below, EFGK is a cyclic quadrilateral with $\hat{F} = 90^0$.
EK and FG are produced to meet at H. HJ is drawn parallel to FE. GK produced meets HJ at J.



7.1 Prove that:

7.1.1 $J\hat{H}F = 90^0$ (2)

7.1.2 $\hat{K}_2 = 90^0$ (2)

7.1.3 Δ HKG ||| Δ JHG (3)

7.2 Calculate JG and KG if HG = 5cm and JH = 10cm. (4)

[11]

QUESTION 8 (ANSWER THIS QUESTION WITHOUT THE USE OF CALCULATOR)

8.1 Show

$$\frac{\sin(90° + x)\cos x \tan(-x)}{\cos(180° + x)} = \sin x$$

(4)

8.2 If $\sin 36^0 = m$, and $\cos 24^0 = n$, determine the following in terms of m and /or n:

8.2.1 $\cos 36^0$ (3)

8.2.2 $\sin 12^0$ (4)

8.3 Simplify:

$$\frac{2\cos 285° \cos 15°}{\cos(45° - x)\cos x - \sin(45° - x)\sin x}$$

(5)

8.4 Calculate the value of

$$(\sin 3x - \cos 3x)^2 \text{ if } \sin 6x = -\tfrac{2}{5}$$

(4)

[20]

QUESTION 9

Given $f(x) = \sin x + 1$ and $g(x) = \cos 2x$

9.1 Show that $f(x) = g(x)$ can be written as $(2 \sin x + 1) \sin x = 0$. (2)

9.2 Hence or otherwise determine the general solution of $\sin x + 1 = \cos 2x$. (6)

9.3 Write down the range of $g(x) - 1$. (1)

9.4 Consider the following geometric series

$$1 + 2\cos 2x + 4cos^2 2x + \cdots$$

Determine the values of x for the interval $0° \leq x \leq 90°$ for which the series will converge. (5)

[14]

TOTAL MARKS: [100]

INFORMATION SHEET: MATHEMATICS

INLIGTINGSBLAD: WISKUNDE

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$A = P(1+ni)$ $\quad A = P(1-ni)$ $\quad A = P(1-i)^n$ $\quad A = P(1+i)^n$

$T_n = a + (n-1)d$ $\quad S_n = \frac{n}{2}\left(2a + (n-1)d\right)$

$T_n = ar^{n-1}$ $\quad S_n = \frac{a\left(r^n - 1\right)}{r-1}$; $\quad r \neq 1$ $\quad S_\infty = \frac{a}{1-r}$; $-1 < r < 1$

$F = \frac{x\left[(1+i)^n - 1\right]}{i}$ $\quad P = \frac{x[1-(1+i)^{-n}]}{i}$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $\quad \mathrm{M}\left(\frac{x_1 + x_2}{2} ; \frac{y_1 + y_2}{2}\right)$

$y = mx + c$ $\quad y - y_1 = m(x - x_1)$ $\quad m = \frac{y_2 - y_1}{x_2 - x_1}$ $\quad m = \tan\theta$

$$(x-a)^2 + (y-b)^2 = r^2$$

In ΔABC: $\quad \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$ $\quad a^2 = b^2 + c^2 - 2bc.\cos A$ $\quad area\,\Delta ABC = \frac{1}{2}ab.\sin C$

$\sin(\alpha + \beta) = \sin\alpha.\cos\beta + \cos\alpha.\sin\beta$ $\quad \sin(\alpha - \beta) = \sin\alpha.\cos\beta - \cos\alpha.\sin\beta$

$\cos(\alpha + \beta) = \cos\alpha.\cos\beta - \sin\alpha.\sin\beta$ $\quad \cos(\alpha - \beta) = \cos\alpha.\cos\beta + \sin\alpha.\sin\beta$

$\cos 2\alpha = \begin{cases} \cos^2\alpha - \sin^2\alpha \\ 1 - 2\sin^2\alpha \\ 2\cos^2\alpha - 1 \end{cases}$ $\quad \sin 2\alpha = 2\sin\alpha.\cos\alpha$

$\bar{x} = \frac{\sum fx}{n}$ $\quad \sigma^2 = \frac{\sum_{i=1}^{n}\left(x_i - \bar{x}\right)^2}{n}$

$\mathrm{P}(A) = \frac{n(\mathrm{A})}{n(\mathrm{S})}$ $\quad \mathrm{P(A\ or\ B) = P(A) + P(B) - P(A\ and\ B)}$

$\hat{y} = a + bx$ $\quad b = \frac{\sum(x - \bar{x})(y - \bar{y})}{\sum(x - \bar{x})^2}$

education

Department:
Education
PROVINCE OF KWAZULU-NATAL

MATHEMATICS

MARKING GUIDELINE

COMMON TEST

MARCH 2019

NATIONAL
SENIOR CERTIFICATE

GRADE 12

MARKS: 100

This marking guideline consists of 10 pages.

QUESTION 1

1.1	26 ; 37	AA✓✓ correct values	2
1.2	2 → 5 → 10 → 17 3 5 7 2 2 $\boldsymbol{2a = 2}$ $\boldsymbol{a = 1}$ $\boldsymbol{3a + b = 3}$ $\boldsymbol{b = 0}$ $\boldsymbol{a + b + c = 2}$ $\boldsymbol{c = 1}$ $\boldsymbol{T_n = n^2 + 1}$ **OR** $2a = 2$ $a = 1$ $3a + b = 3$ $b = 0$ $T_0 = c = 1$ $T_n = n^2 + 1$ **OR** $T_n = T_1 + (n-1)d_1 + \frac{(n-1)(n-2)}{2}d_2$ $= 2 + (n-1)(3) + \frac{(n-1)(n-2)}{2}(2)$ $= 2 + 3n - 3 + n^2 - 3n + 2$ $= n^2 + 1$ **OR** $T_n = \frac{n-1}{2}[2a + (n-2)d] + T_1$ $= \frac{n-1}{2}[2(3) + (n-2)(2)] + 2$ $= (n-1)[3 + n - 2] + 2$ $= (n-1)(n+1) + 2$ $= n^2 - 1 + 2 = n^2 + 1$	A✓ a value CA✓b value CA✓c value CA✓general term **OR** A✓ a value CA✓b value A✓c value CA✓general term **OR** A✓formula A✓substituting first and second difference values CA✓simplifying CA✓general term **OR** A✓formula A✓correct substitution into formula CA✓simplifying CA✓general term	4 4 4 4
			[6]

QUESTION 2

2.1	$3n+1=1051$ $3n=1050$ $n=350$ 1051 is in the 700th position	A✓equating *n*th term to 1051 CA✓value of *n* CA✓conclusion	3
2.2.1	$a=3,\ d=5$ $S_n=\frac{n}{2}[2a+(n-1)d]$ $=\frac{n}{2}[2(3)+(n-1)(5)]$ $=\frac{n}{2}[5n+1]$	A✓ $d=5$ CA✓substitution of *a* & *d* into sum formula	2
2.2.2	$S_{n-3}=\frac{n-3}{2}[5(n-3)+1]$ $=\frac{n-3}{2}[5n-14]$	A✓ $n-3$ CA✓substituting into sum formula	2
2.2.3	$S_n-S_{n-3}=699$ $\frac{n}{2}[5n+1]-\frac{n-3}{2}[5n-14]=699$ $n(5n+1)-(n-3)(5n-14)=1398$ $5n^2+n-5n^2+29n-42=1398$ $30n=1440$ $n=48$	CA✓forming equation CA✓simplification CA✓answer (*n* must be a natural number)	3
	OR	OR	
	$T_n = 5n - 2$ $T_{n-1} = 5(n-1) - 2$ $T_{n-2} = 5(n-2) - 2$ $15n - 21 = 699$ $n = 45$	CA✓forming equations CA✓simplification CA✓answer (*n* must be a natural number)	3
			[10]

QUESTION 3

3.1.1	$T_6 = 80 + T_5$ $ar^5 = 80 + ar^4$ $ar^5 - ar^4 = 80$ $a(r^5 - r^4) = 80$ $a = \frac{80}{r^5 - r^4}$	A✓forming equation A✓factorizing	2
3.1.2	$T_5 + T_6 = 240$ $ar^4 + ar^5 = 240$ $a(r^4 + r^5) = 240$ $a = \frac{240}{r^4 + r^5}$ $\frac{80}{r^5 - r^4} = \frac{240}{r^5 + r^4}$ $80r^5 + 80r^4 = 240r^5 - 240r^4$ $320r^4 = 160r^5$ $r = 2$ **OR** Let the fifth term be = *x* and sixth term = *y*, then $x + y = 240 \rightarrow (1)$ $-x + y = 80 \rightarrow (2)$ $(1) + (2)$ $2y = 320$ $y = 160;\ x = 80$ $r = \frac{160}{80} = 2$	A✓forming equation CA✓factorizing CA✓equating CA✓simplifying CA✓r – value **OR** A✓forming equation (1) A✓forming equation (2) CA✓ y – value CA✓x – value CA✓ r – value	5 5
3.2	$\sum_{k=1}^{130} 9\left(\frac{1}{3}\right)^{k-1}$	A✓upper and lower limit values A✓ k^{th} term	2
			[9]

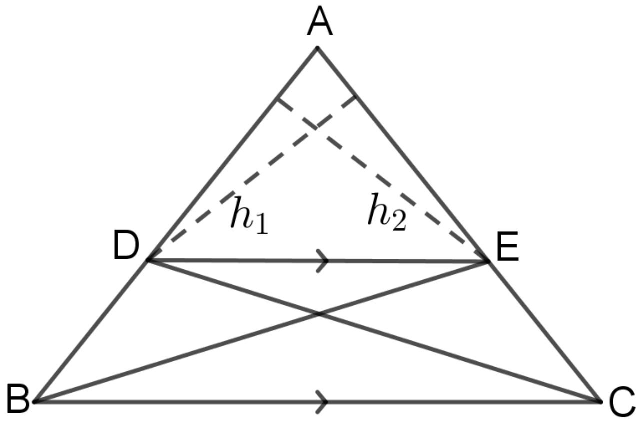
QUESTION 4

4.1	$\hat{B}_2 = \hat{E}_1 = x$ …….. (BC = CE) $\hat{D}_1 = \hat{B}_2 = x$ … (Ext. ∠ of cyclic quad = interior opposite angles) $\therefore \hat{D}_1 = \hat{E}_1$ ∴FD = FE	A✓ S/R A✓ S A✓R A✓ S	(4)
4.2	$\hat{D}_2 = \hat{D}_1 = x$ …… given $\hat{D}_2 = \hat{B}_1 = x$ … subtended by arc AF $\therefore \hat{B}_1 = \hat{B}_2$ ∴EB bisects $C\hat{B}A$	A✓ S A✓ S A ✓R A✓ S	(4)
4.3	$\hat{A}_1 = \hat{F}_2$ …… subtended by arc $\hat{F}_2 = 2x$ ….. ext. ∠ of ΔDFE $\therefore \hat{A}_1 = 2x = C\hat{B}A$	A✓ S A✓R A✓ S A✓R	(4)
			[12]

QUESTION 5

5.1	…….. equal to the angle subtended by the chord in the opposite circle segment.	A✓S	(1)
5.2.1	$\hat{P}_2 = 23°$ …… (ON = OP; radii)	A✓ S/R	(1)
5.2.2	$P\hat{O}Q = 2\hat{N}_2 = 46°$ …… (∠at centre) / (Ext ∠ofΔ)	A✓ S A✓R	(2)
5.2.3	$\hat{L}_2 = \hat{N}_2 = 23°$…… (subt. by arc PQ)	A✓ S A✓R	(2)
5.2.4	$N\hat{L}Q = 90°$ …… (subt. by diameter NQ)	A✓S/R	(1)
5.2.5	$\hat{L}_3 = 90° - 23°$ $= 67°$ **OR** $P\hat{O}N = 134°$ …..(angles of triangle) $P\hat{O}N = 2\hat{L}$ …..(angle at centre theorem) $= 67°$	CA✓ S CA✓ answer **OR** CA✓ S CA✓ answer	(2) (2)
5.2.6	$P\hat{L}K = L\hat{N}P$ …… (tan-chord theorem) $= 32° + 23°$ $= 55°$	A✓ S/R A✓ answer	(2)
			[11]

QUESTION 6

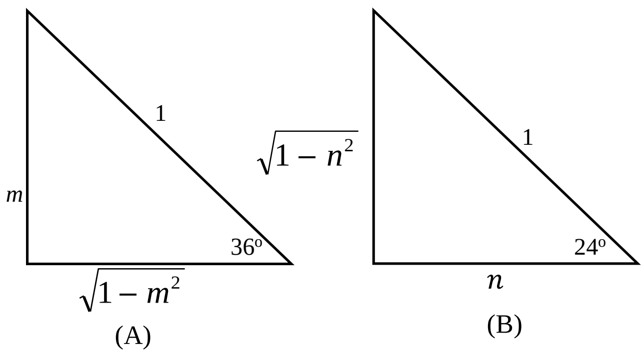


Construction: Draw line $h_1 \perp AC$ and $h_2 \perp AB$	A✓ construction	
Proof: $\frac{Area\ of\ \Delta ADE}{Area\ of\ \Delta BDE} = \frac{\frac{1}{2}.\ AD.h_2}{\frac{1}{2}.\ DB.h_2}$ *same height* h_2	A✓ S A✓ R	
and $\frac{Area\ of\ \Delta ADE}{Area\ of\ \Delta CED} = \frac{\frac{1}{2}.\ AE.h_1}{\frac{1}{2}.\ EC.h_1}$ *same height* h_1	A✓ S/R	
but $area\ of\ \Delta BDE = area\ of\ \Delta CED$ $\because$ $(same\ base\ DE; the\ same\ height;\ DE \parallel BC)$	AA✓ S/✓ R	
$\therefore \frac{Area\ of\ \Delta ADE}{Area\ of\ \Delta BDE} = \frac{Area\ of\ \Delta ADE}{Area\ of\ \Delta CED}$	A✓ S	
$\therefore \frac{AD}{DB} = \frac{AE}{EC}$		**[7]**

QUESTION 7

7.1 7.1.1	$J\hat{H}F + \hat{F} = 180°$ …. Co-interior angles; JH // EF ∴ $J\hat{H}F = 90°$ …… $\hat{F} = 90°$ (given)	A✓ S/R A✓ S	 (2)
7.1.2	$\hat{K}_2 = \hat{F} = 90°$ …… ext. ∠ of cyclic quad	AA✓✓ S/R	(2)
7.1.3	In ΔHKG and ΔJHG $\widehat{G_1} = \widehat{G_1}$…… common $\hat{K}_2 = J\hat{H}G = 90°$ …… proved ∴ $K\hat{H}G = H\hat{J}G$ …… remaining ∠ ∴ $\Delta HKG /// \Delta JHG$ …… (∠∠∠)	 A✓ S/R A✓ S/R A✓ R	 (3)
7.2	$JG^2 = HJ^2 + HG^2$ …… Pythagoras $= 10^2 + 5^2$ $= 125cm^2$ $JG = \sqrt{125cm^2}$ $= 5\sqrt{5}$cm $\frac{KG}{HG} = \frac{HG}{JG}$ $KG = \frac{HG^2}{JG}$ $= \frac{5^2}{5\sqrt{5}}$ $= \frac{5}{\sqrt{5}}$ $= \frac{5\sqrt{5}}{5}$ $= \sqrt{5}$ cm $= 2{,}24$ cm	 A✓ $5\sqrt{5}$ A✓ $\Delta HKG /// \Delta JHG$ CA✓ substitution CA✓ answer in any form	 (4) **[11]**

QUESTION 8

8.1	$\dfrac{(\cos x)(\cos x)(-\tan x)}{-\cos x}$ $= \dfrac{(\cos x)(\cos x)\left(\frac{\sin x}{\cos x}\right)}{\cos x}$ $= \sin x$	A✓$-\tan x$ A✓$\cos x$ A✓$\frac{\sin x}{\cos x}$ A✓$\cos x$	(4)
8.2 8.2.1	 $\cos 36° = \sqrt{1-m^2}$	A✓ A ✓diagrams CA✓ $\sqrt{1-m^2}$ answer	(3)
8.2.2	$\sin 12° = \sin(36° - 24°)$ $= \sin 36° \cos 24° - \cos 36° \sin 24°$ $= m\,n - \left(\sqrt{1-m^2}\right)\left(\sqrt{1-n^2}\right)$ **OR** $\sin 12° = \sqrt{\frac{1-cos24°}{2}}$ $= \sqrt{\frac{1-n}{2}}$	A✓ compound angle A✓ expansion CACA✓✓ answer **OR** A✓ A✓ CACA✓✓ answer	(4) (4)
8.3	$\dfrac{2\cos 75° \cos 15°}{\cos[45° - x + x]}$ $= \dfrac{2\sin 15° \cos 15°}{\cos 45°}$ $= \dfrac{\sin 30°}{\cos 45°}$ $= \dfrac{1}{2} \div \dfrac{1}{\sqrt{2}}$ $= \dfrac{\sqrt{2}}{2}$	A✓$\cos 75°$ A✓$\cos[45°]$ A✓$\sin 15°$ A✓$\frac{\sin 30°}{\cos 45°}$ CA✓answer in any form	(5)
8.4	$sin^2 3x - 2\sin 3x \cos 3x + cos^2 3x$ $= 1 - \sin 2(3x)$ $= 1 - \sin 6x$ $= 1 - \left(\frac{-2}{5}\right)$ $= 1\frac{2}{5}$	A✓expansion A✓$1 - \sin 2(3x)$ A✓$1 - \left(\frac{-2}{5}\right)$ A✓answer in any form	(4)
			[20]

QUESTION 9

9.1	$\sin x + 1 = 1 - 2sin^2x$ $2sin^2x\ +\ \sin x = 0$ $\sin x\,(2\sin x + 1) = 0$	A✓equating A✓standard form	(2)
9.2	$\sin x\,(2\sin x + 1) = 0$ $\sin x = 0$ or $2\sin x = -1$ $x = 0° + k.360°;\ \ k \in \mathbb{Z}$ $\quad \sin x = -\frac{1}{2}$ $x = 180° + k.360°;\ \ k \in \mathbb{Z}$ $\quad x = 210° + k.360°;\ \ k \in \mathbb{Z}$ $x = 330° + k.360°;\ \ k \in \mathbb{Z}$	A✓$\sin x = 0$ A✓$\sin x = -\frac{1}{2}$ A✓$0° + k.360°$ A✓ $k \in \mathbb{Z}$ CA✓$210° + k.360°$ CA✓$330° + k.360°$	(6)
9.3	$-2 \leq y \leq 0$	A✓answer	(1)
9.4	$r = 2\cos 2x$ $-1 < 2\cos 2x < 1$ $\frac{-1}{2} < \cos 2x < \frac{1}{2}$; If $\cos 2x = \frac{1}{2}$ $\therefore 2x = 60°$ then $x = 30°$ $\therefore 30° < x < 90°$	A✓$r = 2\cos 2x$ A✓$-1 < r < 1$ CA✓ $\frac{-1}{2} < \cos 2x < \frac{1}{2}$ CA✓ $x = 30°$ CA✓answer $30° < x < 90°$	(5)
			[14]

TOTAL MARKS: [100]