**NATIONAL
SENIOR CERTIFICATE**

GRADE 12

MATHEMATICS

JUNE CONTROL TEST

MARKING GUIDELINE

MARKS: 150
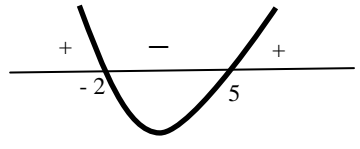
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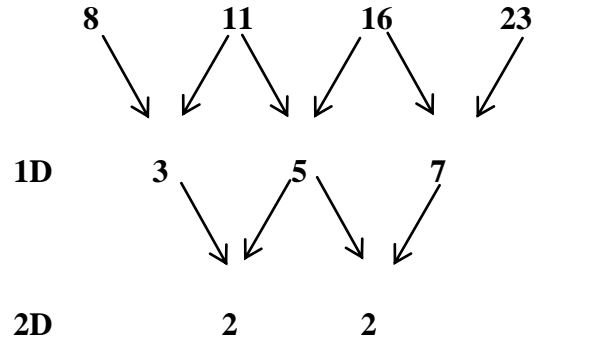**This marking guideline consists of 11 pages.**

QUESTION 1

1.1.1	$x = 4 \quad or \quad x = 10$	A✓ 4 A✓ 10	(2)
1.1.2	$x^2 - 8x - 6 = 0$ $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ $x = \dfrac{-(-8) \pm \sqrt{(-8)^2 - 4(1)(-6)}}{2(1)}$ $x = 8{,}69 \quad or \quad x = -0{,}69$	A✓ standard form CA✓ substitution in correct formula CACA✓✓ answers (penalize 1 mark if rounding off is incorrect-once for whole paper)	(4)
1.1.3	$3^{x+2} + 3^{2-x} = 82$ $9 \cdot 3^x + \dfrac{9}{3^x} = 82$ $9 \cdot 3^{2x} - 82 \cdot 3^x + 9 = 0$ $(9 \cdot 3^x - 1)(3^x - 9) = 0$ $3^x = \dfrac{1}{9} = 3^{-2} \quad or \quad 3^x = 9 = 3^2$ $3^x = 3^{-2} \quad or \quad 3^x = 3^2$ $x = -2 \quad or \quad x = 2$	A✓ standard form CA✓ factors CA✓ $3^x = \frac{1}{9} \quad and \quad 3^x = 9$ CA✓ Exponential forms CA✓ answers	(5)
1.1.4	$\sqrt{7x + 2} + 2x = 0$ $\sqrt{7x + 2} = -2x$ $7x + 2 = 4x^2$ $4x^2 - 7x - 2 = 0$ $(4x + 1)(x - 2) = 0$ $x = -\dfrac{1}{4} \quad or \quad x = 2$ $\qquad\qquad n/a$	A✓ Isolating surd and squaring CA✓ standard form CA✓ factors CA✓ x – values and rejecting	(4)

1.2	$3^{y-x} = 9$ $\quad\quad\quad\quad\quad\quad\quad\quad \rightarrow (1)$		
	$x^2 + 2xy - 4 = 0 \quad\quad\quad\quad\quad \rightarrow (2)$		
	From (1):		
	$3^{y-x} = 3^2 \quad \therefore y = x + 2 \ \rightarrow (3)$	A✓ making x/y the subject	
	Substituting (3) into (2):	CA✓ substitution into equation (2)	
	$x^2 + 2x(x + 2) - 4 = 0$		
	$x^2 + 2x^2 + 4x - 4 = 0$		
	$3x^2 + 4x - 4 = 0$	CA✓ standard form	
	$(3x - 2)(x + 2) = 0$	CA✓ factors	
	$x = \dfrac{2}{3} \quad\quad or \quad\quad x = -2$	CA✓ y/x values	
	$y = 2\dfrac{2}{3} \quad\quad or \quad\quad y = 0$	CA✓ x/y values	(6)
1.3	$x(x - 3) - 10 \geq 0$	A✓ standard form	
	$x^2 - 3x - 10 \geq 0$		
	$(x + 2)(x - 5) \geq 0$	CA✓ factors	
	$x \leq -2 \quad or \quad x \geq 5$		(4)
	OR	CACA✓✓ answer	
	$x(x - 3) - 10 \geq 0$	**OR**	
	$x^2 - 3x - 10 \geq 0$	A✓ standard form	
	$(x + 2)(x - 5) \geq 0$	CA✓ factors	
		A✓ graph with signs and critical values	
	$x \leq -2 \quad or \quad x \geq 5$	CA✓ answer	(4)
			[25]

QUESTION 2

2.1	**32 ; 43**	A✓answers	(1)
2.2	$2a = 2 \qquad \therefore a = 1$ $3a + b = 3 \qquad \therefore b = 0$ $a + b + c = 8 \qquad \therefore c = 7$ $T_n = n^2 + 7$	A✓ a value CA✓b value CA✓ c value CA✓answer	(4)
2.2	$T_n = n^2 + 7 = 2311$ $n^2 = 2311 - 7 = 2304$ $n = \sqrt{2304} = 48$	A✓equating n^{th} term to 2311 CA✓making n^2 the subject CA✓ answer	(3)
			[8]

QUESTION 3

3.1	$4n - 1 = 671$ $4n = 672$ $n = 168$	CA✓equating n^{th} term to 671 ✓answer	(2)
3.2	Pattern : 3 ; 15 ; 27 ; 39 ; ... ; 663 $12n - 9 = 663$ $n = 56$ $S_{56} = \dfrac{56}{2}[3 + 663] = 18648$ $S_{168} = \dfrac{168}{2}[3 + 671] = 56616$ Terms not divisible by 3 $= 56616 - 18648 = 37968$	A✓ setting up and equating A✓ equating n^{th} term to 663 CA✓n – value CA✓18648 CA✓56616 CA✓answer	(6)
			[8]

QUESTION 4

4.1	$\frac{1}{2}r^4 = 40{,}5$ $r^4 = 81 = 3^4$ $r = 3$	A✓$\frac{1}{2}r^4 = 40{,}5$ CA✓$r^4 = 3^4$ CA✓answer	(3)
4.2	$\frac{1}{2}(3)^{n-1} = \frac{59049}{2}$ $(3)^{n-1} = 59040$ $3^{n-1} = 3^{10}$ $n = 11$	A✓equating nth term to $\frac{59049}{2}$ CA✓exponential form CA✓answer	(3)
4.3	AS: 8 ; (8+d) ; (8+2d) ; … GS: 8 ; 8r ; 8r^2; …. $T_5 = 8r^4 = 2048$ $r^4 = 256 = 4^4$ $r = 4$ Sum = 8+12+16 = 36	A✓forming both sequences A✓$8r^4 = 2048$ CA✓$r^4 = 256$ CA✓r – value CA✓answer	(5)
			[11]

QUESTION 5

5.1	$x = -2$ and $y = -1$	AA✓✓both asymptote equations	(2)
5.2	$y = \dfrac{a}{x+p} + q$ $y = \dfrac{a}{x+2} - 1$ $(-5; -3):$ $-3 = \dfrac{a}{-5+2} - 1$ $a = 6$	CA✓substitution of p and q values CA✓substitution of point (-5 ; -3) CA✓a – value	(3)

5.3	$y = \dfrac{6}{x+2} - 1 = 0$ $\dfrac{6}{x+2} = 1$ $x = 4$	A✓Equating to 0 CA✓transposing 1 to RHS CA✓answer	
			(3)
5.4	$g(x) = -\left(\dfrac{6}{x+2} - 1\right) + 2$ $g(x) = -\dfrac{6}{x+2} + 3$	CACA✓✓answer	(2)
			[10]
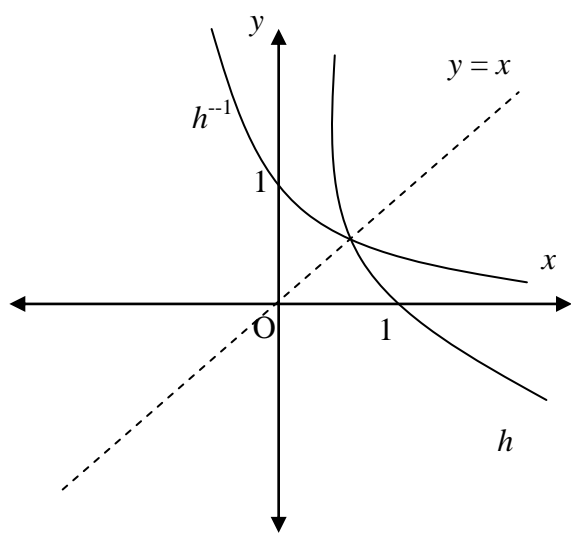
QUESTION 6

6.1	$(2\ ;\ 4)$	A✓answer	(1)
6.2	$y \le 4$ **OR** $y \in (-\infty; 4]$	CA✓answer **OR** CA✓answer	(1) (1)
6.3	$S(0\,;1)$ $1 = a(0-2)^2 + 4$ $-3 = 4a$ $a = -\dfrac{3}{4}$	A✓coordinates of S A✓substitution of the point S A✓$-3 = 4a$	(3)
6.4	$f(x) = -\dfrac{3}{4}(x-2)^2 + 4$ $f(2x) = -\dfrac{3}{4}(2x-2)^2 + 4$ $f(2x) = -3(x-1)^2 + 4$ T.P. $(1\,;4)$	 A✓substitution of $2x$ CA✓$f(x) = -3(x-1)^2 + 4$ CA✓x – value CA✓y – value	(4)
6.5	$y = -\dfrac{1}{2}x + 1$	CA✓gradient CA✓y - intercept	(2)

6.6	$L = \left(-\dfrac{3}{4}(x-2)^2 + 4\right) - \left(-\dfrac{1}{2}x + 1\right)$	A✓ subtraction of equations of both graphs	
	$L = -\dfrac{3}{4}x^2 + \dfrac{7}{2}x$	A✓ simplified expression for length	
	$L' = -\dfrac{3}{2}x + \dfrac{7}{2} = 0$	CA✓ derivative and equal to 0	
	$-3x = 7$		
	$x = \dfrac{7}{3}$	CA✓ answer	(4)
			[15]

QUESTION 7

7.1	$x > 0$	A✓ answer	(1)
7.2	$y = \left(\dfrac{1}{4}\right)^x$	AA✓✓ answer	(2)
7.3	$-\log_4 x \geq -2$ $\log_4 x \leq 2$ $x \leq 16$ $0 < x \leq 16$	A✓ change of inequality when multiplying by − 1 CA✓ value of 16 CA✓ answer	(3)
7.4		Exponential: A✓ shape A✓ y – intercept Log graph: A✓ shape A✓ x – intercept	(4)
			[10]

QUESTION 8

8.1	$A = P(1 + i)^n$ $$800\,000 = P\left(1 + \frac{9{,}45\,\%}{12}\right)^{60}$$ $$P = \frac{800\,000}{\left(1 + \frac{9{,}45\,\%}{12}\right)^{60}}$$ $P = R499\,677{,}30$	A✓ substitution into formula CA✓ making P the subject CA✓ answer	(3)
8.2.1	$A = P(1 - i)^n$ $A = 700\,000(1 - 6\%)^5$ R513 732,82	A✓ formula A✓ substitution of P and i CA✓ answer	(3)
8.2.2	$A = P(1 + i)^n$ $A = 700\,000(1 + 8\%)^5$ R1 028 529,65	A✓ formula A✓ substitution of P and i CA✓ answer	(3)
8.2.3	$F = P(1 + i)^n$ $$514\,796{,}83 = P\left(1 + \frac{9\%}{12}\right)^{60}$$ $P = R328\,800{,}58$	A✓ F – value and n value A✓ i – value CA✓ answer	(3)
			[12]

QUESTION 9 (penalize 1 mark once for incorrect notation in this question)

9.1	$f'(x) = \lim\limits_{n \to 0} \dfrac{f(x+h) - f(x)}{h}$	A✓ formula	
	$f'(x) = \lim\limits_{n \to 0} \dfrac{(x+h)^2 - 8(x+h) - (x^2 - 8x)}{h}$	A✓ substitution	
	$f'(x) = \lim\limits_{n \to 0} \dfrac{x^2 + 2xh + h^2 - 8x - 8h - x^2 + 8x}{h}$	CA✓ simplification of numerator	
	$f'(x) = \lim\limits_{n \to 0} \dfrac{2xh + h^2 - 8h}{h}$		
	$f'(x) = \lim\limits_{n \to 0} \dfrac{h(2x + h - 8)}{h}$	CA✓ factorization	
	$f'(x) = 2x - 8$	CA✓ answer	(5)
	OR $f(x+h) = (x+h)^2 - 8(x+h)$	**OR**	
	$\qquad = x^2 + 2xh + h^2 - 8x - 8h$	A✓ $f(x+h)$ value	
	$f(x+h) - f(x) = 2xh + h^2 - 8h$	CA✓ $f(x+h) - f(x)$ value	
	$\dfrac{f(x+h) - f(x)}{h} = \dfrac{h(2x + h - 8)}{h} = (2x + h - 8)$		
	$f'(x) = \lim\limits_{n \to 0}(2x + h - 8)$	CA✓ $\dfrac{f(x+h)}{h}$ value	
	$f'(x) = 2x - 8$	A✓ formula	
		CA✓ answer	(5)
9.2.1	$g(x) = \left(5 - \sqrt{x}\right)^2$		
	$g(x) = 25 - 10x^{\frac{1}{2}} + x$	A✓ squaring	
	$g'(x) = -5x^{-\frac{1}{2}} + 1$	CACA✓✓ derivatives **OR**	(3)
	OR		
	Chain Rule:		
	$g(x) = \left(5 - \sqrt{x}\right)^2$	A✓ 2	
		A✓ $\left(5 - \sqrt{x}\right)$	
	$g'(x) = 2\left(5 - \sqrt{x}\right).-\dfrac{1}{2}x^{-\frac{1}{2}}$	CA✓ $-\dfrac{1}{2}x^{-\frac{1}{2}}$	
			(3)

9.2.2	$D_x\left[\dfrac{x^2-4x}{\sqrt[3]{x}}\right]$		
	$= D_x\left[x^{\frac{5}{3}} - 4x^{\frac{2}{3}}\right]$	AA✓✓writing in exponential form	
	$= \dfrac{5}{3}x^{\frac{2}{3}} - \dfrac{8}{3}x^{-\frac{1}{3}}$	CACA✓✓each derivative	(4)
			[12]

QUESTION 10

10.1	$q = 3$	A✓answer	(1)
10.2	$f(x) = px^3 - 5x^2 - 4x + q$	A✓substitution of $q = 3$	
	$0 = p(-1)^3 - 5(-1)^2 - 4(-1) + 3$	A✓substitution of $(-1 ; 0)$	
	$0 = -p - 5 + 4 + 3$	A✓simplification	
	$p = -5 + 4 + 3 = 2$		(3)
10.3	$f(x) = 2x^3 - 5x^2 - 4x + 3$		
	$f'(x) = 6x^2 - 10x - 4 = 0$	CA✓derivative and equal to 0	
	$3x^2 - 5x - 2 = 0$		
	$(3x + 1)(x - 2) = 0$	CA✓substitution into formula	
	$x = -\dfrac{1}{3}$ $\quad$ or $\quad$ $x = 2$	CA✓ $x-$ values	
	$y = \dfrac{100}{27}$ or $3,7$ $\quad$ or $y = -9$	CA✓ $y-$ values	(4)
10.4	$-9 < k < 3$	CA✓end points A✓interval	(2)
10.5	$f''(x) = 12x - 10 > 0$ $x > \dfrac{5}{6}$	CA✓derivative and greater than 0 CA✓answer	(2)
10.6	$D'\left(\dfrac{1}{3} ; 3,7\right)$	CA✓ $x-$ value CA✓ $y-$ value	(2)
10.7	$f'(x) = 6x^2 - 10x - 4 = -8$	A✓derivative equal to -8	
	$3x^2 - 5x + 2 = 0$ $(3x - 2)(x - 1) = 0$ $x = \dfrac{2}{3}$ $\quad$ or $\quad$ $x = 1$	CA✓factors CACA✓✓each x- value	(4)
			[18]

QUESTION 11

11.1	$r^2 + \square^2 = 12^2$ $r = \sqrt{144 - \square^2}$	A✓T.O.P CA✓answer	(2)
11.2	$V = \dfrac{1}{3}\pi r^2 h$ $V = \dfrac{1}{3}\pi (144 - h^2) h$ $V = 48\pi h - \dfrac{1}{3}\pi h^3$	A✓formula CA✓expression in h	(2)
11.3	$V = 48\pi h - \dfrac{1}{3}\pi h^3$ $V' = 48\pi - \pi h^2 = 0$ $48 - h^2 = 0$ $\square = \sqrt{48} = 4\sqrt{3} = 6{,}93 \ cm$	CA✓derivative A✓derivative equal to 0 CA✓answer	(3)
11.4	$V = 48\pi h - \dfrac{1}{3}\pi h^3$ $V = 48\pi (6{,}93) - \dfrac{1}{3}\pi (6{,}93)^2$ $V = \ 696{,}5 \ cm^3$	CA✓substitution of 6,93 or $4\sqrt{3}$ CA✓answer	(2)
			[9]

QUESTION 12

S = 240

12.1		AAA✓✓✓ each set ✓26	(4)
12.2	$122 + 36 + 70 - x + 26 = 240$ $254 - x = 240$ $x = 14$	✓equation ✓simplifying ✓answer	(3)
12.3			
a)	$\dfrac{26}{240} = 0.1083 = 10.83\%$	✓answer	(2)
b)	$\dfrac{144}{240} = 0.6 = 60\%$	✓answer	(2)
c)	$\dfrac{46}{240} = 0.1917 = 19.17\%$	✓answer	(1)
			[12]

Total Marks : 150