KWAZULU-NATAL PROVINCE

EDUCATION
REPUBLIC OF SOUTH AFRICA

NATIONAL SENIOR CERTIFICATE

GRADE 12

MATHEMATICS

COMMON TEST

APRIL 2021

MARKS: 100

TIME: 2 hours

N.B. This question paper consists of 6 pages, an answer sheet, 1 diagram sheet and an information sheet.

INSTRUCTIONS AND INFORMATION

Read the following instructions carefully before answering the questions.

1. This question paper consists of 7 questions.

2. Answer **ALL** questions.

3. Clearly show **ALL** calculations, diagrams, graphs, et cetera that you have used in determining your answers.

4. Answers only will not necessarily be awarded full marks.

5. An approved scientific calculator (non-programmable and non-graphical) may be used, unless stated otherwise.

6. If necessary, answers should be rounded off to TWO decimal places, unless stated otherwise.

7. Diagrams are NOT necessarily drawn to scale.

8. Number the answers correctly according to the numbering system used in this question paper. Write neatly and legibly.

QUESTION 1

Given the quadratic sequence: $44;\ 52;\ 64;\ 80;\ \ldots$

1.1 Write down the next two terms of the sequence. (2)

1.2 Determine the n^{th} term of the quadratic sequence. (4)

1.3 Calculate the 30^{th} term of the sequence. (2)

1.4 Prove that the quadratic sequence will always have even terms. (3)

[11]

QUESTION 2

The 8^{th} term of an arithmetic sequence is 31 and the sum of the first 30 terms is 1830.
Determine the first three terms of the sequence. **[7]**

QUESTION 3

3.1 The second term of a geometric sequence $\frac{5}{128}$ and the ninth term is 5.
Determine the value of the common ratio. (5)

3.2 Calculate the value of m if

$$\sum_{k=1}^{m}(-8).(0.5)^{k-1} = -\frac{255}{16}$$

(4)

3.3 Given: $\frac{24}{x} + 12 + 6x + 3x^2 + \cdots\ ;\ x \neq 0.$

3.3.1 Determine the value of x for which the series converges. (3)

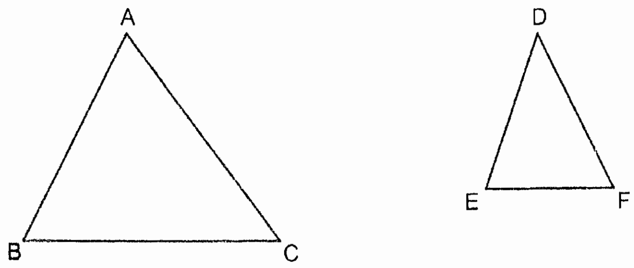
3.3.2 Write down the value of x for which the series is increasing. (2)

[14]

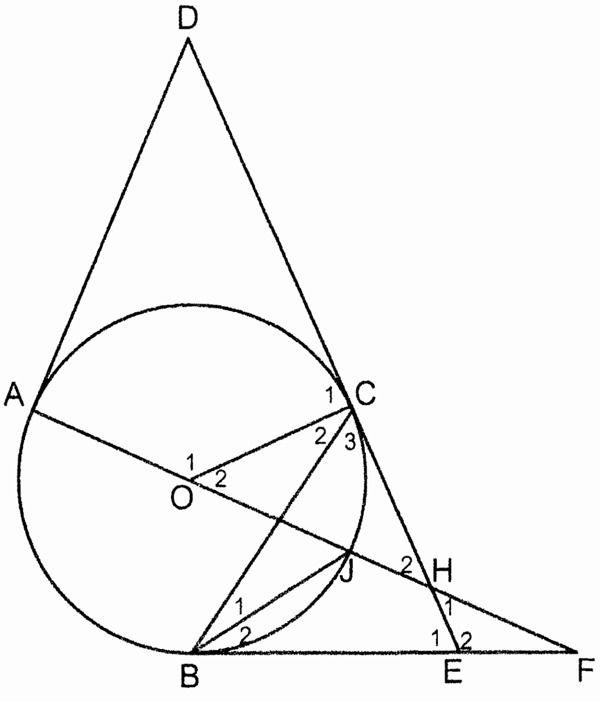
QUESTION 4

4.1 Given ΔABC and ΔDEF with $\hat{A} = \hat{D}, \hat{B} = \hat{E}$ and $\hat{C} = \hat{F}$.

Prove that $\frac{AB}{DE} = \frac{AC}{DF}$ (7)



4.2 In the figure AD, DC and BE are tangents to the circle at A, C and B respectively. O is the centre of the circle. DE and AF intersect at H. AH produced meets BE produced in F. AJ, BC and BJ are chords.



Prove that:

4.2.1 ΔDAH ||| ΔOCH. (4)

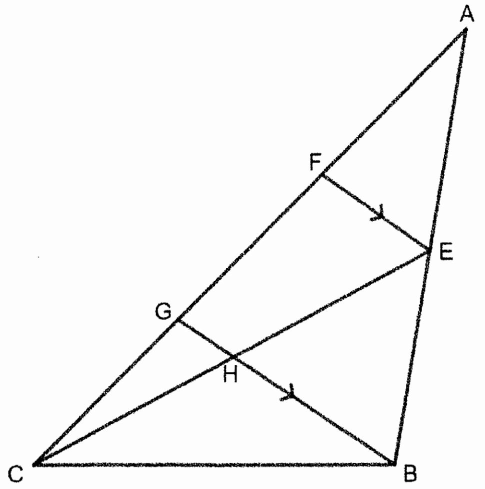
4.2.2 $OH = \frac{AO.DH}{DC}$ (6)

4.2.3 If BA is drawn, then $BF^2 = JF \cdot AF$ (6)

[23]

QUESTION 5

In the figure AF = 2CG and FE || GB. $\frac{AE}{AB} = \frac{2}{5}$.



Determine (with reasons):

5.1 $\frac{AF}{FG}$ (2)

5.2 $\frac{CH}{HE}$ (4)

5.3 $\frac{Area\ of\ \Delta BCG}{Area\ of\ \Delta AFE}$ (4)

[10]

QUESTION 6

6.1 Given $\cos 26^\circ = \frac{1}{p}$

Without using a calculator, calculate the value of the following in terms of *p*.

6.1.1 $\cos 52^\circ$ (4)

6.1.2 $\sin 71^\circ$ (4)

6.2 Simplify without using into a single trigonometric ratio.

$$\frac{\cos(-180^\circ).\tan\theta.\cos 690^\circ.\sin(\theta-180^\circ)}{\cos^2(\theta-90^\circ)}$$ (5)

6.3 Show that

$\cos 0^\circ + \cos 1^\circ + \cos 2^\circ + \cdots + \cos 178^\circ + \cos 179^\circ + \cos 180^\circ + 6\sin 90^\circ = 6$ (4)

[17]

QUESTION 7

7.1 Prove the following identity:

$$\frac{1-\sin 2x}{\sin x-\cos x}=\sin x-\cos x \qquad (3)$$

7.2 Determine the general solution of:

$$\tan 3x \, . \, \frac{1}{\tan 24^{\circ}}-1=0 \qquad (5)$$

7.3 Determine the maximum value of $\sqrt{3}\sin x+\cos x$, without the use of a calculator. (4)

7.4 Given: $f(x)=2\cos(x-30^{\circ})$

7.3.1 Sketch the graph of f for the domain $x\in[-90^{\circ};270^{\circ}]$ on the axes provided. (2)

7.3.2 Use the letters P and Q to indicate on the graph the solution of the equation $\cos(x-30^{\circ})=0{,}5$ and the x – coordinates of P and Q. (4)

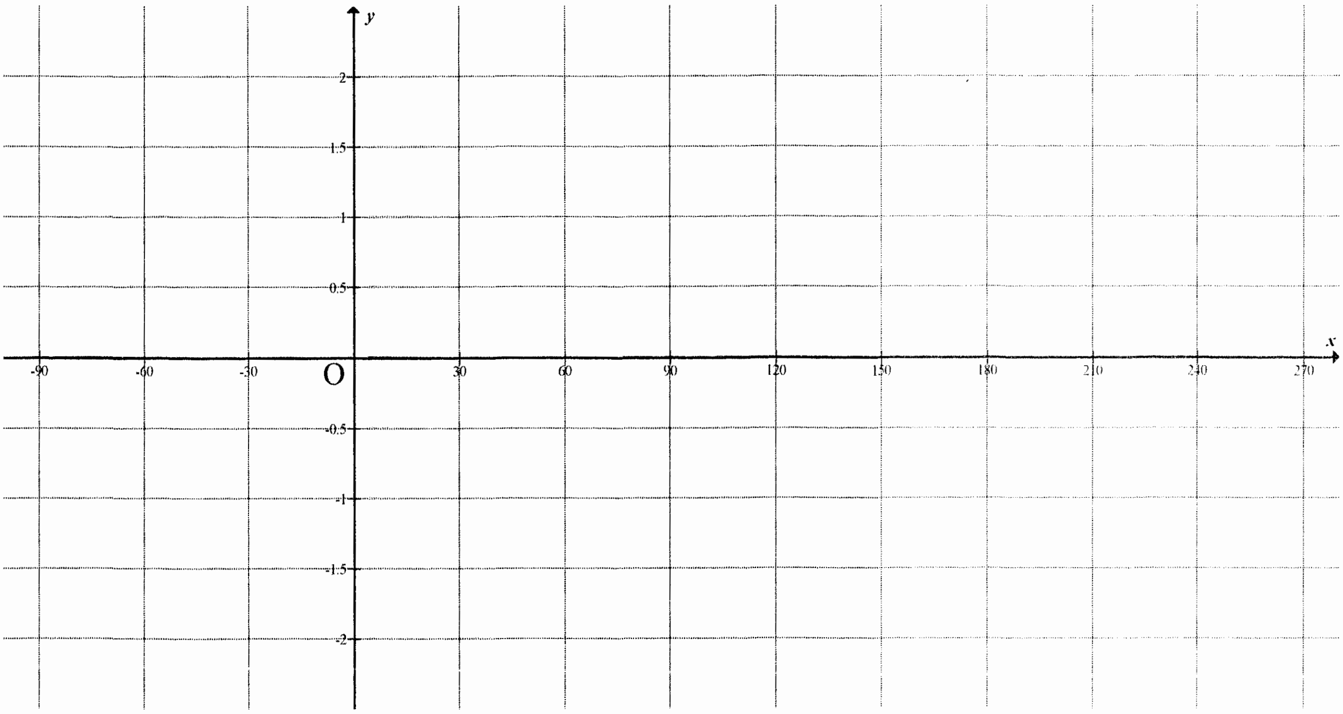
[18]

TEAR-OFF PAGE

NAME: ______________________ **GRADE:** __________

ANSWER SHEET

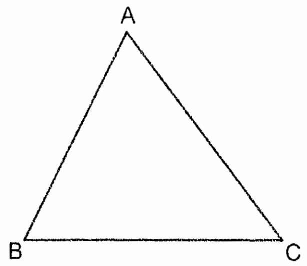
Question 7.3.1

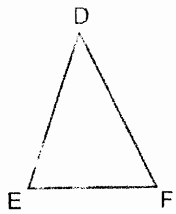


NAME: ______________________________ **GRADE:** __________

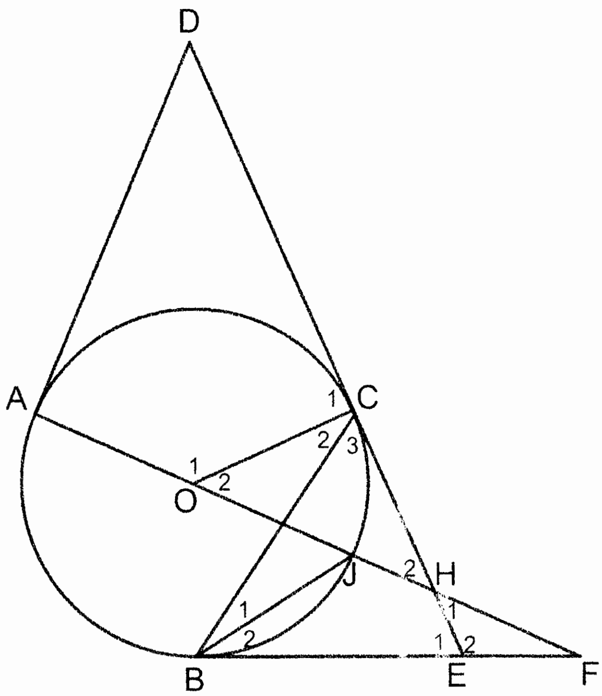
DIAGRAM SHEET

QUESTION 4.1

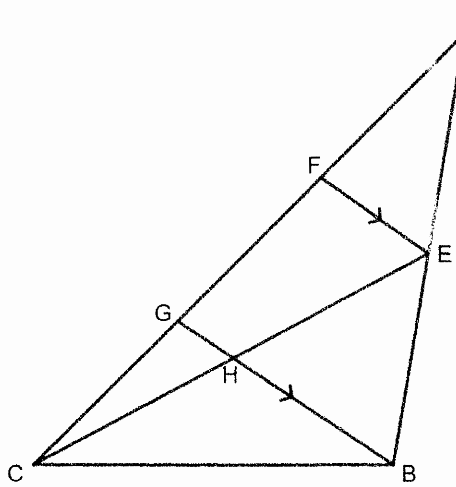




QUESTION 4.2



QUESTION 5



INFORMATION SHEET: MATHEMATICS
INLIGTING BLADSY

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$A = P(1+ni)$ $A = P(1-ni)$ $A = P(1-i)^n$ $A = P(1+i)^n$

$T_n = a + (n-1)d$ $S_n = \frac{n}{2}(2a + (n-1)d)$

$T_n = ar^{n-1}$ $S_n = \frac{a(r^n - 1)}{r-1}$; $r \neq 1$ $S_\infty = \frac{a}{1-r}$; $-1 < r < 1$

$F = \frac{x[(1+i)^n - 1]}{i}$ $P = \frac{x[1-(1+i)^{-n}]}{i}$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $M\left(\frac{x_1 + x_2}{2}; \frac{y_1 + y_2}{2}\right)$

$y = mx + c$ $y - y_1 = m(x - x_1)$ $m = \frac{y_2 - y_1}{x_2 - x_1}$ $m = \tan\theta$

$$(x-a)^2 + (y-b)^2 = r^2$$

$In\ \Delta ABC: \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$ $a^2 = b^2 + c^2 - 2bc.\cos A$ $area\ \Delta ABC = \frac{1}{2}ab.\sin C$

$\sin(\alpha + \beta) = \sin\alpha.\cos\beta + \cos\alpha.\sin\beta$ $\sin(\alpha - \beta) = \sin\alpha.\cos\beta - \cos\alpha.\sin\beta$

$\cos(\alpha + \beta) = \cos\alpha.\cos\beta - \sin\alpha.\sin\beta$ $\cos(\alpha - \beta) = \cos\alpha.\cos\beta + \sin\alpha.\sin\beta$

$\cos 2\alpha = \begin{cases} \cos^2\alpha - \sin^2\alpha \\ 1 - 2\sin^2\alpha \\ 2\cos^2\alpha - 1 \end{cases}$ $\sin 2\alpha = 2\sin\alpha.\cos\alpha$

$\bar{x} = \frac{\sum f.x}{n}$ $\sigma^2 = \frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n}$

$P(A) = \frac{n(A)}{n(S)}$ $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$

$\hat{y} = a + bx$ $b = \frac{\sum(x - \bar{x})(y - \bar{y})}{\sum(x - \bar{x})^2}$

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GRADE 12

MATHEMATICS

MARKING GUIDELINE

COMMON TEST

APRIL 2021

MARKS: 100

This memorandum consists of 9 pages.

QUESTION 1

1.1	100 ; 124	AA✓✓answers	(2)
1.2	44 52 64 80 1D 8 12 16 2D 4 4 $\boldsymbol{2a = 4 \quad \therefore a = 2}$ $\boldsymbol{3a + b = 8 \quad \therefore b = 2}$ $\boldsymbol{a + b + c = 44 \quad \therefore c = 40}$ $\boldsymbol{T_n = 2n^2 = 2n + 40}$ **OR** $\boldsymbol{2a = 4 \quad \therefore a = 2}$ $\boldsymbol{3a + b = 8 \quad \therefore b = 2}$ $\boldsymbol{\therefore c = T_0 = 40}$ $\boldsymbol{T_n = 2n^2 + 2n + 40}$ **OR** $\boldsymbol{T_n = T_1 + (n-1)d_1 + (n-1)(n-2)d_2}$ **OR** $\boldsymbol{T_n = \frac{(n-1)}{2}[2a + (n-2)d] + T_1}$	A✓*a* value CA✓*b* value CA✓*c* value CA✓nth term **OR** A✓*a* value CA✓*b* value CA✓*c* value CA✓nth term **OR** **OR**	(4) (4)
1.3	$\boldsymbol{T_{30} = 2(30)^2 + 2(30) + 40 = 1900}$	CA✓substitution CA✓answer	(2)
1.4	$\boldsymbol{T_n = 2n^2 + 2n + 40}$ $\boldsymbol{T_n = 2(n^2 + n + 20)}$ $\boldsymbol{2(n^2 + n + 20)}$ is even for all $\boldsymbol{n \in Z}$	A✓Taking out common factor of 2 A✓Rewriting nth term A✓ is even for all $\boldsymbol{n \in Z}$ **Note: Mark CA provided** $\boldsymbol{T_n}$ **(from1.2) is a factor of 2**	(3)
			[11]

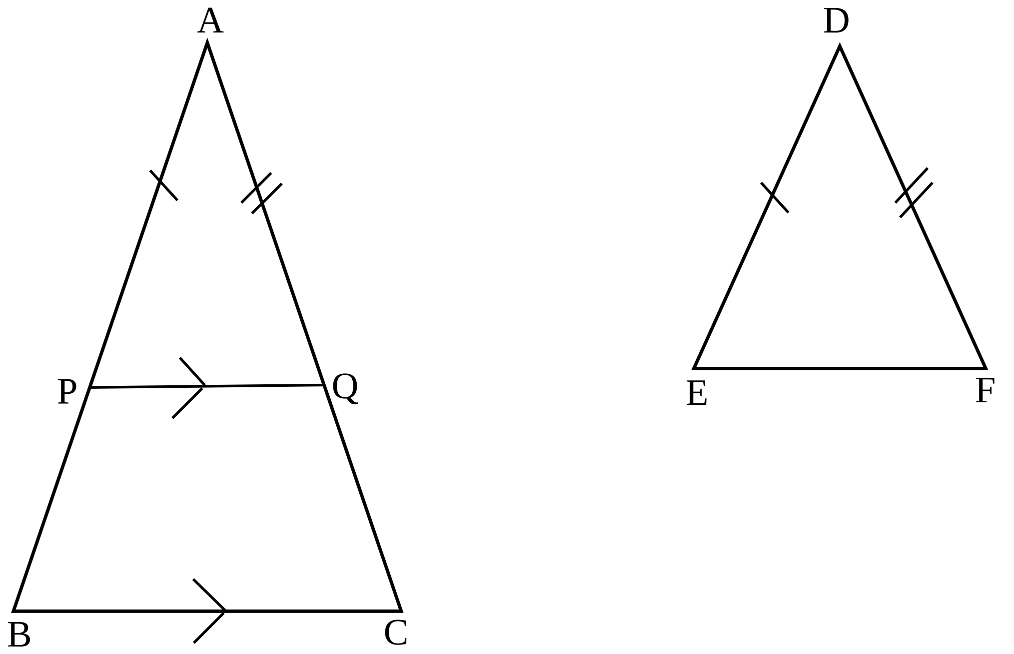
QUESTION 2

2.	$a+7d=31 \quad \rightarrow (1)$ $15(2a+29d)=1830$ $2a+29d=122 \quad \rightarrow (2)$ $a=31-7d \quad \rightarrow (3)$ $2(31-7d)+29d=122$ $62-14d+29d=122$ $15d=60$ $d=4$ $a=3$ $3\,;7\,;\ 11\,;\ \ldots$	A✓equation (1) A✓equation (2) CA✓making *a* the subject CA✓correct substitution of *a* CA✓*d* value CA✓*a* value CA✓sequence	**[7]**

QUESTION 3

3.1	$ar=\frac{5}{128} \quad \rightarrow (1)$ $ar^8=5 \quad \rightarrow (2)$ $r^7=128$ $r^7=2^7$ $r=2$	A✓equation (1) A✓equation (2) CA✓ $r^7=128$ CA✓exponential form CA✓answer	(5)
3.2	$(-8)+(-8)(0.5)+(-8)(0.5)^2+\cdots$ $\frac{-8(0.5^m-1)}{0.5-1}=-\frac{255}{16}$ $0.5^m-1=-\frac{255}{256}$ $0.5^m=\frac{1}{256}=0.5^8$ $m=8$	A✓generating series CA✓correct substitution into correct formula CA✓writing in exponential form or using logs CA✓answer	(4)
3.3.1	$-1<r<1$ $-1<\frac{x}{2}<1$ $-2<x<2$	A✓condition for convergence A✓*r* value CA✓answer	(3)
3.3.2	$x<-2 \ \ or\ x>2$	CACA✓✓answer	(2)
			[14]

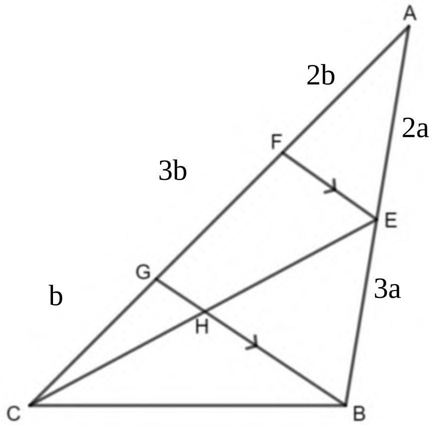
QUESTION 4

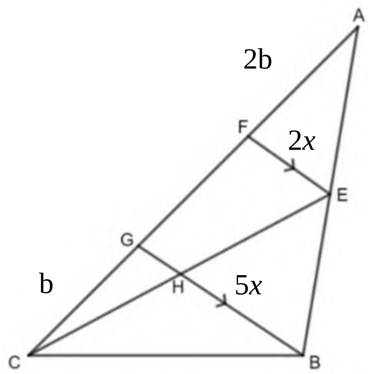


4.1	Draw AP = DE and AQ = DF In $\Delta \mathbf{ABC}$ and $\Delta \mathbf{DEF}$ **1.** AP = DE (Construction) **2.** AQ = DF (Construction) **3.** $\hat{\mathbf{A}} = \hat{\mathbf{D}}$ (Given) $\therefore \Delta \mathbf{APQ} \equiv \Delta \mathbf{DEF}$ (SAS) Now $\mathbf{A\hat{P}Q} = \mathbf{D\hat{E}F}$ But $\mathbf{D\hat{E}F} = \hat{\mathbf{B}}$ (Given) $\therefore \mathbf{A\hat{P}Q} = \hat{\mathbf{B}}$ PQ ∥ BC (Corresponding angles =) $\frac{\mathbf{AB}}{\mathbf{AP}} = \frac{\mathbf{AC}}{\mathbf{AY}}$ (Prop. Thm. PQ∥BC) $\frac{\mathbf{AB}}{\mathbf{DE}} = \frac{\mathbf{AC}}{\mathbf{DF}}$ (Construction AP = DE and AQ = DF)	✓S Construction (or could be shown on diagram) ✓S/R ✓S ✓S ✓S/R ✓S/R ✓R	(7)
4.2.1	In $\Delta \mathbf{DAH}$ and $\Delta \mathbf{OCH}$ 1. $D\hat{A}H = O\hat{C}H = 90°$ (Radius ⊥Tangent) 2. $\hat{\mathbf{H}}_2$ is common 3. $\mathbf{A\hat{D}H} = \mathbf{C\hat{O}}H$ (Remaining angles) $\therefore \Delta \mathbf{DAH}\ \|\|\|\ \Delta \mathbf{OCH}$ (AAA)	✓ S ✓ R ✓S ✓R(AAA)	(4)

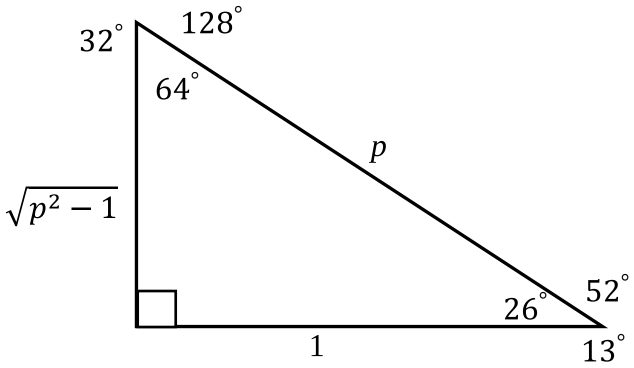
4.2.2	$\frac{DA}{OC}=\frac{DH}{OH}=\frac{AH}{CH}$ (Δ**DAH** ||| Δ**OCH**) $OH=\frac{DH\times OC}{DA}$ DA = DC (Tangents drawn from common point equal) AO = OC (Radii of a circle) Therefore $OH=\frac{AO.DH}{DC}$	✓S/R ✓S ✓S✓R ✓S✓R	(6)
4.2.3	In Δ**ABF and** Δ**BJF** 1. $B\hat{A}F = J\hat{B}F$ (Tangent – Chord Theorem) 2. $\hat{F}$ is common) 3. $A\hat{B}F = B\hat{J}F$ (Remaining angles) ∴ Δ**ABF** ||| Δ**BJF** (AAA) ∴ $\frac{AB}{BJ}=\frac{BF}{JF}=\frac{AF}{BF}$ (Δ**ABF** ||| Δ**BJF**) $BF^2 = JF.AF$	✓S✓R ✓S ✓S✓R ✓S	(6)
			[23]

QUESTION 5



5.1	Let AE = 2a therefore EB = 3a $\frac{\mathbf{AF}}{\mathbf{FG}}=\frac{\mathbf{2}}{\mathbf{3}}$ (Prop. Thm.; FE//GB) or (Line // one side of Δ)	✓S ✓R	(2)
5.2	Let AF = 2b and FG = 3b Then CG = b (Given AF = 2CG) $\frac{\mathbf{CH}}{\mathbf{HE}}=\frac{\mathbf{CG}}{\mathbf{GF}}=\frac{\boldsymbol{b}}{\mathbf{3}\boldsymbol{b}}$ (Prop. Thm.; GH//FE) or (Line // one side of Δ) $\therefore \frac{\mathbf{CH}}{\mathbf{HE}}=\frac{\mathbf{1}}{\mathbf{3}}$	✓S ✓S ✓R ✓S	(4)
5.3	$\frac{\mathbf{AE}}{\mathbf{AB}}=\frac{\mathbf{AF}}{\mathbf{AG}}=\frac{\mathbf{FE}}{\mathbf{GB}}=\frac{\mathbf{2}}{\mathbf{5}}$ …. (Prop. Thm; FE ‖ GB) or (Line // one side of Δ) $\mathbf{C\hat{G}B=G\hat{F}E}$ ….. (Corresp Angles ; FE ‖ GB) Let FE = 2x and GB = 5x Then $\frac{\textbf{Area of }\Delta\textbf{ BCG}}{\textbf{Area of }\Delta\textbf{ AFE}}=\frac{\frac{1}{2}(\boldsymbol{b})(5\boldsymbol{x})\sin \mathbf{C\hat{G}B}}{\frac{1}{2}(2\boldsymbol{b})(2\boldsymbol{x})\sin \mathbf{A\hat{F}E}}$ $=\frac{\frac{1}{2}(\boldsymbol{b})(5\boldsymbol{x})\sin \mathbf{C\hat{G}B}}{\frac{1}{2}(2\boldsymbol{b})(2\boldsymbol{x})\sin(180^\circ-\mathbf{C\hat{G}B})}$ $=\frac{\mathbf{5}}{\mathbf{4}}$  **OR** $\textbf{Area of }\Delta\textbf{ BCG}=\frac{1}{6}\textbf{ Area of }\Delta\textbf{ABC}$ …(Equal Heights) $\textbf{Area of }\Delta\textbf{ AFE}=\frac{1}{3}\textbf{ Area of }\Delta\textbf{AEC}$ …(Equal Heights) $\textbf{Area of }\Delta\textbf{ AEC}=\frac{2}{5}\textbf{ Area of }\Delta\textbf{ABC}$ …(Equal Heights) $\textbf{Area of }\Delta\textbf{ AFE}=\frac{\mathbf{2}}{\mathbf{15}}\textbf{ Area of }\Delta\textbf{ABC}$ $\frac{\textbf{Area of }\Delta\textbf{ BCG}}{\textbf{Area of }\Delta\textbf{ AFE}}=\frac{\mathbf{15}}{\mathbf{12}}=\frac{\mathbf{5}}{\mathbf{4}}$	✓S/R ✓S ✓S ✓S **OR** ✓S/R ✓S/R ✓S ✓S	(4) (4)
			[10]

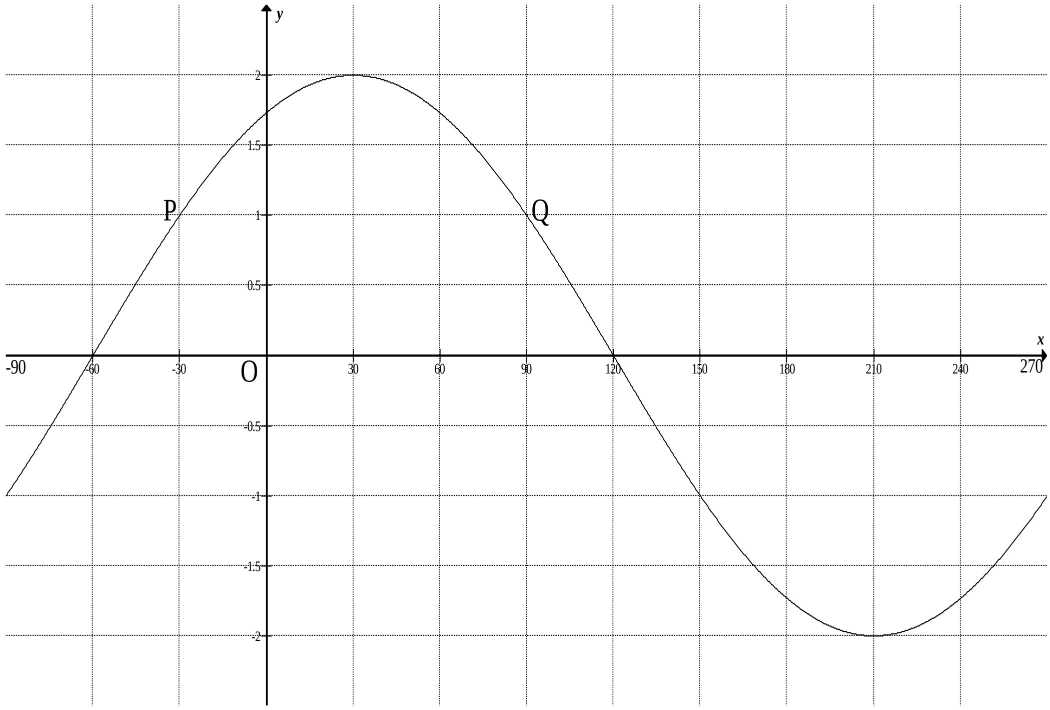
QUESTION 6

6.1.1	 $\cos 52^\circ = \cos[2(26^\circ)]$ $= 2\cos^2 26^\circ - 1$ $= 2(\frac{1}{p})^2 - 1$	A✓diagram A✓writing as double angle A✓expansion CA✓answer	(4)
6.1.2	$\sin 71^\circ = \sin(45^\circ + 26^\circ)$ $= \sin 45^\circ \cos 26^\circ + \cos 45^\circ \sin 26^\circ$ $= \frac{\sqrt{2}}{2} . \frac{1}{p} + \frac{\sqrt{2}}{2} . \frac{\sqrt{p^2-1}}{p}$	A✓$\sin(45^\circ + 26^\circ)$ A✓compound angle expansion CA CA ✓✓each term	(4)
6.2	$\frac{\cos(-180^\circ).\tan\theta.\cos 690^\circ.\sin(\theta - 180^\circ)}{\cos^2(\theta - 90^\circ)}$ $= \frac{\cos(180^\circ) \times \frac{\sin\theta}{\cos\theta}.\cos 30^\circ.(-\sin\theta)}{\sin^2\theta}$ $= \frac{(-1) \times \frac{\sin\theta}{\cos\theta}.\left(\frac{\sqrt{3}}{2}\right)(-\sin\theta)}{\sin^2\theta}$ $= \frac{\frac{\sqrt{3}\sin^2\theta}{2\cos\theta}}{\sin^2\theta}$ $= \frac{\sqrt{3}}{2\cos\theta}$	A✓$\frac{\sin\theta}{\cos\theta}$ A✓$\cos 30^\circ$. A✓$-\sin\theta$ CA✓$\frac{\sqrt{3}}{2}$ or 0,866 CA✓answer	(5)
6.3	LHS = $\cos 0^\circ + \cos 1^\circ + \cos 2^\circ + \cdots + \cos 178^\circ + \cos 179^\circ + \cos 180^\circ + 6\sin 90^\circ$ $= \cos 0^\circ + \cos 1^\circ + \cos 2^\circ + \ldots - \cos 2^\circ - \cos 1^\circ - \cos 0^\circ + 6\sin 90^\circ$ $= 6 =$ RHS **OR** LHS = $(\cos 0^\circ + \cos 180^\circ) + (\cos 1^\circ + \cos 179^\circ) + (\cos 2^\circ + \cos 178^\circ) \ldots\ldots\ldots + 6\sin 90^\circ$ LHS = $(0) + (0) + (0) \ldots\ldots\ldots + 6\sin 90^\circ$ LHS = 6	A✓$-\cos 2^\circ$ A✓$-\cos 1^\circ$ A✓$-\cos 0^\circ$ A✓All terms cancel except 6 A✓$(\cos 0^\circ + \cos 180^\circ)$ A✓$(\cos 1^\circ + \cos 179^\circ)$ A✓$(\cos 2^\circ + \cos 178^\circ)$ A✓All terms cancel except 6	(4)
			[17]

QUESTION 7

7.1	$\frac{1-\sin 2x}{\sin x-\cos x}$ $= \frac{\sin^2 x-2\sin x\cos x+\cos^2 x}{\sin x-\cos x}$ $= \frac{(\sin x-\cos x)^2}{\sin x-\cos x}$ $= \sin x-\cos x$ = RHS	A✓ $\sin^2 x + \cos^2 x = 1$ A✓ $2\sin x\cos x$ A✓ $(\sin x - \cos x)^2$	(3)
7.2	$\tan 3x \cdot \frac{1}{\tan 24^\circ} - 1 = 0$ $\tan 3x = \tan 24^\circ$ $3x = 24^\circ + 180k\ ; k \in Z$ $x = 8^\circ + 60k\ ; k \in Z$	A✓transposing and forming equation A✓$3x = 24^\circ$ A✓$180k$ A✓ $k \in Z$ CA✓ $x = 8^\circ + 60k$ **Note: If calculator is used maximum marks may be allocated**	(5)
7.3	$2\left[\frac{\sqrt{3}}{2}\sin x + \frac{1}{2}\cos x\right]$ $= 2[\sin 60^\circ \sin x + \cos 60^\circ \cos x]$ $= 2[\cos(x - 60^\circ)]$ ∴ Maximum value is 2, since maximum value of $\cos(x - 60^\circ) = 1$ **OR** $2\left[\frac{\sqrt{3}}{2}\sin x + \frac{1}{2}\cos x\right]$ $= 2[\cos 30^\circ \sin x + \sin 30^\circ \cos x]$ $= 2[\sin(x + 30^\circ)]$ ∴ Maximum value is 2, since maximum value of $\sin(x + 30^\circ) = 1$	A✓ $2\left[\frac{\sqrt{3}}{2}\sin x + \frac{1}{2}\cos x\right]$ A✓ $[\sin 60^\circ \sin x + \cos 60^\circ \cos x]$ A✓ $\cos(x - 60^\circ)$ CA✓ answer **OR** A✓ $2\left[\frac{\sqrt{3}}{2}\sin x + \frac{1}{2}\cos x\right]$ A✓ $2[\cos 30^\circ \sin x + \sin 30^\circ \cos x]$ A✓ $\sin(x + 30^\circ)$ CA✓ answer	(4) (4)

7.4.1



		A✓for both *x* – intercepts A✓for both turning points	(2)
7.4.2	$\mathbf{cos}(x-30^\circ)=0,5$ $\mathbf{2cos}(x-30^\circ)=1$ $x-30^\circ=60^\circ$ *or* $x-30^\circ=-60^\circ$ $x=90^\circ$ *at Q* *or* $x=-30^\circ$ *at P*	A✓$x-30^\circ=60^\circ$ and $x-30^\circ=-60^\circ$ CA✓ 90° **and** -30° CACA✓✓for P and Q	(4)
			[18]