education

Department:
Education
PROVINCE OF KWAZULU-NATAL

NATIONAL SENIOR CERTIFICATE

GRADE 12

MATHEMATICS P1

PREPARATORY EXAMINATION

SEPTEMBER 2022

MARKING GUIDELINE

MARKS: **150**
TIME: **3 hours**

NOTE:

- If a candidate answered a QUESTION TWICE, mark only the FIRST attempt.
- If a candidate crossed out an answer and did not redo it, mark the crossed-out answer.
- Consistent accuracy applies to ALL aspects of the marking guidelines.
- Assuming values/answers in order to solve a problem is unacceptable.

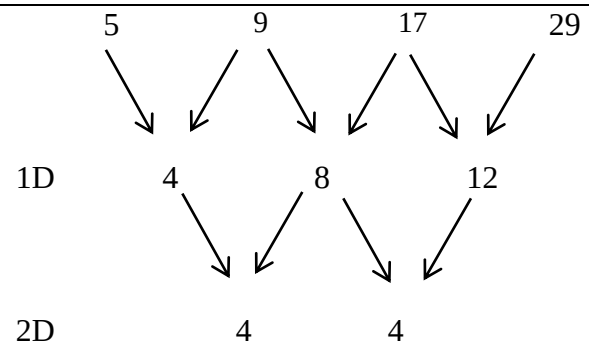
This marking guideline consists of 12 pages.

QUESTION 1

1.1.1	$x = -5 \quad or \quad x = \frac{1}{2}$	A✓ – 5 A✓ $\frac{1}{2}$	(2)
1.1.2	$-3x^2 - 7x + 8 = 0$ $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ $x = \frac{-(-7) \pm \sqrt{(-7)^2 - 4(-3)(8)}}{2(-3)}$ $x = -3{,}17 \quad or \quad 0{,}84$ (Penalize 1 mark if rounding off is incorrect – once here for entire paper)	A✓ standard form CA✓ correct substitution into quadratic formula CA✓CA✓answers	(4)
1.1.3	$\sqrt{x+5} + 1 = x$ $\sqrt{x+5} = x - 1$ $\left(\sqrt{x+5}\right)^2 = (x-1)^2$ $x + 5 = x^2 - 2x + 1$ $x^2 - 3x - 4 = 0$ $(x+1)(x-4) = 0$ $x = -1 \; or \; x = 4$ *n/a*	A✓ isolating surd CA✓ standard form CA✓ factors CA✓ $x \neq -1$ CA✓ $x = 4$	(5)
1.1.4	$-5 \leq x \leq \frac{3}{2}$	A✓ critical value – 5 A✓ critical value $\frac{3}{2}$ CA✓ interval notation	(3)

1.2	$x + 3y = 5 \quad \rightarrow (1)$ $xy + y^2 - 3 = 0 \quad \rightarrow (2)$ From (1): $x = 5 - 3y \quad \rightarrow (3)$ Substituting (3) into (2): $y(5 - 3y) + y^2 - 3 = 0$ $-2\,y^2 + 5y - 3 = 0$ $2\,y^2 - 5y + 3 = 0$ $(2y - 3)(y - 1) = 0$ $y = \frac{3}{2} \ or \ y = 1$ $x = \frac{1}{2} \ or \ x = 2$ **OR** $x + 3y = 5 \quad \rightarrow (1)$ $xy + y^2 - 3 = 0 \quad \rightarrow (2)$ From (1): $y = \frac{5-x}{3} \quad \rightarrow (3)$ Substituting (3) into (2): $x\left(\frac{5-x}{3}\right) + \left(\frac{5-x}{3}\right)^2 - 3 = 0$ $3x(5 - x) + 25 - 10x + x^2 - 27 = 0$ $-2\,x^2 + 5x - 2 = 0$ $2\,x^2 - 5x + 2 = 0$ $(2x - 1)(x - 2) = 0$ $x = \frac{1}{2} \ or \ x = 2$ $y = \frac{3}{2} \ or \ y = 1$	A✓ making *x* the subject CA✓ substitution CA✓ standard form CA✓ factors CA✓ *y* – values CA✓ *x* – values **OR** A✓ making *y* the subject CA✓ substitution CA✓ standard form CA✓ factors CA✓ *x* – values CA✓ *y* – values	(6) (6)
1.3	$\sqrt[n]{\frac{10^n + 2^{n+2}}{5^{2n} + 4.5^n}}$ $= \sqrt[n]{\frac{2^n.5^n + 2^n.2^2}{5^{2n} + 4.5^n}}$ $= \sqrt[n]{\frac{2^n(5^n + 4)}{5^n(5^n + 4)}}$ $= \sqrt[n]{\frac{2^n}{5^n}}$ $= \frac{2}{5}$	A✓ factorising numerator A✓ factorising denominator CA✓ simplifying CA✓ answer	(4)
			[24]

QUESTION 2

2.1	45 ; 65	AA✓✓ answers	(2)
2.2	 $2a = 4 \quad \therefore a = 2$ $3a + b = 4 \quad \therefore b = -2$ $a + b + c = 5 \quad \therefore c = 5$ $T_n = 2n^2 - 2n + 5$	A✓ $2a = 4$ A✓ $a = 2$ A✓ $3a + b = 4$ A✓ $a + b + c = 5$	(4)
2.3	$T_n = 2n^2 - 2n + 5 = 2023$ $2n^2 - 2n - 2018 = 0$ $n^2 - n - 1009 = 0$ $n = \frac{1 \pm \sqrt{1 + 4036}}{2} = 32.27 \ or \ -31.27$ Since *n* is not a Natural Number, 2023 is not a term of the sequence.	A✓ equating n^{th} term to 2023 CA✓ standard form CA✓ *n* – values CA✓ conclusion	(4)
			[10]

QUESTION 3

	26 ; 22 ; 18 ; ... $S_n = \frac{n}{2}[2a + (n-1)d]$ $S_{50} = \frac{50}{2}[2(26) + (50-1)(-4)]$ $S_{50} = -3600$ **OR** $S_n = \frac{n}{2}[a + T_n]$ $S_{50} = \frac{50}{2}[26 + (-170)]$ $S_{50} = -3600$	A✓ *n* – value A✓ *a* – value A✓ *d* – value CA✓ answer **OR** A✓ *n* – value A✓ *a* – value A✓ T_{50} – value CA✓ answer	(4) (4)
			[4]

QUESTION 4

4.1	$S_n = a + ar + ar^2 + ar^3 + \cdots + ar^{n-1} \quad \rightarrow (1)$ $rS_n = ar + ar^2 + ar^3 + \cdots + ar^{n-1} + ar^n \quad \rightarrow (2)$ (2) – (1): $rS_n - S_n = ar^n - a$ $S_n(r-1) = a(r^n - 1)$ $S_n = \frac{a(r^n-1)}{r-1}\ ; r \neq 1$	A✓ equation (1) A✓ equation (2) A✓ subtracting LHS and RHS terms A✓ factorising	(4)
4.2.1	AS: $12 + d\ ; 12 + 2d\ ;$ .... GS: $12r\ ; 12r^2\ ;$	A✓ AS set up of terms A✓ GS set up of terms	(2)
4.2.2	$12 + d = 12r \quad \rightarrow (1)$ $36 + 3d + 3 = 12 + 12r + 12r^2 \quad \rightarrow (2)$ From (1): $d = 12r - 12 \quad \rightarrow (3)$ Substituting (3) into (2), we have $36 + 3(12r - 12) + 3 = 12 + 12r + 12r^2$ $12 + (12r - 12) + 1 = 4 + 4r + 4r^2$ $4r^2 - 8r + 3 = 0$ $(2r - 1)(2r - 3) = 0$ $r = \frac{1}{2}\ or\ r = \frac{3}{2}$	A✓ equation (1) and (2) A✓ making *d* the subject CA✓ standard quadratic form CA✓ factors CA✓ answers	(5)
			[11]

QUESTION 5

5.1	$y = 1 - \frac{1}{x-2}$ $x = 2\ and\ y = 1$	A✓ $x = 2$ A✓ $y = 1$	(2)
5.2	$y - \text{intercept}: \left(0\ ; 1\frac{1}{2}\right)$ $x - \text{intercept}: x = 3$ $(3;\ 0)$	A✓ *y*-intercept A✓ $x = 3$ CA✓ coordinate form	(3)
5.3	$y = x - 1$	A✓ Gradient value A✓ *y* – intercept	(2)
5.4	$y \in R; y \neq 1$ **OR** $y \in (-\infty; 1) \cup (1; \infty)$	A✓ answer **OR** A✓ answer	(1) (1)
			[8]

QUESTION 6

6.1	$0 = -2(6) + d$ $d = 12$	A ✓ substitution of point T(6 ; 0) A✓ answer **Answer only full marks**	(2)
6.2	$c = 12$ $y = a(x + 2)(x - 6)$ $12 = a(0 + 2)(0 - 6)$ $12 = -12a$ $a = -1$ $y = -1(x + 2)(x - 6)$ $f(x) = -x^2 + 4x + 12$ **OR** $x = 2$ …Axis of symmetry $y = a(x - 2)^2 + q$ …..(A) Substituting (0;12) into (A) $12 = a(0 - 2)^2 + q \quad \rightarrow 12 = 4a + q$ ….(1) Substituting (6;0) into (A) $0 = a(6 - 2)^2 + q \quad \rightarrow 0 = 16a + q$ ….(2) (1) – (2) : $12 = -12a$ $-1 = a$ $16 = q$ $y = -1(x - 2)^2 + 16$ $f(x) = -x^2 + 4x + 12$	A✓ subst. x – intercepts A✓ substitution R(0 ; 12) A✓ a – value A✓ a – value and factors **OR** A✓ axis of symmetry value A✓ substitution R(0 ; 12) A✓ a – value A✓ a – value and TP	(4) (4)
6.3	$-2x + 4 = 0$ **or** $x = -\frac{4}{2(-1)}$ **or** $x = \frac{-2+6}{2}$ $x = 2$ $y = -(2)^2 + 4(2) + 12 = 16$ $(2\,;16)$	A✓ Using calculus/formula/midpoint CA✓ x = 2 (provided it is +ve) CA✓ y – value	(3)
6.4.1	$0 \leq x \leq 6$	AA✓✓answer (penalty 1 mark for incorrect notation)	(2)
6.4.2	$-2 < x < 0 \quad or \quad x > 6$	AA✓✓$-2 < x < 0$ A✓ $x > 6$	(3)
6.5	$R'(2\,;-12\,)$	A✓x – value A✓ y – value	(2)
			[17]

QUESTION 7

7.1	$A(0\,;1)$	AA✓✓answer	(2)
7.2	$y = a^x$ $32 = a^5$ $a = 2$	A✓ substitution of point T(5;32) A✓ answer	(2)
7.3	$x \in R$ **OR** $x \in (-\infty; \infty)$	A✓ answer **OR** A✓ answer	(1) (1)
7.4	$y = \log_2 x$	CACA✓✓answer	(2)
7.5	$\log_2 x = 5$ $x = 2^5 = 32$ $0 < x \leq 32$	CA✓ end points A✓ interval **Can be solved by log inequalities**. **Answer Only – Full marks**	(2)
			[9]

QUESTION 8

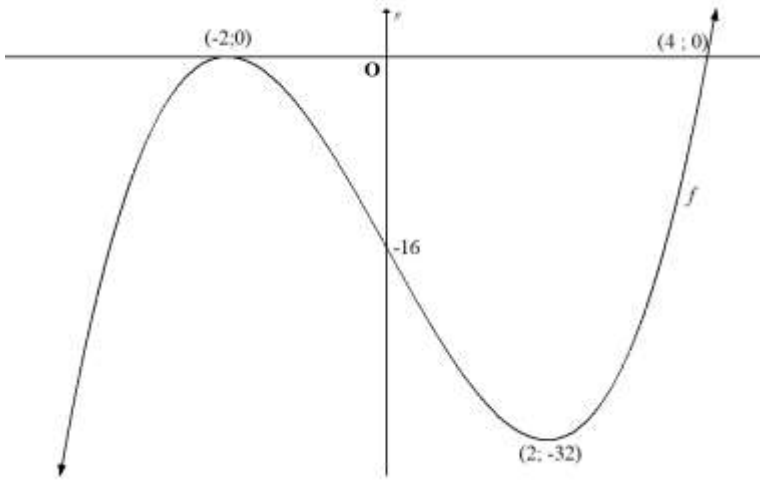
8.1	<table><tr><td>1 Jun 2021</td><td>31 Jul 2021</td><td>31Aug 2021</td><td>…….</td><td>30 Apr 2023</td></tr><tr><td>5000</td><td></td><td></td><td></td><td></td></tr><tr><td>600</td><td>600</td><td>600</td><td>……..</td><td>600</td></tr></table> Dipinda's final amount in the account: $= \mathrm{P}(1+i)^n + \frac{x[(1+i)^n - 1]}{i}$ $= 5000\left(1 + \frac{14.25\%}{12}\right)^{23} + \frac{600\left[\left(1 + \frac{14.25\%}{12}\right)^{23} - 1\right]}{\frac{14.25\%}{12}}$ $= \mathrm{R}22\,321{,}54$	<u>A – formula</u> A✓ value of *n* A✓ value of *i* <u>FV – formula</u> A✓ value of *n* CA✓correct substitution into A CA✓ correct substitution into Fv CA✓ answer	(6)
8.2.1	$\mathrm{P} = \frac{x[1-(1+i)^{-n}]}{i}$ $800\,000 = \frac{10000\left[1 - \left(1 + \frac{13.35\,\%}{12}\right)^{-n}\right]}{\frac{13.35\%}{12}}$ $\left(1 + \frac{13.35\,\%}{12}\right)^{-n} = \frac{11}{100} = 0{,}11$ $-n = \log_{\left(1+\frac{13.35\,\%}{12}\right)} 0.11$ $n = 199{,}5083362$ Therefore the loan will be paid off in 200 months. **N.B. Candidates can also substitute the value of 200 into the Pv formula to show that the loan will be paid in 200 months.**	A✓ value of P, *x* and value of *i* A✓ substitution into formula A✓ use of logs A✓ decimal value	(4)

8.2.2a	$\text{P} = \frac{x[1-(1+i)^{-n}]}{i}$ $= \frac{10\,000\left[1-\left(1+\frac{13.35\,\%}{12}\right)^{-80,5083362}\right]}{\frac{13.35\,\%}{12}}$ $= R530\,009,55$ **If n = 81 is used and P = $R532\,010,58$** **Give a maximum of 2/3 marks** **N.B. Candidates can also use the method of A – Fv**	A✓ value of *n* A✓ value of *i* CA✓ answer	(3)
8.2.2b	$\text{A} = P(1+i)^n$ $\text{A} = R530\,009,55\left(1+\frac{13.35\,\%}{12}\right)^4$ $\text{A} = R\ 553\,991,4839$ $\text{P} = \frac{x[1-(1+i)^{-n}]}{i}$ $553\,991,4839 = \frac{x\left[1-\left(1+\frac{13.35\,\%}{12}\right)^{-77}\right]}{\frac{13.35\,\%}{12}}$ $x = R10\,748,55$	A✓ value of *n* CA✓ answer A✓ value of *n* CA✓ answer	(4)
			[17]

QUESTION 9 (penalize 1 mark once for incorrect notation in this question)

9.1	$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$ $f'(x) = \lim_{h\to 0} \frac{\frac{2}{3(x+h)} - \frac{2}{3x}}{h}$ $f'(x) = \lim_{h\to 0} \frac{2x-2(x+h)}{3x(x+h)} \times \frac{1}{h}$ $f'(x) = \lim_{h\to 0} \frac{2x-2x-2h}{3x(x+h)} \times \frac{1}{h}$ $f'(x) = \frac{-2}{3x^2}$ **OR** $f(x+h) = \frac{2}{3(x+h)}$ $f(x+h) - f(x) = \frac{2}{3(x+h)} - \frac{2}{3x}$ $\frac{f(x+h)-f(x)}{h} = \frac{-2}{3x(x+h)}$ $f'(x) = \lim_{h\to 0} \frac{-2}{3x(x+h)}$ $f'(x) = \frac{-2}{3x^2}$	A✓ formula A✓ substitution CA✓ LCD CA✓ simplification of numerator CA✓ answer **OR** A✓ value of $f(x+h)$ CA✓ value of $f(x+h)-f(x)$ CA✓ value of $\frac{f(x+h)-f(x)}{h}$ A✓ formula CA✓ answer	(5) (5)
9.2.1	$g(x) = (x+7)^3$ $g(x) = x^3 + 21x^2 + 147x + 343$ $g'(x) = 3x^2 + 42x + 147$	AA✓✓ (two terms correct 1 mark, all terms correct 2 marks) CACACA✓✓✓each term	(5)
9.2.2	$y = \sqrt{x^5} - \frac{4}{9x^2}$ $y = x^{\frac{5}{2}} - \frac{4}{9}x^{-2}$ $\frac{dy}{dx} = \frac{5}{2}x^{\frac{3}{2}} + \frac{8}{9}x^{-3}$	AA✓✓ writing in exponential form CACA✓✓ each term	(4)
			[14]

QUESTION 10

10.1.1	$x^3 - 12x - 16 = 0$ $(x+2)(x^2 - 2x - 8) = 0$ $(x+2)(x+2)(x-4) = 0$ $x = -2 \ or \ x = 4$	A✓ binomial factor AA✓✓factors CA CA ✓✓ answers	(5)
10.1.2	$f(x) = x^3 - 12x - 16$ $f'(x) = 3x^2 - 12 = 0$ $x^2 - 4 = 0$ $(x+2)(x-2) = 0$ $x = -2 \ or \ x = 2$ $y = 0 \quad or \quad y = -32$	A✓ derivative and equating to 0 CA✓ factors CA✓ x – values CA✓ y – values	(4)
10.1.3		CA✓ Maximum and Minimum points CA✓ x – intercepts A✓ y – intercept A✓ shape	(4)
10.1.4	$f''(x) = 6x > 0$ $x > 0$ Answer only: full marks	A✓ 2nd derivative A✓ answer	(2)
10.2	$p'(x) = -3x^2 - 8$ $-3x^2 \leq 0$ for all $x \epsilon R$ $-3x^2 - 8 \leq 0$ The gradient of all tangents to the graph of p is always negative.	A✓ derivative A✓ reasoning A✓ reasoning	(3)
			[18]

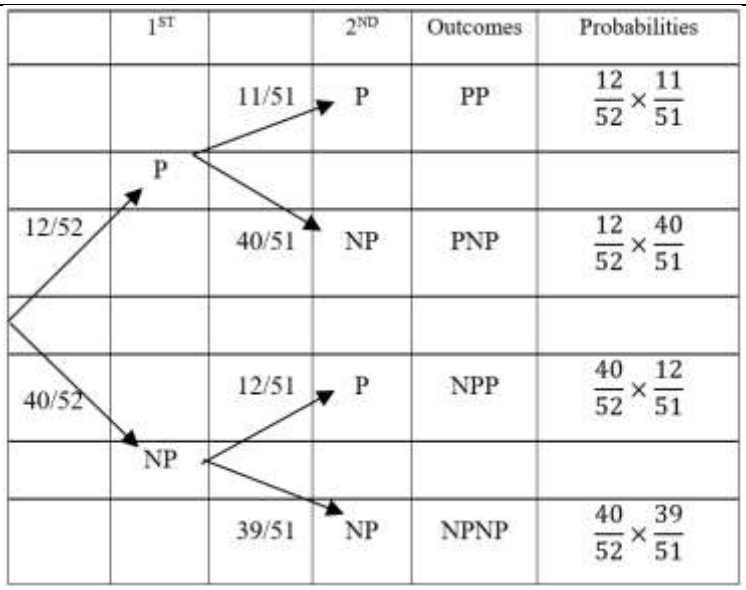
QUESTION 11

11.1	$L = 1000 + 6t - \frac{t^2}{4}$ $\frac{dL}{dt} = 6 - \frac{1}{2}t$	AA✓✓ for each term	(2)
11.2	For greatest lead: $\frac{dL}{dt} = 0$ $6 - \frac{1}{2}t = 0$ $t = 12$ minutes	CA✓ $\frac{dL}{dt} = 0$ or equating derivative to 0 CA✓ answer	(2)
11.3	$\frac{dL}{dt}_{t=60} = 6 - \frac{1}{2}(60)$ $\frac{dL}{dt}_{t=60} = -24$ The runner's lead is decreasing at 24 metres per minute	CA✓ substitution of t = 60 into derivative **and** value of – 24 CA✓ conclusion (provided the derivative is – ve)	(2)
			[6]

QUESTION 12

12.1	7! or 5 040	A✓ A✓ 7! or 5040	(2)
12.2	$6! \times 2!$ = 1440	AA✓✓ $6! \times 2!$ A✓ 1440	(3)
			[5]

QUESTION 13

13.1	P (A or B) = P(A) + P(B) 0,63 = 3P(B)+P(B) 4P(B) = 0,63 P(B) = 0,16	A✓ condition for mutually exclusive events A✓ correct substitution A✓ P(B) value	(3)
13.2.1	$P(\text{Both Picture cards}) = \frac{12}{52} \times \frac{11}{51}$ $= \frac{11}{221} = 0{,}0498 = 4{,}98\ \%$	A✓ $\frac{12}{52} \times \frac{11}{51}$ A ✓ $\frac{11}{221} = 0{,}0498 = 4{,}98\ \%$	(2)
13.2.2	 P (at least 1 picture card) = 1 – P (no picture card) $= 1 - \left(\frac{40}{52} \times \frac{39}{51}\right)$ $= \frac{7}{17} = 0{,}4118 = 41{,}18\ \%$ **OR** P (at least 1 picture) $= \left(\frac{12}{52} \times \frac{11}{51}\right) + \left(\frac{12}{52} \times \frac{40}{51}\right) + \left(\frac{40}{52} \times \frac{12}{51}\right)$ $= \frac{7}{17} = 0{,}4118 = 41{,}18\ \%$	A✓ Method A✓ Correct Substitution A✓ $\frac{7}{17} = 0{,}4118 = 41{,}18\ \%$ **OR** AA✓✓probabilities A✓ $\frac{7}{17} = 0{,}4118 = 41{,}18\ \%$	(3) (3)
			[8]

TOTAL: 150