education

Department:
Education
PROVINCE OF KWAZULU-NATAL

GRADE 12

NATIONAL SENIOR CERTIFICATE

MATHEMATICS P2

COMMON TEST

JUNE 2019

MARKS: 150

TIME: 3 hours

N.B. This question paper consists of 10 pages, 1 information sheet and an answer book with 18 pages.

INSTRUCTIONS AND INFORMATION

Read the following instructions carefully before answering the questions.

1. This question paper consists of 10 questions.
2. Answer ALL the questions.
3. Number the answers correctly according to the numbering system used in this question paper.
4. Clearly show ALL calculations, diagrams, graphs, et cetera that you have used in determining your answers.
5. Answers only will not necessarily be awarded full marks.
6. You may use an approved scientific calculator (non-programmable and non-graphical), unless stated otherwise.
7. If necessary, round off answers to TWO decimal places, unless stated otherwise.
8. Diagrams are NOT necessarily drawn to scale.
9. Write neatly and legibly.

QUESTION 1

The time taken, in minutes, to complete a 5 km race by 10 athletes of an Athletic Club is given below.

16	18	19	20	21	22	23	24	28	29

1.1 Calculate the mean time taken to complete the race. (2)

1.2 Calculate the standard deviation. (2)

1.3 Draw a box and whisker diagram to represent the five number summary of the above information. (4)

1.4 How many runners completed the race within one standard deviation of the mean time? (3)

[11]

QUESTION 2

A certain company develops a new product and does some market research. The table below is a summary of the ages of people who say they will buy the product.

Age	Frequency	Cumulative Frequency
$5 < x \leq 15$	200	200
$15 < x \leq 25$	**A**	450
$25 < x \leq 35$	20	470
$35 < x \leq 45$	32	**B**
$45 < x \leq 55$	23	525
$55 < x \leq 65$	300	825
$65 < x \leq 75$	475	1300

2.1 Calculate the values of A and B in the table above. (2)

2.2 Calculate the estimated mean age of the people who say they will buy the new product. (3)

2.3 Find the modal class interval. (2)

2.4 Sketch the ogive on the grid provided on the diagram sheet. (3)

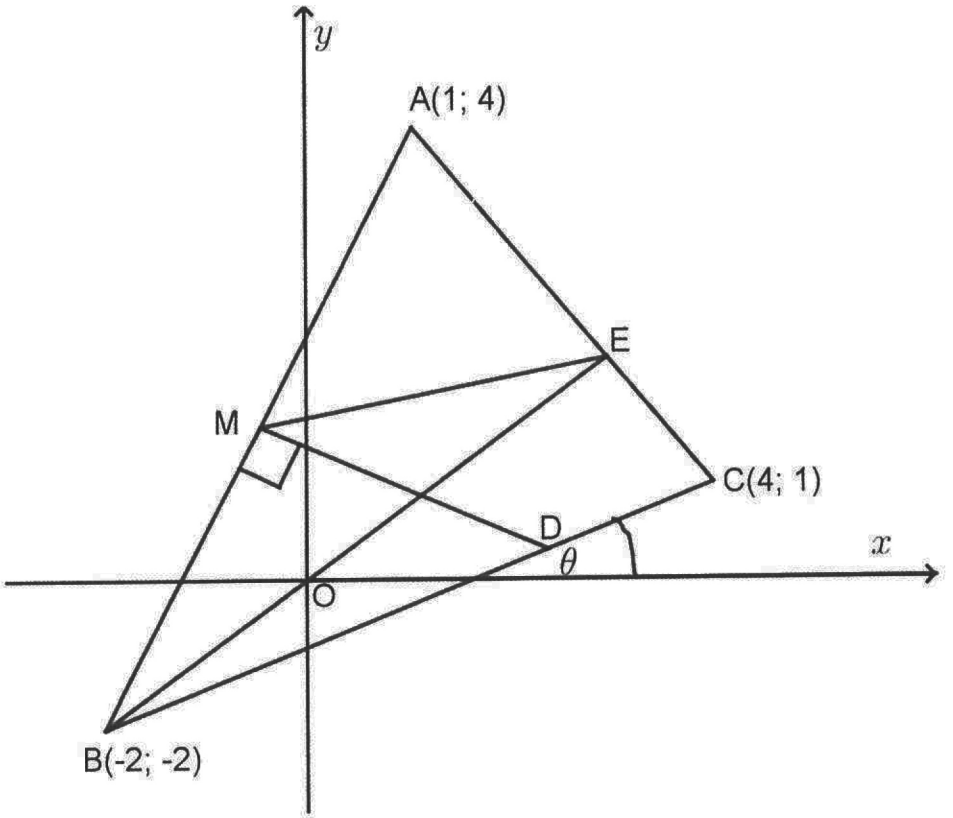
2.5 Use your sketch to answer the following question:

Is the data normally distributed? Give a reason for your answer. (2)

[12]

QUESTION 3

In the diagram, A(1; 4); B(−2; −2) and C(4; 1) are vertices of Δ ABC in a Cartesian plane. M is the midpoint of AB. E is on AC, D is on BC such that DM $\perp$ AB. ME and BE are joined. The inclination of line CB is θ.



3.1 Calculate the length of BC, in simplified surd form. (2)

3.2 Determine the co-ordinates of M, the midpoint of AB. (2)

3.3 Determine the equation of MD. (4)

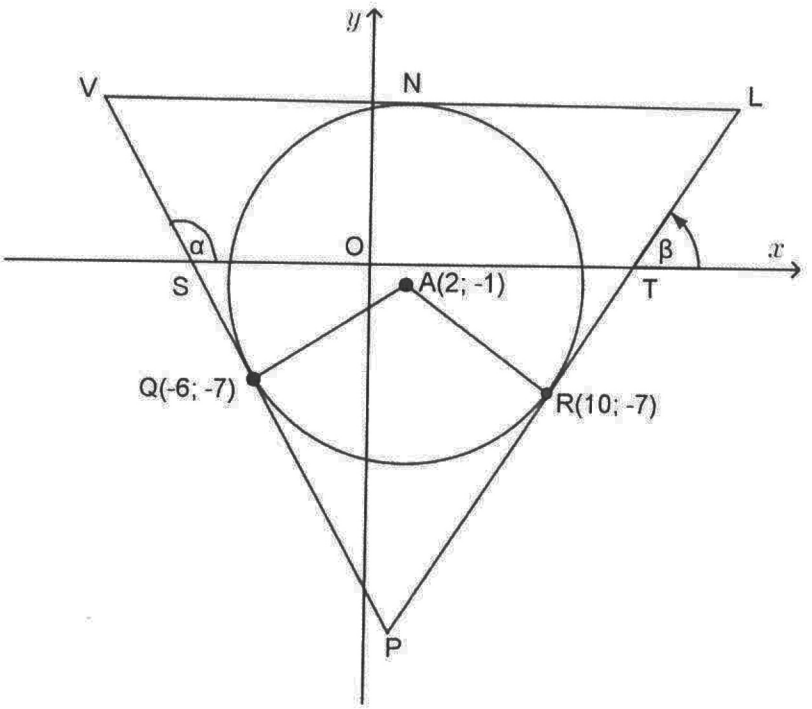
3.4 If ME $\parallel$ BC, calculate the co-ordinates of E, with reasons. (3)

3.5 Determine the equation of BE. (3)

3.6 Calculate the size of θ. (4)
[18]

QUESTION 4

PV, PL and VL are tangents to the circle, centre A(2; −1) at Q, R and N respectively. The co-ordiantes of Q and R are (−6; −7) and (10; −7) respectively. The inclination of PL is β and PV is α.



4.1 Calculate the length of AQ. (2)

4.2 Write down the equation of the circle whose centre is A in the form $(x-a)^2+(y-b)^2=r^2$ (2)

4.3 Calculate the gradients of:

4.3.1 QP. (2)

4.3.2 PR. (2)

4.4 Determine the equations of:

4.4.1 QP. (2)

4.4.2 PR. (2)

4.5.1 Calculate the co-ordinates of P. (4)

4.5.2 Express $\hat{P}$ in terms of α and β. (2)

4.5.3 If $\tan(\alpha-\beta)=\frac{\sin(\alpha-\beta)}{\cos(\alpha-\beta)}$, show that $\tan P=\frac{24}{7}$. (4)

[22]

QUESTION 5

5.1 If $4\tan A = 3$ and $3\sin B - 1 = 0$, where $180° \leq A \leq 360°$, $0° \leq B \leq 90°$. Use a sketch to determine the value of the following without using a calculator.

5.1.1 $\cos 2A$ (3)

5.1.2 $\sin(A + B)$ (3)

5.2 Simplify, without the use of a calculator:

$$\sin 20° \cos 320° + \cos(-20°) \sin(400°)$$

(3)

5.3 Prove the identity:

$$\frac{\cos^2(90° + \theta)}{\cos(-\theta) + \sin(90° - \theta)\cos\theta} = \frac{1}{\cos\theta} - 1$$ (5)

5.4 It is given that

$$p = \cos\alpha + \sin\alpha$$
$$q = \cos\alpha - \sin\alpha$$

deduce the following trigonometric ratios in terms of p and q.

5.4.1 $\cos 2\alpha$ (2)

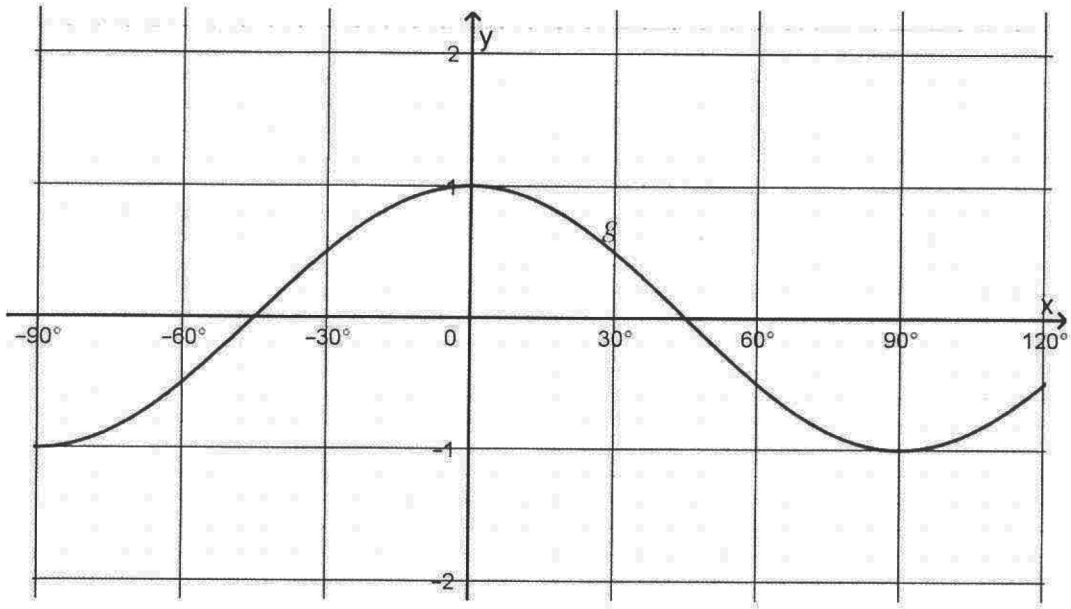
5.4.2 $\dfrac{1 + \sin 2\alpha}{\cos 2\alpha}$ (5)

5.5 Determine the general solution of $6\cos^2 x + \sin x - 5 = 0$. (6)

[27]

QUESTION 6

In the diagram below, the graph of $g(x) = \cos 2x$, for $x \in [-90°; 120°]$ is drawn.



6.1 Draw the graph of $f(x) = \sin(x + 30°)$ for $x \in [-90°; 120°]$ on the set of axes provided in the ANSWER BOOK. (3)

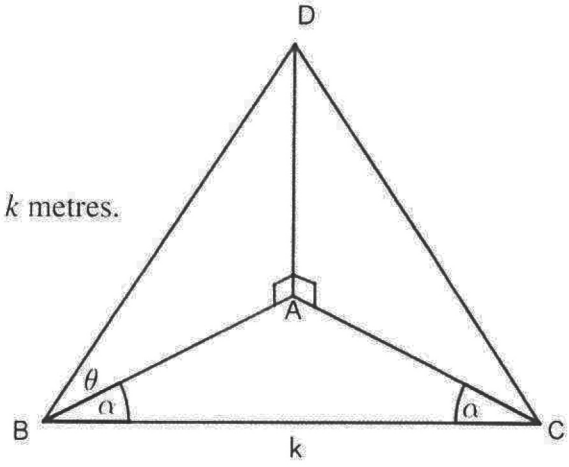
6.2 Determine the value(s) of x, $x \in [-90°; 120°]$ for which both graphs are decreasing (2)

6.3 Consider $h(x) = f(x + 60°)$.
Describe the transformation the graph of f to obtain the graph of h. (2)

[7]

QUESTION 7

A, B and C are three points in the same horizontal plane.
DA is a vertical cliff.
The angle of elevation of the top of the cliff from D is θ.
If $A\hat{B}C = A\hat{C}B = \alpha$ and the distance between B and C is k metres.



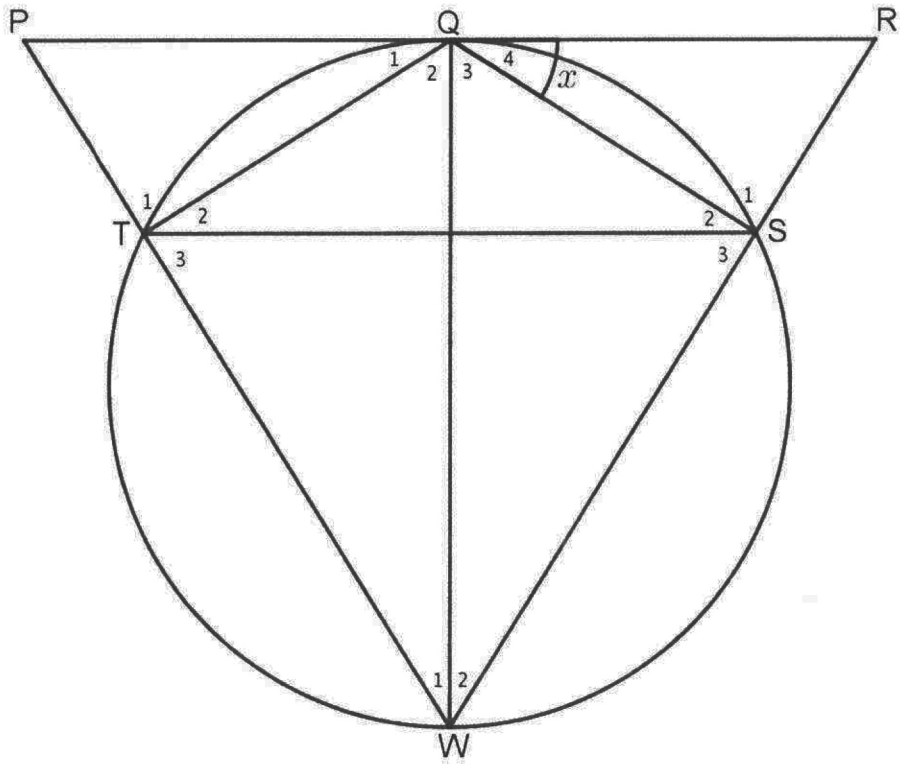
Prove that

$$AD = \frac{k \tan\theta}{2\cos\alpha}$$

[6]

QUESTION 8

In the figure, TQSW is a cyclic quadrilateral with tangent PR touching the circle at Q.
WQ bisects $P\hat{W}R$. $\hat{Q}_4 = x$



8.1 Name with reasons 5 other angles each equal to *x*. (5)

8.2 Prove that:

8.2.1 TS // PR (2)

8.2.2 $\hat{Q}_3 = \hat{P}$ (3)

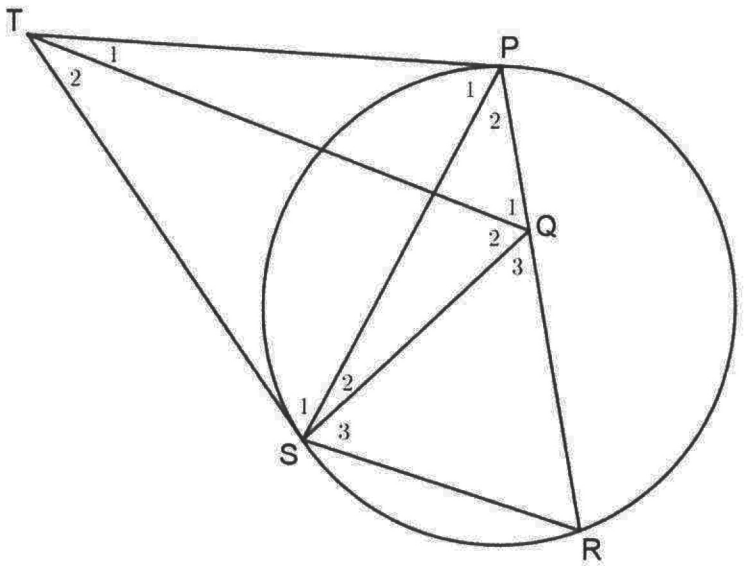
8.2.3 Δ TQS is an isosceles triangle (4)

8.2.4 $W\hat{Q}P = \hat{T}_1$ (3)

[17]

QUESTION 9

9.1 In the figure, TP and TS are tangents to the circle. R is a point on the circle and RS are drawn. Q is a point on PR such that $T\widehat{Q}P = T\widehat{P}S$. SQ is drawn.



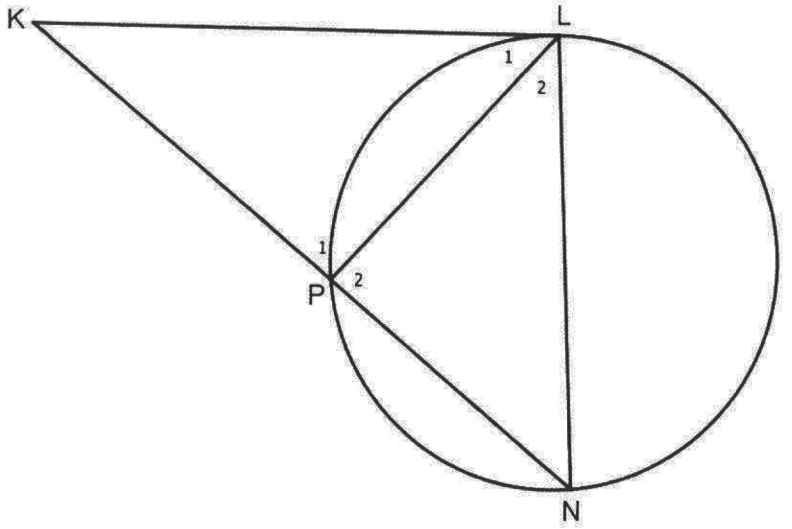
Prove that:

9.1.1 TQ ∥ SR (3)

9.1.2 QPTS is a cyclic quadrilateral (3)

9.1.3 TQ bisects $S\widehat{Q}P$. (3)

9.2 In the figure, LN is the diameter of the circle. KL is the tangent to the circle at L.



9.2.1 Prove that Δ LPK /// Δ NPL (4)

9.2.2 Show that $PL^2 = NP.\ PK$ (3)

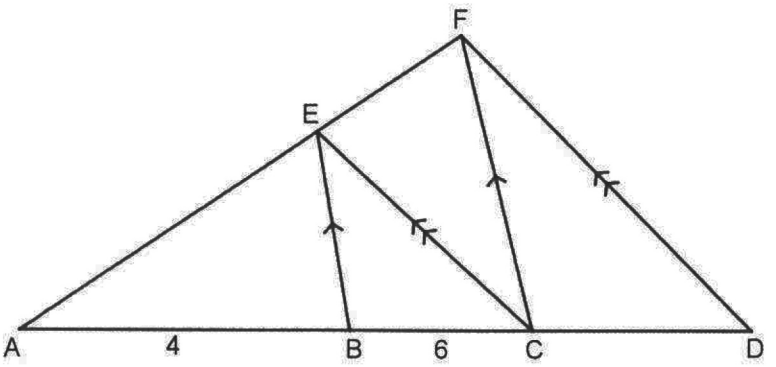
9.2.3 Name another triangle which is similar to Δ NPL (1)

9.2.4 Calculate the area of the circle if NP = 10 cm and PK = 6 cm (6)

[23]

QUESTION 10

In ΔADF, DF ∥ CE and CF ∥ BE. If AB = 4 units and BC = 6 units, then calculate



10.1 the length of CD (3)

10.2 the numerical value of

$$\frac{\text{area of } \Delta FEC}{\text{area of } \Delta FAD}$$ (4)

[7]

TOTAL: 150

INFORMATION SHEET

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$A = P(1+ni)$ $A = P(1-ni)$ $A = P(1-i)^n$ $A = P(1+i)^n$

$T_n = a + (n-1)d$ $S_n = \frac{n}{2}[2a + (n-1)d]$

$T_n = ar^{n-1}$ $S_n = \frac{a(r^n - 1)}{r-1}$; $r \neq 1$ $S_\infty = \frac{a}{1-r}$; $-1 < r < 1$

$F = \frac{x[(1+i)^n - 1]}{i}$ $P = \frac{x[1-(1+i)^{-n}]}{i}$

$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $M\left(\frac{x_1 + x_2}{2}; \frac{y_1 + y_2}{2}\right)$

$y = mx + c$ $y - y_1 = m(x - x_1)$ $m = \frac{y_2 - y_1}{x_2 - x_1}$ $m = \tan\theta$

$(x-a)^2 + (y-b)^2 = r^2$

In ΔABC: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

$a^2 = b^2 + c^2 - 2bc.\cos A$

$area\,\Delta ABC = \frac{1}{2}ab.\sin C$

$\sin(\alpha + \beta) = \sin\alpha.\cos\beta + \cos\alpha.\sin\beta$ $\sin(\alpha - \beta) = \sin\alpha.\cos\beta - \cos\alpha.\sin\beta$

$\cos(\alpha + \beta) = \cos\alpha.\cos\beta - \sin\alpha.\sin\beta$ $\cos(\alpha - \beta) = \cos\alpha.\cos\beta + \sin\alpha.\sin\beta$

$$\cos 2\alpha = \begin{cases} \cos^2\alpha - \sin^2\alpha \\ 1 - 2\sin^2\alpha \\ 2\cos^2\alpha - 1 \end{cases}$$

$\sin 2\alpha = 2\sin\alpha.\cos\alpha$

$\bar{x} = \frac{\Sigma f.x}{n}$ $\sigma^2 = \frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n}$

$P(A) = \frac{n(A)}{n(S)}$ $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$ $\hat{y} = a + bx$

$b = \frac{\sum(x - \bar{x})(y - \bar{y})}{\sum(x - \bar{x})^2}$

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MATHEMATICS P2

ANSWER BOOK

COMMON TEST

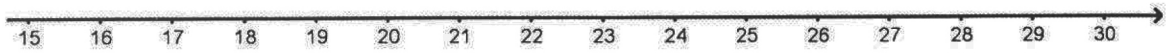
JUNE 2019

MARKS: 300

These answer book consist of 18 pages.

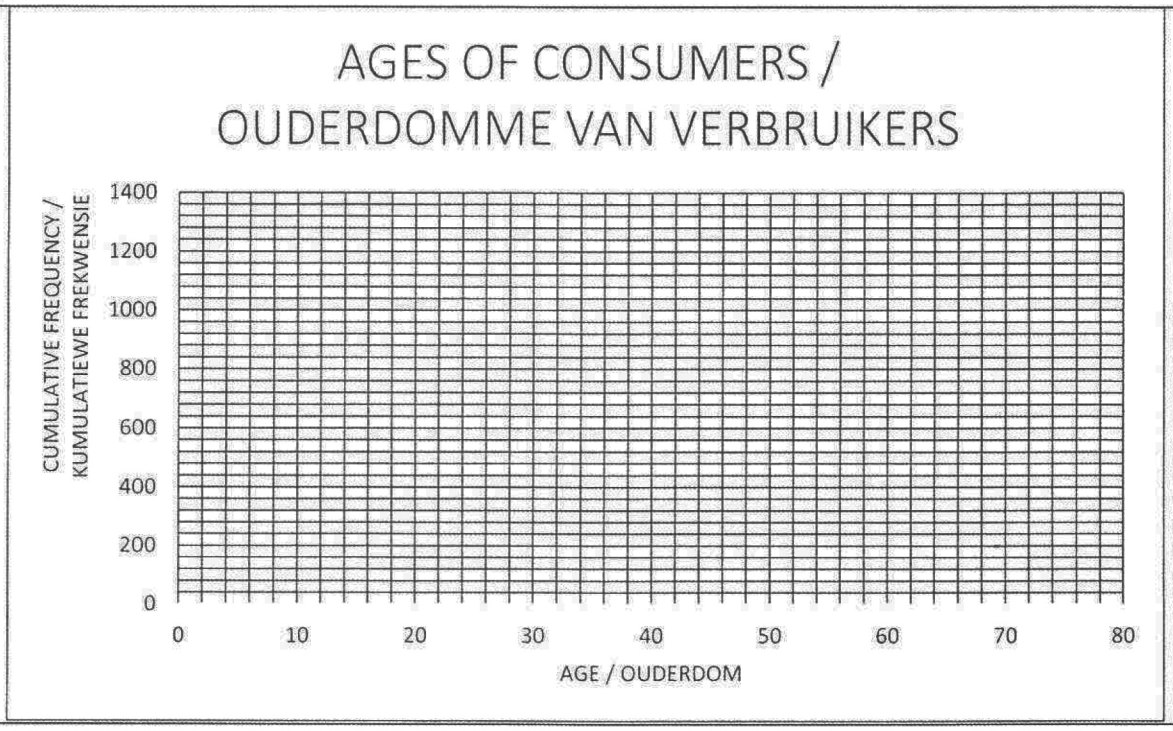
QUESTION / VRAAG 1

16	18	19	20	21		22	23	24	28	29

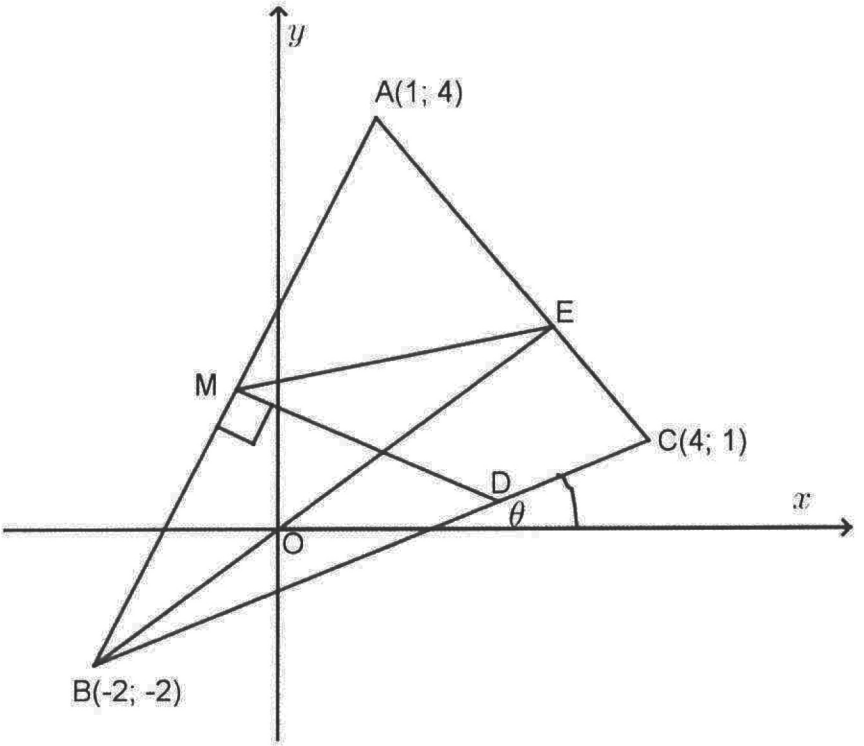
	Solution/*Oplossing*	**Marks/ *Punte***
1.1		(2)
1.2		(2)
1.3		(4)
1.4		(3)
		[11]

QUESTION / VRAAG 2

Age Ouerdom	Frequency Frekwensie	Cumulative Frequency Kumulatiewe frekwensie
$5 < x \leq 15$	200	200
$15 < x \leq 25$	**A**	450
$25 < x \leq 35$	20	470
$35 < x \leq 45$	32	**B**
$45 < x \leq 55$	23	525
$55 < x \leq 65$	300	825
$65 < x \leq 75$	475	1300

	Solution/*Oplossing*	**Marks/*Punte***
2.1		(2)
2.2		(3)
2.3		(2)
2.4		(3)
2.5		(2)
		[12]

QUESTION / VRAAG 3

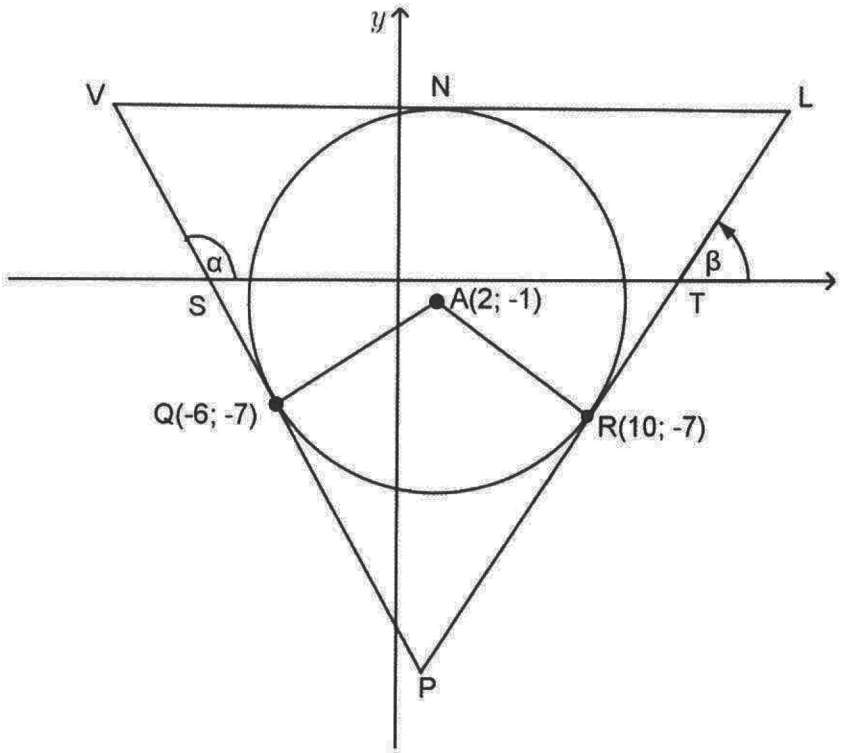


	Solution/*Oplossing*	**Marks/ *Punte***
3.1		(2)
3.2		(2)

	Solution/*Oplossing*	Marks/ *Punte*
3.3		(4)
3.4		(3)

	Solution/*Oplossing*	**Marks/ *Punte***
3.5		(3)
3.6		(4)
		[18]

QUESTION / VRAAG 4



	Solution/*Oplossing*	Marks/ *Punte*
4.1		(2)
4.2		(2)
4.3.1		(2)

	Solution/*Oplossing*	**Marks/ *Punte***
4.3.2		(2)
4.4.1		(2)
4.4.2		(2)
4.5.1		(4)

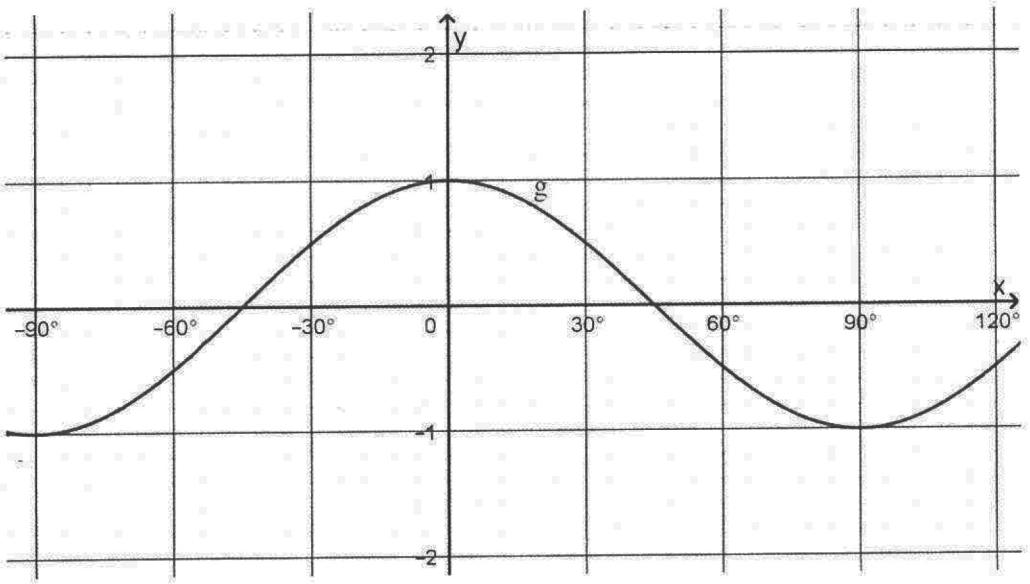
	Solution/*Oplossing*	Marks/ *Punte*
4.5.2		(2)
4.5.3		(4)
		[22]

QUESTION / VRAAG 5

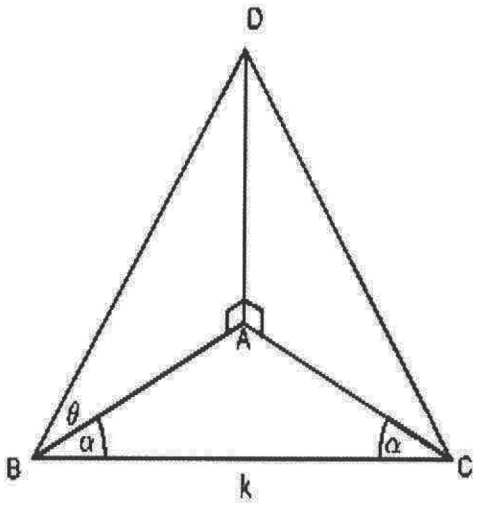
	Solution/*Oplossing*	Marks/ *Punte*
5.1.1		(3)
5.1.2		(3)
5.2		(3)

	Solution/*Oplossing*	**Marks/ *Punte***
5.3		(5)
5.4.1		(2)
5.4.2		(5)
5.5		(6)
		[27]

QUESTION / VRAAG 6

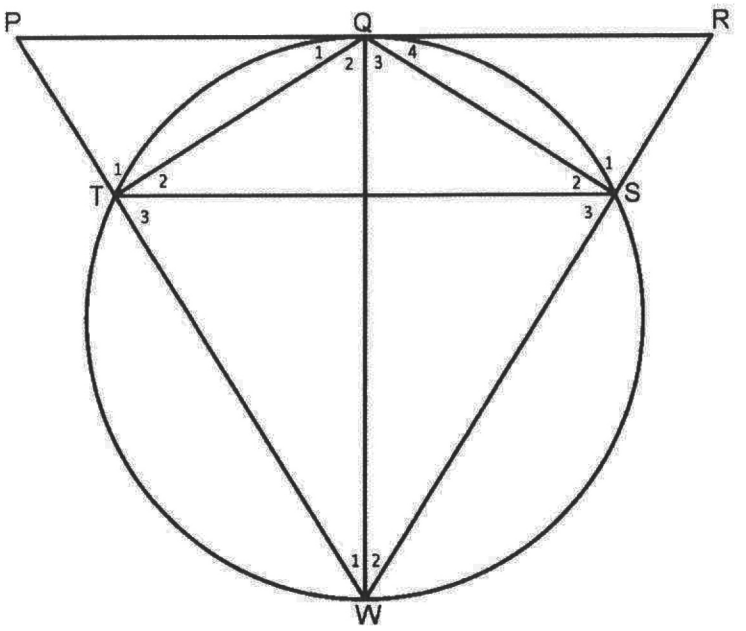
	Solution/*Oplossing*	Marks/ *Punte*
6.1		(3)
6.2		(2)
6.3		(2)
		[7]

QUESTION / VRAAG 7



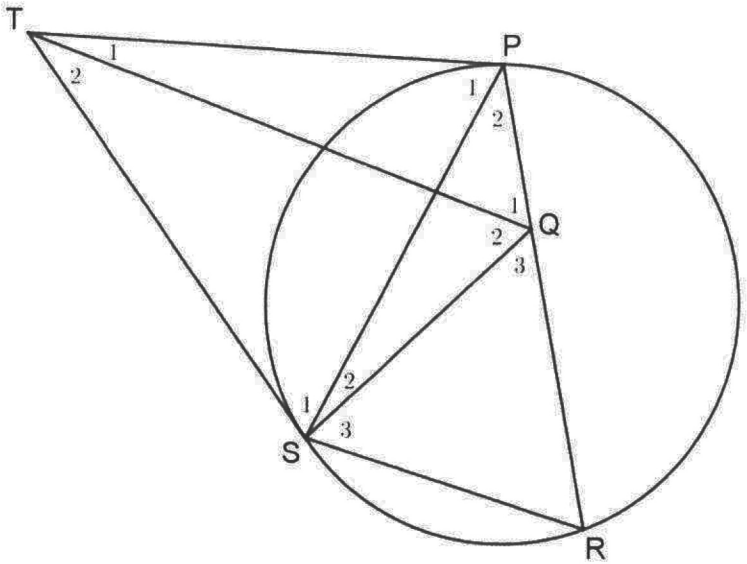
[6]

QUESTION / VRAAG 8

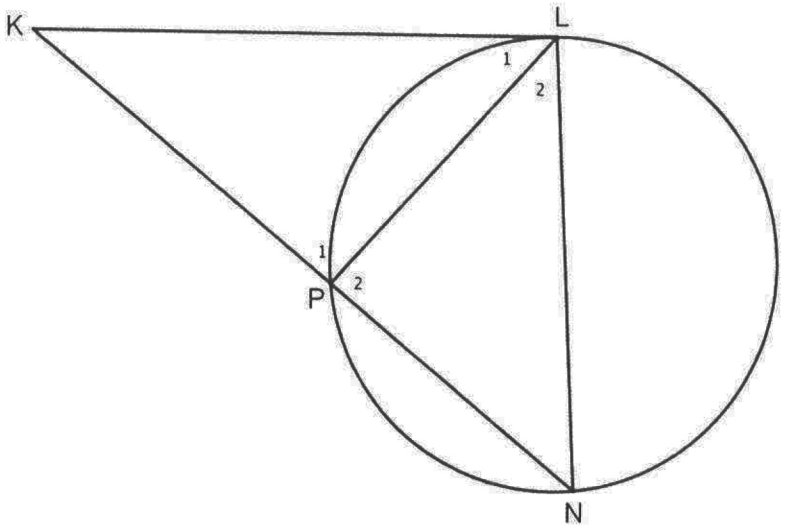


	Solution/*Oplossing*	**Marks/ *Punte***
8.1		(5)
8.2.1		(2)
8.2.2		(3)
8.2.3		(4)
8.2.4		(3)
		[17]

QUESTION / VRAAG 9



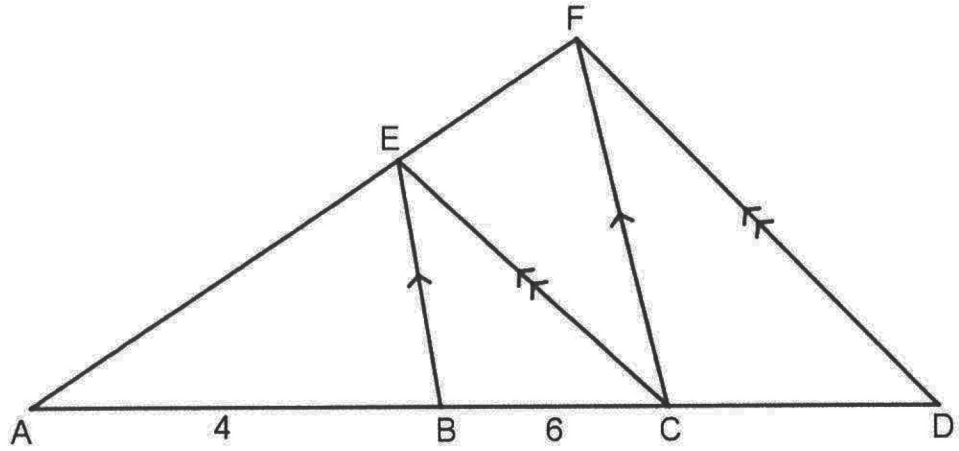
	Solution/*Oplossing*	**Marks/ *Punte***
9.1.1		(3)
9.1.2		(3)
9.1.3		(3)



	Solution/*Oplossing*	**Marks/ *Punte***
9.2.1		(4)
9.2.2		(3)
9.2.3		(1)

	Solution/*Oplossing*	Marks/ *Punte*
9.2.4		
		(6)
		[23]

QUESTION / VRAAG 10



10.1	**Solution/*Oplossing***	**Marks/ *Punte***
		(3)
10.2		(4)
		[7]

TOTAL MARKS: 150

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MATHEMATICS P2

MARKING GUIDELINE

COMMON TEST

JUNE 2019

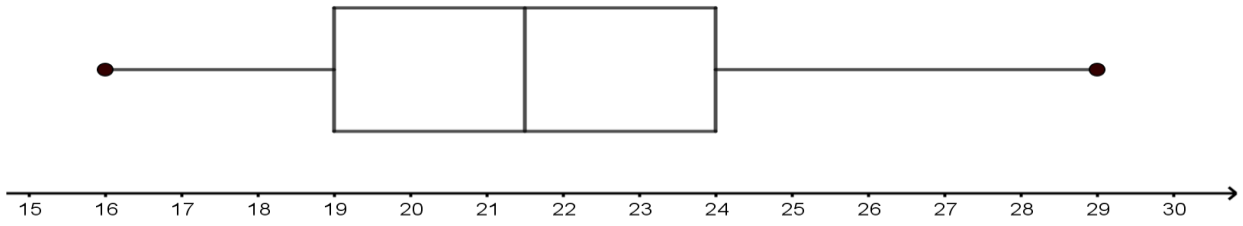
NATIONAL
SENIOR CERTIFICATE

GRADE 12

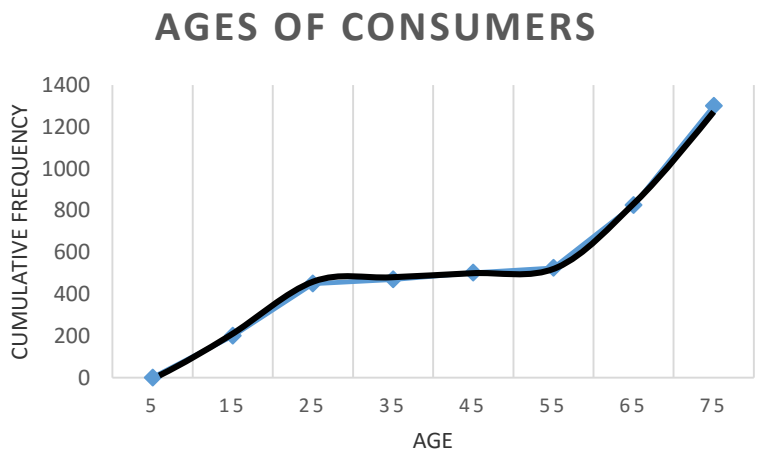
MARKS: 150

N.B. This marking guidelines consists of 14 pages.

QUESTION 1

1.1	$\bar{x} = \frac{220}{10} = 22$	✓A 220 ✓ CA answer Answer only full marks	(2)
1.2	σ = 3,95	✓✓ AA answer If formula is used 1CA mark for substitution and 1CA mark for answer.	(2)
1.3		✓ A minimum & maximum value ✓ A quartile 1 value ✓ A median value ✓A quartile 3 value If No Diagram No marks (4)	
1.4	$(\bar{x} - \sigma; \bar{x} + \sigma)$ (22 – 3,95; 22 + 3,95) (18,05; 25, 95) 6 runners (answer only – full marks)	✓CA 18,05 ✓ CA 25,95 ✓ CA answer	(3)
			[11]

QUESTION 2

2.1	A = 250 B = 502	✓ (A)A ✓ (A) B	(2)
2.2	$\bar{x} = \frac{2000 + 5000 + 600 + 1280 + 1150 + 18000 + 33250}{1300}$ $\bar{x} = \frac{61280}{1300}$ $\bar{x} = 47{,}14$ (answer only – full marks)	✓CA sum ✓CA 61280 ✓CA answer	(3)
2.3	$65 < x \leq 75$	✓✓AA answer	(2)
2.4		✓CA upper limits ✓CA grounding (5; 0) ✓CA joining points with a smooth curve	(3)
2.5	Not a normal distribution. Highest frequency is found between the ages 55 to 75. Mean < median, therefore skewed to the left.	✓A No ✓A Reason	(2)
			[12]

QUESTION 3

3.1	$BC = \sqrt{(x^2 - x^1)^2 + (y^2 - y^1)^2}$ $= \sqrt{(-2-1)^2 + (-2-4)^2}$ $= \sqrt{9+36}$ $= \sqrt{45}$ $= 3\sqrt{5}$	✓A substitution ✓CA answer (2)
3.2	$M\left(\frac{1-2}{2}; \frac{4-2}{2}\right)$ $M\left(-\frac{1}{2}; 1\right)$	✓A $\frac{-1}{2}$ ✓A 1 (2)
3.3	$m_{AB} = \frac{-2-4}{-2-1} = \frac{-6}{-3}$ $= 2$ $m_{MD} = -\frac{1}{2}$ (DM ⊥ AB) $y = mx + c$ $1 = -\frac{1}{2}\left(-\frac{1}{2}\right) + c$ $c = 1 - \frac{1}{4}$ $= \frac{3}{4}$ $y = -\frac{1}{2}x + \frac{3}{4}$	✓ A M_{AB} ✓CA gradient of MD ✓CA subst. $\left(-\frac{1}{2}; 1\right)$ into eq. ✓CA answer (4)

3.4	E is the midpoint since ME ∥ BC. $\text{E}\left(\frac{1+4}{2};\ \frac{4+1}{2}\right)$ $= \text{E}\left(\frac{5}{2};\ \frac{5}{2}\right)$	✓ A S/R ✓A substitution ✓CA answer (provided coordinates are positive) (3)
3.5	$m_{BE} = \frac{-2-\frac{5}{2}}{-2-\frac{5}{2}}$ $m_{BE} = 1$ $y - y_1 = m(x - x_1)$ OR $y = mx + c$ $y-(-2) = 1(x-(-2))$ $-2 = 1(-2) + c$ $y + 2 = x + 2$ $0 = c$ $y = x$ $y = x$	✓CA m_{BE} (must be positive) ✓CA substitution ✓CA answer (must be positive) (3)
3.6	$m_{BC} = \frac{-2-1}{-2-4} = \frac{1}{2}$ $m_{BC} = \tan\theta = \frac{1}{2}$ $\theta = 26{,}57°$	✓ A Substitution ✓CA $m_{BC} = \frac{1}{2}$ ✓CA $\tan\theta = \frac{1}{2}$ ✓CA answer (4)
		[18]
	QUESTION 4	
4.1	$AQ = \sqrt{(-6-2)^2 + \left(-7-(-1)\right)^2}$ $AQ = \sqrt{(-8)^2 + (-6)^2}$ $AQ = \sqrt{64+36}$ $AQ = \sqrt{100}$ $\therefore\ AQ = 10$	✓A subst. into dist. formula ✓CA answer (2)

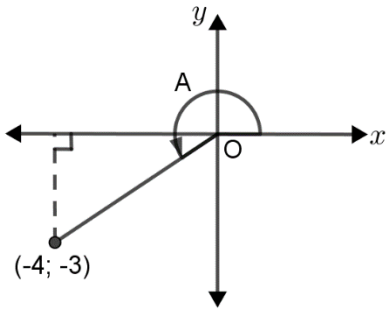
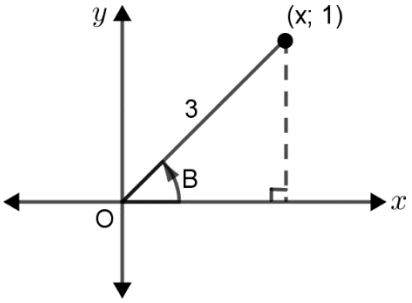
4.2	$(x-a)^2 + (y-b)^2 = r^2$ $(x-2)^2 + (y+1)^2 = 100$	✓A $(x-2)^2 + (y+1)^2$ ✓CA 100 (2)
4.3.1	$m_{AQ} = \frac{-7+1}{-6-2} = \frac{3}{4}$ $\therefore \ m_{QP} = -\frac{4}{3}$ $rad. \perp tan$	✓ A $\frac{3}{4}$ ✓CA $-\frac{4}{3}$ (2)
4.3.2	$m_{AR} = \frac{-7+1}{10-2} = -\frac{3}{4}$ $\therefore \ m_{PR} = \frac{4}{3}$ $rad. \perp tan$	✓A $\frac{-3}{4}$ ✓CA $\frac{4}{3}$ (2)
4.4.1	$m_{QP} = -\frac{4}{3}$ $y - y_1 = m(x - x_1)$ $y + 7 = -\frac{4}{3}(x+6)$ $y = -\frac{4}{3}x - 15$	✓CA substitution ✓CA answer (2)
4.4.2	$m_{PR} = \frac{4}{3}$ $y - y_1 = m(x - x_1)$ $y + 7 = \frac{4}{3}(x - 10)$ $y = \frac{4x}{3} - \frac{40}{3} - 7$ $= \frac{4}{3}x - \frac{61}{3}$	✓CA substitution ✓CA answer (2)

4.5.1	$\frac{4}{3}x - \frac{61}{3} = -\frac{4}{3}x - 15$ $\frac{8}{3}x = \frac{16}{3}$ $x = 2$ $y = -\frac{53}{3}$ P$(2; = -\frac{53}{3})$ **OR** The x – co-ordinate of P is 2 (ARPQ is a kite) Subst $x=2$ *in* $y = -\frac{4}{3}x - 15$ $y = -\frac{4}{3}(2) - 15$ $= -\frac{8}{3} - 15$ $= \frac{-8-45}{3}$ $= \frac{-53}{3}$ $P\left(2; -\frac{53}{3}\right)$	✓CA Equating ✓CA x value ✓CA y value ✓CA both co-ordinates ✓A $x = 2$ ✓CA substitution ✓CA y value ✓CA both co-ordinates (4)
4.5.2	In ΔSPR $\alpha = \hat{P} + \beta$ (ext $\angle$ *of* ΔSPR) $\therefore \hat{P} = \alpha - \beta$	✓A S/R ✓A $\hat{P} = \alpha - \beta$ (2)

4.5.3	$\tan(\alpha-\beta) = \frac{\sin(\alpha-\beta)}{\cos(\alpha-\beta)}$ $= \frac{\sin\alpha\cos\beta - \cos\alpha\sin\beta}{\cos\alpha\cos\beta + \sin\alpha\sin\beta}$ $= \frac{\left(\frac{4}{5}\right)\left(\frac{3}{5}\right) - \left(-\frac{3}{5}\right)\left(\frac{4}{5}\right)}{\left(-\frac{3}{5}\right)\left(\frac{3}{5}\right) + \left(\frac{4}{5}\right)\left(\frac{4}{5}\right)}$ $= \frac{\frac{12}{25} + \frac{12}{25}}{\frac{-9}{25} + \frac{16}{25}}$ $= \frac{24}{25} \times \frac{25}{7}$ $= \frac{24}{7}$ OR $tan\beta = \frac{4}{3} \quad \therefore \beta = 53.13°$ $tan\alpha = -\frac{4}{3} \therefore \alpha = 126{,}87°$ $\tan(\alpha-\beta) = \frac{\sin(126.87-53.13)}{\cos(126.87-53.13)}$ $= 3, 42857\ldots\ldots$ $\cong \frac{24}{7}$	✓A expansion ✓A numerator ✓A denominator ✓A Simplification (4) ✓A answer ✓A answer ✓A simplification ✓A Answer (4)

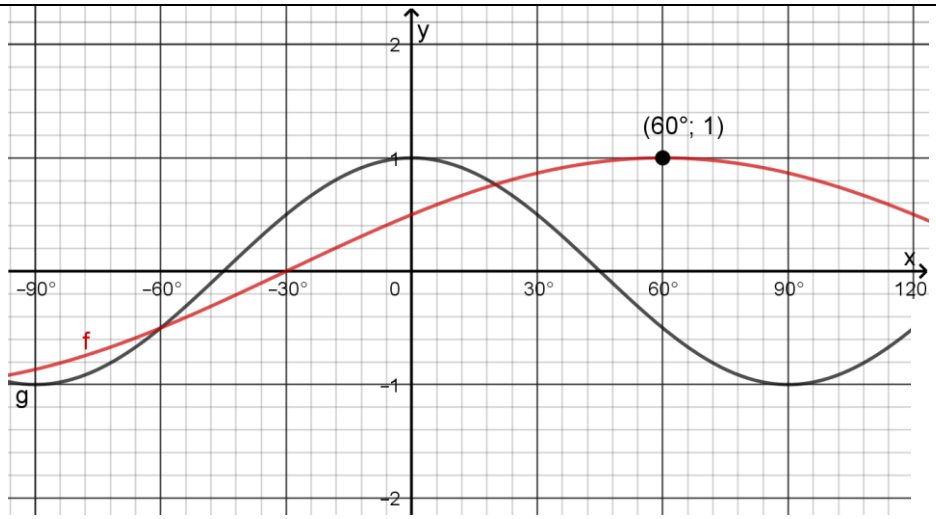
[22]

QUESTION 5

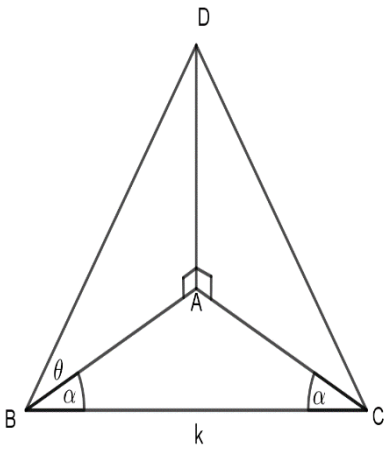
5.1	$\tan A = \frac{3}{4}$; $\sin B = \frac{1}{3}$ Diagram 1  $r^2 = (-4)^2 + (-3)^2$ …. $Pyth$ $r^2 = 25$ $r = 5$ Diagram 2  $x^2 = 3^2 - 1^2$ … $Pyth$ $x^2 = 8$ $x = 2\sqrt{2}$	✓A diagram 1	
5.1.1	$\cos 2A$ $= 2cos^2A - 1$ $= 2\left(\frac{-4}{5}\right)^2 - 1$ $= \frac{7}{25}$	✓A identity ✓CA answer	(3)
5.1.2	$\sin(A + B)$ $= \sin A \cos B + \cos A \sin B$ $= \left(\frac{-3}{5}\right)\left(\frac{2\sqrt{2}}{3}\right) + \left(\frac{-4}{5}\right)\left(\frac{1}{3}\right)$ $= -\frac{4 + 6\sqrt{2}}{15}$	✓A Diagram 2 ✓A expansion ✓CA answer	(3)
5.2	$\sin 20^o \cos 320^o + \cos(-20^o) \sin 400^o$ $= \sin 20^o \cos 40^o + \cos 20^o \sin 40^o$ $= \sin(20^o + 40^o)$ $= \sin 60^o$ $= \frac{\sqrt{3}}{2}$	✓A reduction ✓CA simplify ✓CA answer (provided special angle ratio) ANSWER ONLY = 0	(3)

5.3	$\frac{\cos^2(90^\circ+\theta)}{\cos(-\theta)+\sin(90^\circ-\theta)\cos\theta}$ $=\frac{\sin^2\theta}{\cos\theta+\cos^2\theta}$ $=\frac{\sin^2\theta}{\cos\theta(1+\cos\theta)}$ $=\frac{1-\cos^2\theta}{\cos\theta(1+\cos\theta)}$ $=\frac{1-\cos\theta}{\cos\theta}$ $=\frac{1}{\cos\theta}-1$ $=$ RHS	✓A numerator ✓A denominator ✓A common factor ✓ A difference of squares ✓A simplification	(5)
5.4.1	$\cos 2\alpha=\cos^2\alpha-\sin^2\alpha$ $=(\cos\alpha+\sin\alpha)(\cos\alpha-\sin\alpha)$ $=p.q$	✓A expansion ✓ A answer	(2)
5.4.2	$\frac{1+\sin 2\alpha}{\cos 2\alpha}$ $=\frac{\sin^2\alpha+2\sin\alpha\cos\alpha+\cos^2\alpha}{\cos^2\alpha-\sin^2\alpha}$ $=\frac{(\sin\alpha+\cos\alpha)^2}{(\cos\alpha-\sin\alpha)(\cos\alpha+\sin\alpha)}$ $=\frac{p}{q}$	✓A numerator ✓A denominator ✓A factorise ✓A factorise ✓CA answer	(5)
5.5	$6\cos^2 x+\sin x-5=0$ $6(1-\sin^2 x)+\sin x-5=0$ $6-6\sin^2 x+\sin x-5=0$ $-6\sin^2 x+\sin x+1=0$ $(3\sin x+1)(-2\sin x+1)=0$ $\sin x=-\frac{1}{3}$; $\sin x=\frac{1}{2}$ $x=199{,}47^\circ+k.360^\circ;k\in\mathbb{Z}$ OR $x=30^\circ+k.360^\circ;k\in\mathbb{Z}$ $x=340{,}53^\circ+k.360^\circ;k\in\mathbb{Z}$ OR $x=150^\circ+k.360^\circ;k\in\mathbb{Z}$	✓A identity ✓CA factors ✓CA both ratios ✓A $k\in\mathbb{Z}$ ✓CA both solutions of $\sin x=-\frac{1}{3}$ ✓CA both solutions of $\sin x=\frac{1}{2}$	(6)
			[27]

QUESTION 6

6.1		✓A x-intercept ✓A t. pt. $(60^0; 1)$ ✓A shape	(3)
6.2	$60° < x < 90°$	✓CA end values ✓CA notation	(2)
6.3	Graph of f moves 60^0 left.	✓A shifts 60^0 ✓A to the left	(2)
			[7]

QUESTION 7

7	$\tan\theta = \frac{DA}{AB}$ $\therefore AD = AB\tan\theta$ Also $\frac{AB}{\sin\alpha} = \frac{k}{\sin(180°-2\alpha)}$ $AB\ \sin 2\alpha = k\ \sin\alpha$ $AB = \frac{k\ \sin\alpha}{2\sin\alpha\cos\alpha}$ $AB = \frac{k}{\cos\alpha}$ $\therefore AD = \frac{k.\tan\theta}{2\cos\alpha}$ 	✓A trig ratio ✓ AAD value ✓A substitute into sine rule ✓A $\sin 2\alpha = 2\sin\alpha\cos\alpha$ ✓A $AB = \frac{k.\sin\alpha}{2\sin\alpha\cos\alpha}$ ✓A making AB value - simplified	
			[6]

QUESTION 8

8.1	$\hat{Q}_4 = \hat{W}_2 = x$ … (tan chord theorem) $\hat{W}_2 = \hat{W}_1 = x$ … (WQ bisects PWR.) $\hat{Q}_1 = \hat{W}_1 = x$ … (tan – chord theorem) $\hat{T}_2 = \hat{W}_2 = x$ … ($\angle$s in same segment) $\hat{S}_2 = \hat{W}_1 = x$ … $\angle$s in same segment)	A✓S/R A✓S/R A✓S/R A✓S/R A✓S/R	(5)
8.2.1	$\hat{T}_2 = \hat{Q}_1 = x$ $\therefore$ TS // PR … (alternate $\angle$s equal)	A✓S A✓R	(2)
8.2.2	$\hat{T}_3 = \hat{P}$ … (corresponding $\angle s$; TS//PR) $\hat{T}_3 = \hat{Q}_3$ … ($\angle$s in same segment) $\therefore \hat{P} = \hat{Q}_3$	AA✓S✓R A✓S/R	(3)
8.2.3	In Δ TQS $\hat{T}_2 = x$ $\hat{S}_2 = x$ $\therefore \hat{T}_2 = \hat{S}_2 = x$ $\therefore$ Δ TQS isosceles … ($\angle$s opposite equal sides)	A✓S A✓S A✓S A✓R	(4)
8.2.4	$W\hat{Q}P = W\hat{S}Q$…(tan – chord theorem) $\hat{T}_1 = W\hat{S}Q$ … (ext $\angle$ of cyclic quad)	AA✓S✓R A✓S/R	(3)
			[17]

QUESTION 9

9.1.1	$\hat{P}_1 = \hat{Q}_1$ given $\hat{P}_1 = \hat{R}$ tan-chord theorem $\therefore \hat{Q}_1 = \hat{R}$ $\therefore$ TQ ∥ SR (corr $\angle^s$ *are equal*)	AA✓S✓R A✓R	(3)
9.1.2	$\hat{P}_1 = \hat{Q}_1$ given TS = TP tan from same point $\hat{P}_1 = \hat{S}_1$ equal $\angle^s$ opp equal sides $\therefore \hat{Q}_1 = \hat{S}_1$ ∴QPTS is a cyclic quad converse equal $\angle^s$ subtended by same chord	A✓S/R A✓S/R A✓R	(3)
9.1.3	QPTS is a cyclic quad $\therefore \hat{P}_1 = \hat{Q}_2$ $\angle^s$in same ⊙ segm but $\hat{P}_1 = \hat{Q}_1$ given $\therefore \hat{Q}_1 = \hat{Q}_2$ ∴TQ bisect $S\hat{Q}P$	AA✓S✓R A✓S	(3)
9.2.1	In Δ LPK and Δ NPL $K\hat{L}P = L\hat{N}P$ … tan chord theorem $\hat{P}_2 = 90°$ ... ... $\angle$ in semi ⊙ $\hat{P}_1 = \hat{P}_2$ … both = 90° $P\hat{K}L = N\hat{L}P$ … remaining angle ∴ ΔLPK///ΔNPL … ∠∠∠	AA✓S✓R A✓S/R A✓R	(4)
9.2.2	$\frac{PL}{NP} = \frac{KL}{NL} = \frac{PK}{PL}$… Δ LPK /// Δ NPL $\frac{PL}{NP} = \frac{PK}{PL}$ ……. ΔLPK///ΔNPL $\therefore PL^2 = NP.PK$	AA✓S✓R A✓ proportionality	(3)
9.2.3	Δ NLK	A✓ answer	(1)
9.2.4	ΔNLK///ΔNPL $\therefore \frac{KN}{LN} = \frac{LN}{NP}$[///Δ's] $LN^2 = KN.NP$ = 16 x 10 = 160 LN = $\sqrt{160}$ Radius = $\frac{1}{2}\sqrt{160}$ Area of Circle = πr^2 = $\pi(\frac{1}{2}\sqrt{160})^2$ = 125.66cm^2 **OR**	A✓ S/R A ✓ Substitution ✓CA NL value ✓CA radius = $\frac{1}{2}\sqrt{160}$ ✓CA Substitution ✓CA Answer	(6)

	From Question No. 9.2.2: $PL^2 = NP.PK$ $= 10\text{ cm} \times 6\text{ cm}$ $= 60\text{ cm}^2$ $NL^2 = PL^2 + PN^2$ … Pythagoras $= 60 + 100$ (Δ LPN) $= 160\text{ cm}^2$ $\therefore NL = \sqrt{160}cm$ $\frac{1}{2}NL = \frac{1}{2}$ diameter $= \frac{1}{2}\sqrt{160}\,cm$ Area of circle $= \pi r^2$ $= \pi \times \left(\frac{1}{2}\sqrt{160}\right)^2\ cm^2$ $= 125{,}66\text{ cm}^2$ OR $40\pi\ cm^2$	✓A $PL^2 = 60\text{ cm}^2$ ✓A Pythagoras ✓CA NL value ✓CA radius = $\frac{1}{2}\sqrt{160}$ ✓CA substitution ✓CA answer	
			[23]

QUESTION 10

10.1	$\frac{AE}{EF} = \frac{4}{6}$ …… prop theorem ; EB//FC $\frac{AE}{EF} = \frac{AC}{CD}$ …… prop theorem; EC//FD $\frac{4}{6} = \frac{10}{CD}$ $CD = 15\ units$	A✓ S/R A✓ S/R ✓CA answer	(3)
10.2	$\frac{\Delta FEC}{\Delta CFA} = \frac{3}{5}$ same height $\frac{\text{Area }\Delta CFA}{\text{Area }\Delta FAD} = \frac{10}{25} = \frac{2}{5}$ same height $\frac{\text{Area }\Delta FEC}{\text{Area }\Delta FAD} = \frac{\text{Area }\Delta FEC}{\text{Area }\Delta CFA} \times \frac{\text{Area }\Delta CFA}{\text{Area }\Delta FAD}$ $= \frac{3}{5} \times \frac{2}{5}$ $= \frac{6}{25}$	A✓ S/R A✓ S/R ✓CA simplify ✓CA answer	(4)
			[7]

TOTAL MARKS: 150