GAUTENG PROVINCE
EDUCATION
REPUBLIC OF SOUTH AFRICA

GAUTENG DEPARTMENT OF EDUCATION

PREPARATORY EXAMINATION

2019

10611

MATHEMATICS

PAPER 1

TIME: 3 hours

MARKS: 150

12 pages + 1 information sheet

MATHEMATICS: Paper 1

10611E

X05

P.T.O.

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(Paper 1)

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INSTRUCTIONS AND INFORMATION

Read the following instructions carefully before answering the questions.

1. This question paper consists of 13 questions.
2. Answer ALL the questions.
3. Clearly show ALL calculations, diagrams, graphs, etc. which were used in determining the answers.
4. Answers only will not necessarily be awarded full marks.
5. Use an approved scientific calculator (non-programmable and non-graphical), unless stated otherwise.
6. Where necessary, answers should be rounded-off to TWO decimal places, unless stated otherwise.
7. Diagrams are NOT necessarily drawn to scale.
8. An information sheet is included on Page 13 of the question paper.
9. Number the questions correctly according to the numbering system used in this question paper.
10. Write neatly and legibly.

QUESTION 1

1.1 Solve for x:

1.1.1 $2x^2 + 3 = 8x$ (correct to TWO decimal places) (4)

1.1.2 $4x - 2x(x-3) \leq 0$ (4)

1.1.3 $2^x - 5.2^{x+1} = -144$ (4)

1.2 If $f(2) = 0$ and $f(-6) = 0$, determine an equation for $f(x)$ in the form $f(x) = x^2 + bx + c$. (2)

1.3 Solve for x and y simultaneously:

$$2x + y = 17 \quad \text{and} \quad xy = 8$$ (6)

1.4 Given: $2mx^2 = 3x - 8$ where $m \neq 0$.
Determine the value(s) of m for which the roots of the equation are non-real. (4)

[24]

QUESTION 2

Given the quadratic sequence: $-\frac{1}{2}\ ;\ 2\ ;\ \frac{11}{2}\ ;\ 10\ ;\ \ldots$

2.1 Show that the nth term of this sequence can be written as $T_n = \frac{1}{2}(n^2 + 2n - 4)$. (4)

2.2 Determine the value of $T_{75} - T_{74}$. (2)

2.3 The first differences of the given sequence above forms another number sequence.

2.3.1 Is the sequence of the first differences arithmetic or geometric?
Give a reason for your answer. (2)

2.3.2 Which term in the sequence of the first differences will be equal to $\frac{151}{2}$? (1)

2.3.3 Calculate the value of the 30th first difference. (2)

2.3.4 Calculate the number of terms in the quadratic sequence if the sum of the first n first differences is 2 176. (4)

[15]

QUESTION 3

3.1 The following geometric series is given:

$$2(3x-1)+2(3x-1)^2+2(3x-1)^3 \ \ldots$$

Determine the value(s) of x for which the series converges. (3)

3.2 The first two terms of a convergent geometric series are k and 6 respectively where $k \neq 0$.
The sum of the infinite series is 25.
Calculate the value(s) of k. (5)

3.3 Given the series:

$$(1\times 2)+(5\times 6)+(9\times 10)+(13\times 14)+...+(81\times 82)$$

Write the series in sigma notation. (It is not necessary to calculate the value of the series). (4)

[12]

QUESTION 4

4.1 How many years will it take for an investment of R3 000 to accumulate to R4 500, if it is invested at 8% p.a. compounded monthly? (4)

4.2 Bongani paid off a 20-year loan of R40 000. During the period of the loan the interest rate changed from 24% p.a. compounded monthly for the first five years to 18% p.a. compounded monthly for the remaining years.

4.2.1 Calculate the initial monthly payment before the interest rate changed. (4)

4.2.2 What is the outstanding balance of the loan after the FIRST five years? (4)

4.2.3 Determine the monthly payment after the interest rate changed. (4)

[16]

QUESTION 5

Given: $f(x) = \dfrac{3}{x-1} - 2$

5.1 Calculate the coordinates of the x-intercept of f. (2)

5.2 Calculate the coordinates of the y-intercept of f. (1)

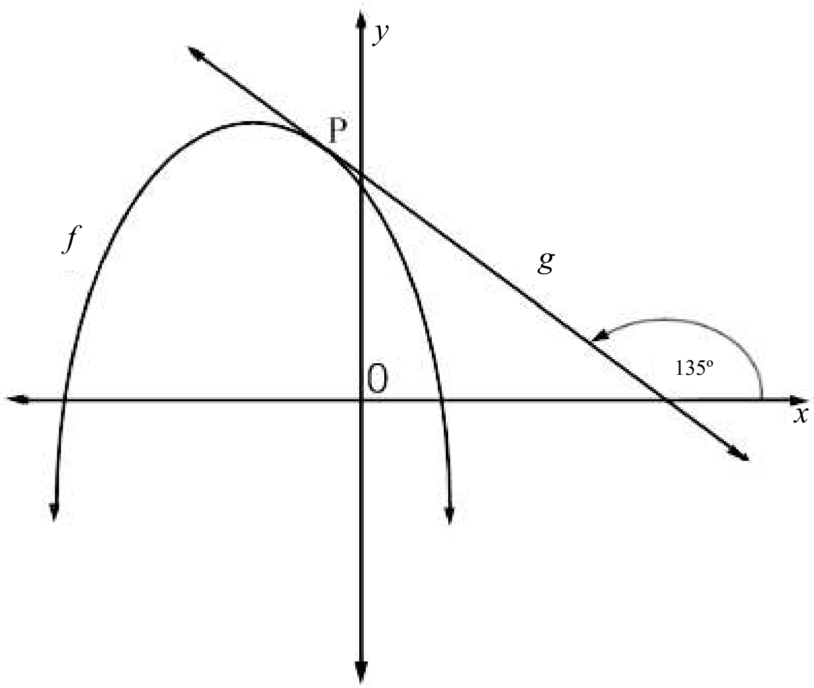
5.3 Sketch the graph of f in your ANSWER BOOK. Clearly show the asymptotes and the intercepts with the axes. (3)

5.4 ONE of the axes of symmetry of f is a decreasing function.
Write down the equation of this axis of symmetry. (3)

[9]

QUESTION 6

The graphs of $f(x) = -2x^2 - 5x + 3$ and $g(x) = ax + q$ are sketched below.
The angle of inclination of g is 135º. Graph g is a tangent to f at point P.



6.1 Calculate the coordinates of the turning point of f. (3)

6.2 Write down the range of f. (1)

6.3 Calculate the coordinates of point P, the point of contact of f and g. (4)

6.4 Determine the value(s) of k for which the straight line $y = k$ is NOT a tangent to $y = 2x^2 + 5x - 3$. (2)

[10]

QUESTION 7

Given $f(x)=a^x$, where $a>0$, passing through the point $(2\ ;\ \frac{1}{4})$ and $g(x)=4x^2$.

7.1 Prove that $a=\frac{1}{2}$. (2)

7.2 Determine the equation of $y=f^{-1}(x)$ in the form $y=\ \ldots$ (2)

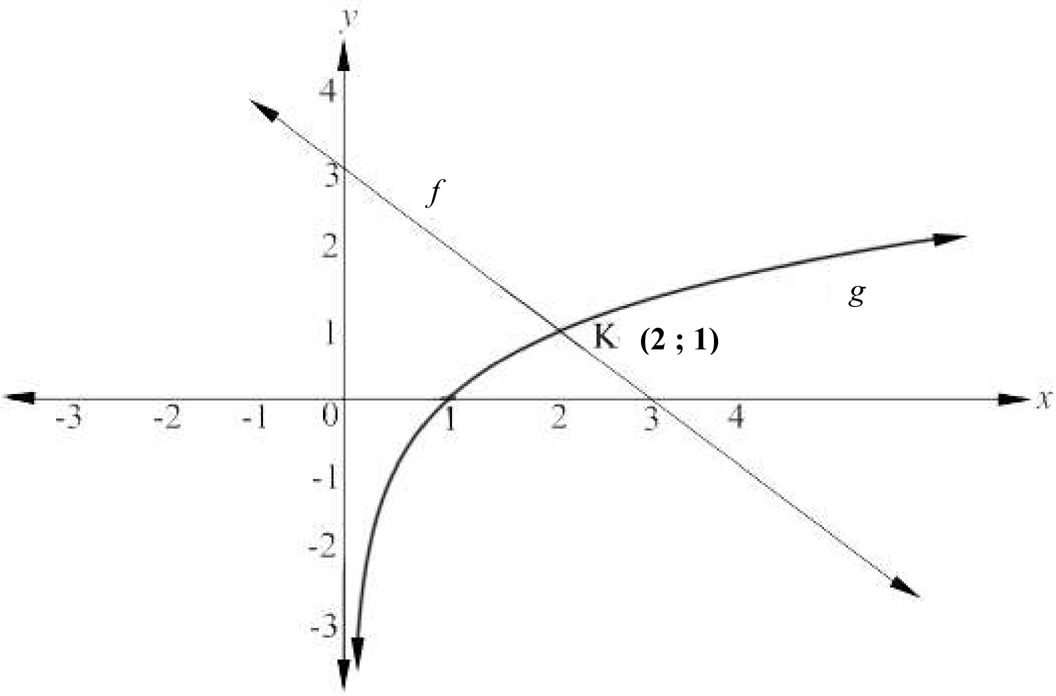
7.3 Determine the equation of $y=h(x)$ where h is the reflection of f in the x-axis. (1)

7.4 How must the domain of $g(x)$ be restricted so that $g^{-1}(x)$ will be a function? (2)

[7]

QUESTION 8

The graphs of $f(x) = -x + 3$ and $g(x) = \log_2 x$ are drawn below.
Graphs f and g intersect at point K(2 ; 1).



8.1 Write down value(s) of x for which:

8.1.1 $f(x) - g(x) > 0$ (2)

8.1.2 $g(x) \,.\, g^{-1}(x) \leq 0$ (2)

8.2 8.2.1 Write down the equation of g^{-1} in the form $y = \ldots$ (2)

8.2.2 Explain how you could use the given sketch to solve the equation $\log_2(3 - x) = x$. (2)

8.2.3 Write down the solution to $\log_2(3 - x) = x$. (1)

[9]

QUESTION 9

9.1 Given: $f(x) = 3x - x^2$

9.1.1 Determine $f'(x)$ from FIRST principles. (5)

9.1.2 Determine the average gradient of f between $x = 1$ and $x = 3$. (3)

9.2 Determine:

9.2.1 $\frac{dy}{dx}$ if $y = \frac{8 - 3x^6}{8x^5}$ (3)

9.2.2 $D_x\left[\sqrt[3]{x^2} + \frac{1}{x} + 2x\right]$ (4)

[15]

QUESTION 10

A cubic function has the following essential properties:

- $f(0) = 8$
- $f(4) = f(1) = 0$
- $f'(3) = f'(1) = 0$
- $f(3) = 8$

10.1 Sketch the graph of f in your ANSWER BOOK clearly indicating the turning point(s) and the points of intersection of the graph with the axes. (3)

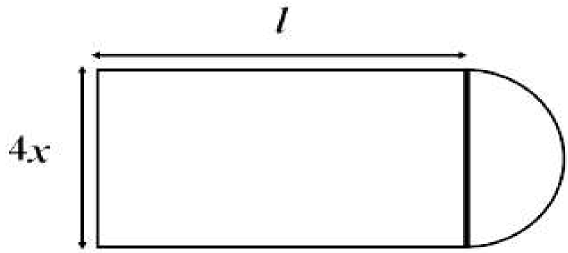
10.2 Show that the defining equation of f is $f(x) = -2x^3 + 12x^2 - 18x + 8$. (4)

10.3 Determine the value(s) of x for which graph of f is concave down. (3)

[10]

QUESTION 11

The diagram below shows a garden in the form of a rectangle and a semi-circle.



The rectangular section of the garden has the dimensions length (l) and width ($4x$).

The perimeter of the garden is $32m$.

11.1 Express the length (l) in terms of x. (3)

11.2 Show that the area of the garden can be written as $\text{A}(x) = -8x^2 - 2\pi x^2 + 64x$. (2)

11.3 Determine the value of x for which the area of the garden is minimum. (3)

[8]

QUESTION 12

12.1 For two events, A and B, it is given that:

- $P(A) = 0{,}3$
- $P(B) = 0{,}4$
- $P(A \text{ or } B) = 0{,}6$

Calculate $P(A \text{ and } B)$. (2)

12.2 A survey was completed among Grade 11 and Grade 12 learners at a certain school to establish the type of cell phone that each learner uses. Some of the results are shown in the table below.

	Gr. 11	**Gr. 12**	**Total**
Android	***A***	33	65
iPhone	53	***B***	101
Total	85	81	166

12.2.1 Calculate values for ***A*** and ***B*** in the table. (2)

12.2.2 If a learner from this group is selected at random, what is the probability that he/she will use an iPhone? (2)

12.2.3 All these Grade 11 and Grade 12 learners attend Mathematics lessons in a particular class at the school. At the end of the day, the Mathematics educator found an iPhone in this class.
What is the probability that the phone belongs to a Grade 12 learner? (2)

[8]

QUESTION 13

Cindy has the following books to arrange on a bookshelf:

- 4 Mathematics books
- 3 Physical Sciences books
- 2 Life Sciences books

13.1 Determine the number of different ways that ALL the books can be arranged. (2)

13.2 Determine the number of different ways that the books can be arranged in order that the books in each learning area are NEXT to each other. (3)

13.3 In how many different ways can all the books be arranged in order of descending height? (2)

[7]

TOTAL: 150

END

INFORMATION SHEET

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$A = P(1+ni)$ $\quad A = P(1-ni)$ $\quad A = P(1-i)^n$ $\quad A = P(1+i)^n$

$\sum_{i=1}^{n} 1 = n$ $\quad \sum_{i=1}^{n} i = \frac{n(n+1)}{2}$ $\quad T_n = a + (n-1)d$ $\quad S_n = \frac{n}{2}\left(2a + (n-1)d\right)$

$T_n = ar^{n-1}$ $\quad S_n = \frac{a\left(r^n - 1\right)}{r-1}$; $\quad r \neq 1$ $\quad S_\infty = \frac{a}{1-r}$; $-1 < r < 1$

$F = \frac{x\left[(1+i)^n - 1\right]}{i}$ $\quad P = \frac{x[1-(1+i)^{-n}]}{i}$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $\quad \mathrm{M}\left(\frac{x_1 + x_2}{2} ; \frac{y_1 + y_2}{2}\right)$

$y = mx + c$ $\quad y - y_1 = m(x - x_1)$ $\quad m = \frac{y_2 - y_1}{x_2 - x_1}$ $\quad m = \tan\theta$

$$(x-a)^2 + (y-b)^2 = r^2$$

In ΔABC: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$ $\quad a^2 = b^2 + c^2 - 2bc.\cos A$ $\quad area\ \Delta ABC = \frac{1}{2}ab.\sin C$

$\sin(\alpha + \beta) = \sin\alpha.\cos\beta + \cos\alpha.\sin\beta$ $\quad \sin(\alpha - \beta) = \sin\alpha.\cos\beta - \cos\alpha.\sin\beta$

$\cos(\alpha + \beta) = \cos\alpha.\cos\beta - \sin\alpha.\sin\beta$ $\quad \cos(\alpha - \beta) = \cos\alpha.\cos\beta + \sin\alpha.\sin\beta$

$\cos 2\alpha = \begin{cases} \cos^2\alpha - \sin^2\alpha \\ 1 - 2\sin^2\alpha \\ 2\cos^2\alpha - 1 \end{cases}$ $\quad \sin 2\alpha = 2\sin\alpha.\cos\alpha$

$$(x ; y) \to (x\cos\theta - y\sin\theta ; y\cos\theta + x\sin\theta)$$

$\bar{x} = \frac{\sum fx}{n}$ $\quad \sigma^2 = \frac{\sum_{i=1}^{n}\left(x_i - \bar{x}\right)^2}{n}$

$P(A) = \frac{n(A)}{n(S)}$ $\quad P(A\ of\ B) = P(A) + P(B) - P(A\ en\ B)$

$\hat{y} = a + bx$ $\quad b = \frac{\sum(x - \bar{x})(y - \bar{y})}{\sum(x - \bar{x})^2}$