KWAZULU-NATAL PROVINCE

EDUCATION
REPUBLIC OF SOUTH AFRICA

NATIONAL SENIOR CERTIFICATE

GRADE 12

MATHEMATICS P2

PREPARATORY EXAMINATION

SEPTEMBER 2022

MARKS: **150**

TIME: **3 hours**

This question paper consists of 13 pages and 1 information sheet and 1 answer sheet.

INSTRUCTIONS AND INFORMATION

Read the following instructions carefully before answering the questions.

1. This question paper consists of 11 questions.
2. Read the questions carefully.
3. Answer ALL the questions.
4. Number your answers exactly as the questions are numbered.
5. Clearly show ALL calculations, diagrams, graphs, etc. which you have used in determining your answers.
6. Answers only will NOT necessarily be awarded full marks.
7. You may use an approved scientific calculator (non-programmable and non-graphical), unless stated otherwise.
8. If necessary, round off answers correct to TWO decimal places, unless stated otherwise.
9. Diagrams are NOT necessarily drawn to scale.
10. An information sheet with formulae is included at the end of the question paper.
11. Write neatly and legibly.

QUESTION 1

The weight (in kg) of 20 boys in the soccer squad of school A are given below:

40	47	48	51	53	57	58	58	59	59
60	60	60	60	61	62	63	64	66	69

1.1 Calculate:

1.1.1 the mean weight of the boys in this soccer squad. (2)

1.1.2 the standard deviation of this data. (1)

1.2 Determine the number of boys that have a weight within one standard deviation of the mean. (2)

1.3 The following information was obtained from the coach of the soccer squad of school B:

$$\sum_{n=1}^{22} x_n = 1320$$

1.3.1 How many boys are in the school B squad? (1)

1.3.2 Calculate the mean weight of a boy in the soccer squad of school B. (1)

1.4 Assume that the mean weight of the boys in the soccer squad at school B is 60 kg. Five boys of equal weight are added to the school A squad so that the means of both school squads are the same. Calculate the weight of each of these five boys. (4)

[11]

QUESTION 2

A survey was done on 250 people to determine the distances they travel to work daily.

The results are shown in the table below.

DISTANCE, d (in km)	FREQUENCY	CUMULATIVE FREQUENCY
$0 < d \leq 5$	8	
$5 < d \leq 10$	41	
$10 < d \leq 15$	63	
$15 < d \leq 20$	52	
$20 < d \leq 25$	41	
$25 < d \leq 30$	38	
$30 < d \leq 35$	7	
TOTAL		

2.1 Complete the cumulative frequency column, on the attached ANSWER SHEET. (3)

2.2 Draw a cumulative frequency graph (ogive) for the given data on the grid provided on the attached ANSWER SHEET. (4)

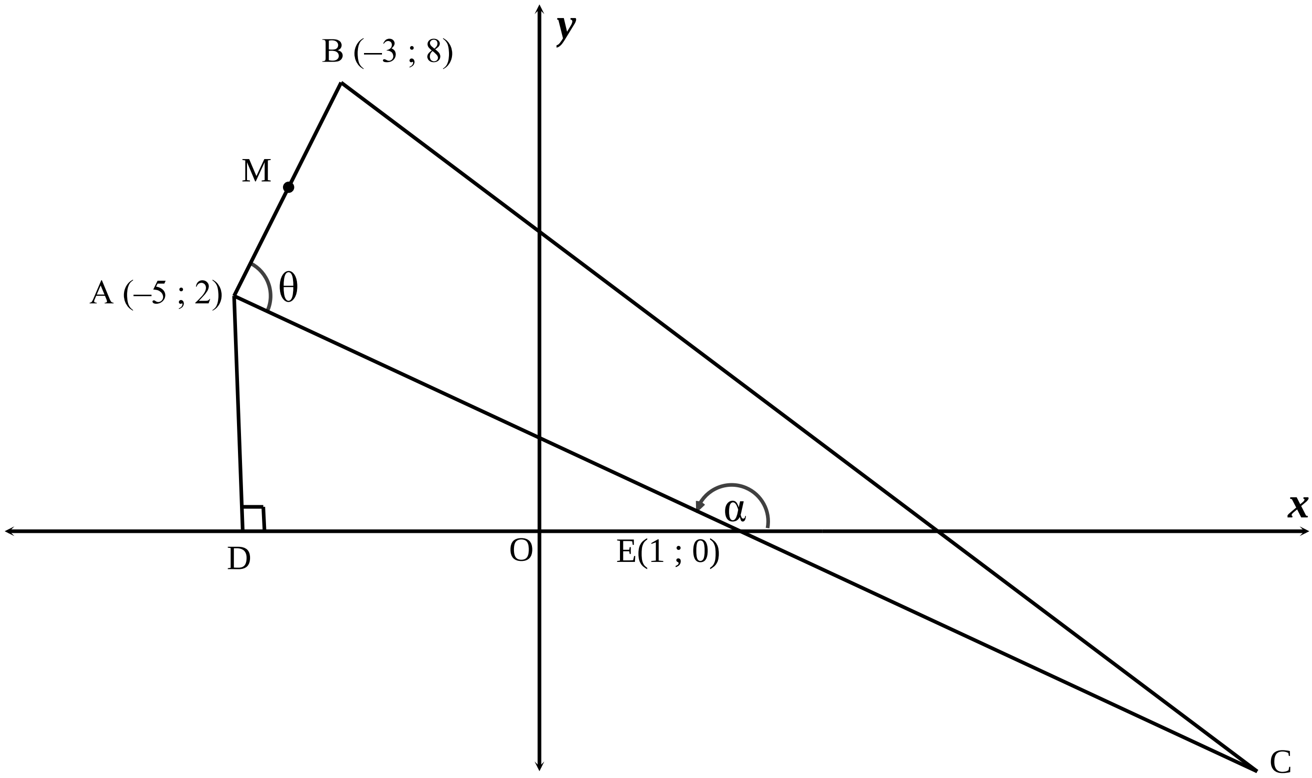
2.3 Use the graph to determine the median distance travelled. Indicate on your graph the median distance. (2)

[9]

QUESTION 3

In the diagram below, A(–5 ; 2) , B(–3 ; 8) and C are vertices of ΔABC.

E(1 ; 0) is the midpoint of AC. D is a point on the x-axis such that AD is a line perpendicular to the x-axis. α is the angle of inclination of AC.

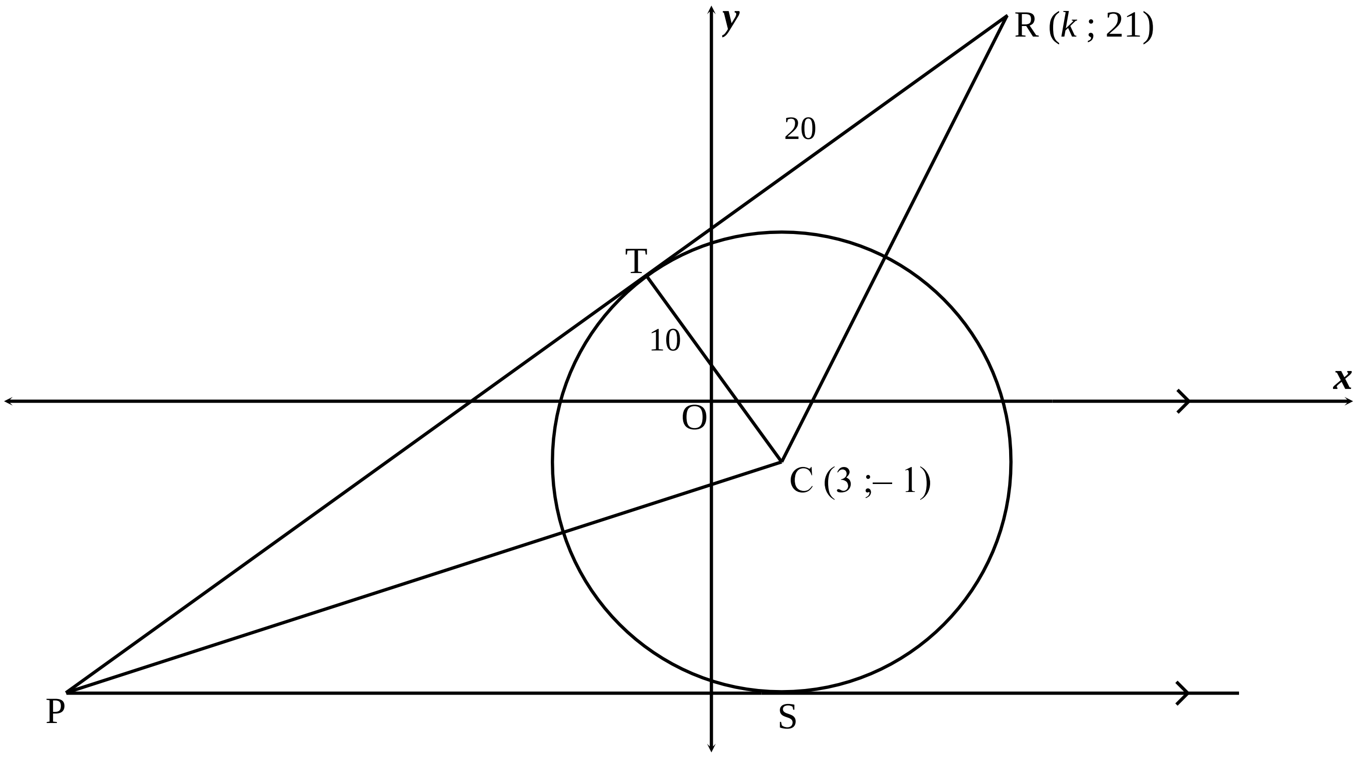


3.1	Determine the coordinates of M , the midpoint of AB.	(2)
3.2	Write down the coordinates of point D.	(1)
3.3	Show that the coordinates of C are (7 ; –2)	(2)
3.4	Calculate the length of line AC. (Leave answer in simplest surd form)	(2)
3.5	Determine the coordinates of F , if F lies in the first quadrant and CABF is a parallelogram.	(2)
3.6	Determine the equation of the perpendicular bisector of AB.	(4)
3.7	Calculate the size of α , the angle of inclination of line AC.	(3)
3.8	Determine the equation of the line parallel to AB passing through E.	(2)
3.9	Calculate the size of angle θ .	(2)
3.10	Calculate the area of ΔABC.	(4)

[24]

QUESTION 4

In the diagram, the circle TS centred at $C(3;-1)$ has a radius CT of 10 units. PTR, where $R(k\,;21)$, is a tangent to the circle at T. PS is a tangent to the circle at S and PS ∥ x-axis. PC, TC and CR are drawn. TR = 20 units.



4.1	Give a reason why $CT \perp TR$.	(1)
4.2	Calculate the value of k, where R is in the first quadrant.	(4)
4.3	Write down the equation of the given circle.	(2)
4.4	Write down the equation of PS.	(1)
4.5	The equation of tangent PTR is $3y = 4x + 35$.	

4.5.1	Calculate the coordinates of P.	(2)
4.5.2	Calculate the length of PT.	(3)

[13]

QUESTION 5

5.1 If $5\cos A = 2\sqrt{6}$ where $A \in [90°;360°]$, calculate, **without using a calculator** and with the aid of a diagram, the values in simplest form of:

5.1.1 $-\sqrt{6}.\tan A$ (4)

5.1.2 $\sin 2A$ (4)

5.2 Given: $\sin 18° = p$
Without using a calculator, determine each of the following in terms of p.

5.2.1 $\cos 18°$ (2)

5.2.2 $\cos 48°$ (5)

5.2.3 $\sin 9°$ (3)
[18]

QUESTION 6

6.1 **Without using a calculator**, simplify the following expression fully:

$$\frac{\sin(180° - x).\tan(x - 180°).\cos(360° + x)}{\sin^2(180° + x) + \sin^2(90° - x)}$$ (6)

6.2 **Without using a calculator**, determine the value of:

$$\frac{\cos 330°.\tan 150°.\sin 12°}{\tan 675°.\cos 258°}$$ (7)

6.3 Given the identity: $\dfrac{\cos\alpha + \cos 2\alpha}{\sin 2\alpha - \sin\alpha} = \dfrac{\cos\alpha + 1}{\sin\alpha}$

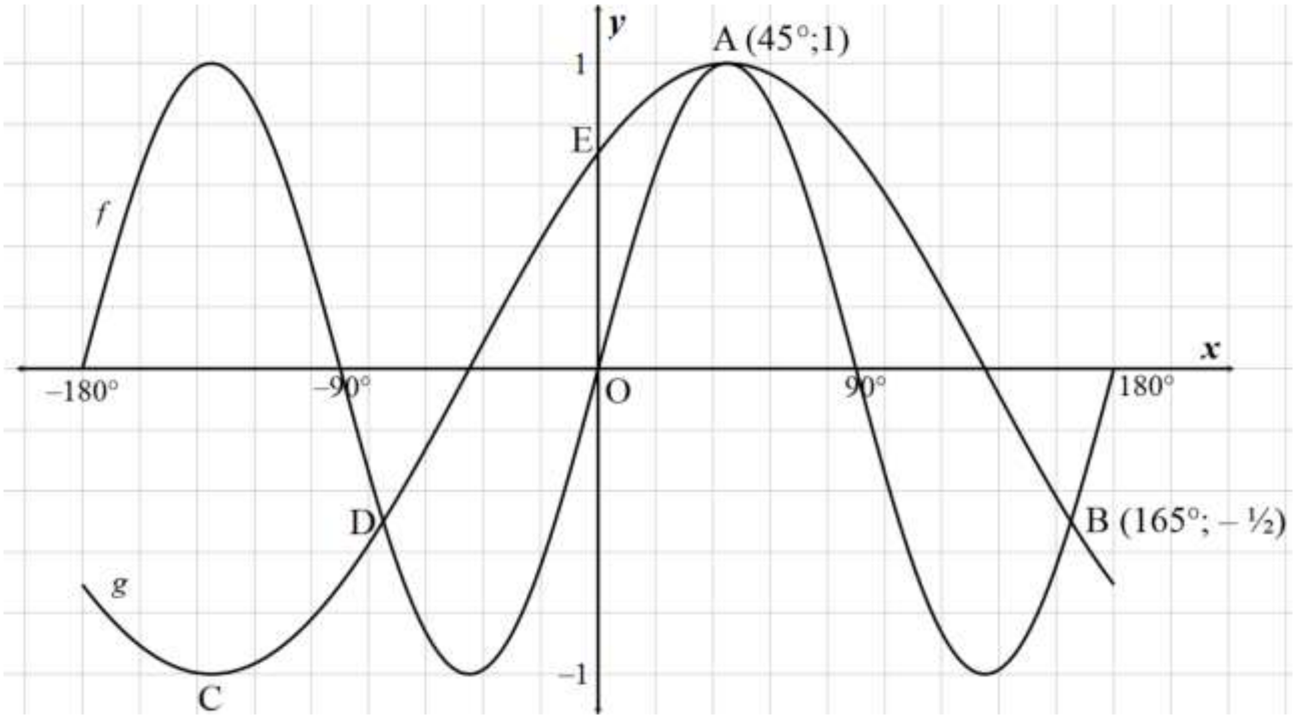
6.3.1 Prove the identity. (4)

6.3.2 For which other values of α is the identity undefined? (5)
[22]

QUESTION 7

Given: $f(x) = \sin 2x$ and $g(x) = \cos(x + a)$ where $x \in [-180^\circ ; 180^\circ]$

The graphs of f and g intersect at B and D. E is the y-intercept of g, and C is a turning point of g. A is a turning point of both f and g.



7.1 Write down the value of a. (1)

7.2 State the period of f. (1)

7.3 Determine the coordinates of C and E. (3)

7.4 Write down the amplitude of h if $h(x) = 3f(x)$. (1)

7.5 Determine for which value(s) of x, if $x \in [0^\circ ; 180^\circ]$, will:

7.5.1 $g(x) > f(x)$ (2)

7.5.2 $g'(x) \cdot f'(x) \geq 0$ (2)

7.6 **Without solving the equation,** use the above graphs to show how you would solve the following equation:

$$\sqrt{2}\sin 2x = \cos x + \sin x$$ (3)

[13]

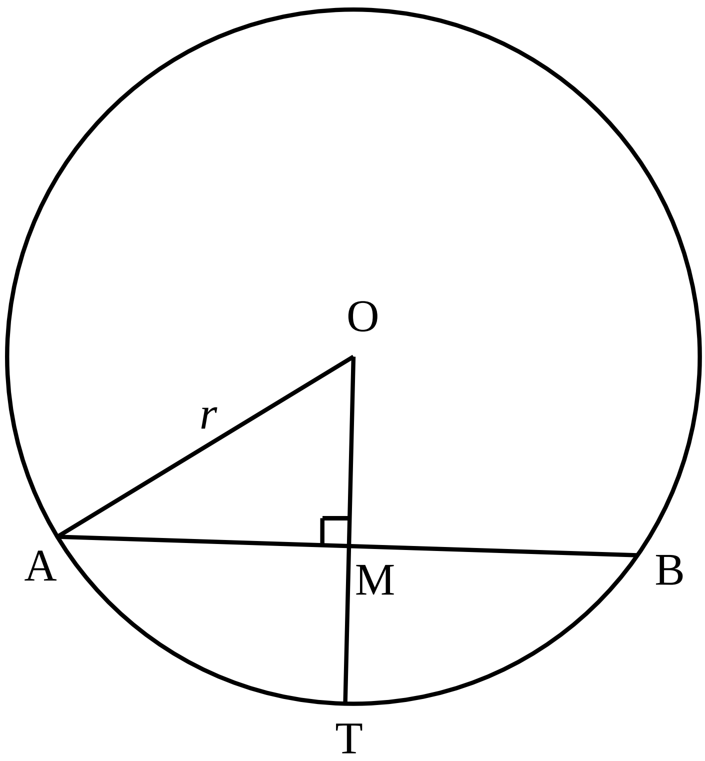
QUESTION 8

8.1 Complete the following statement:

The line drawn from the centre of the circle perpendicular to the chord ……… (1)

8.2 The circle below with centre O has chord AB = 8 cm.

OMT $\perp$ AB with MT = 2 cm. The radius of the circle is r cm.



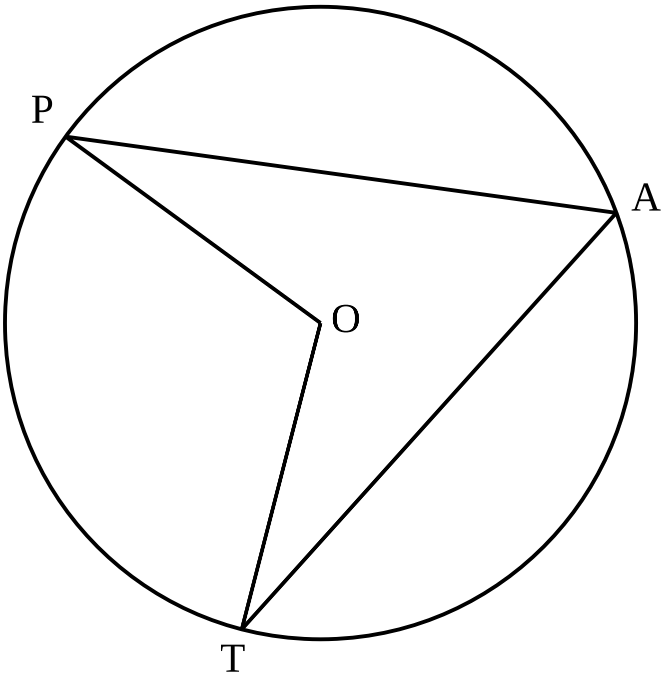
8.2.1 Write down, with a reason, the value of AM. (2)

8.2.2 Calculate the length of the radius of the circle. (4)

[7]

QUESTION 9

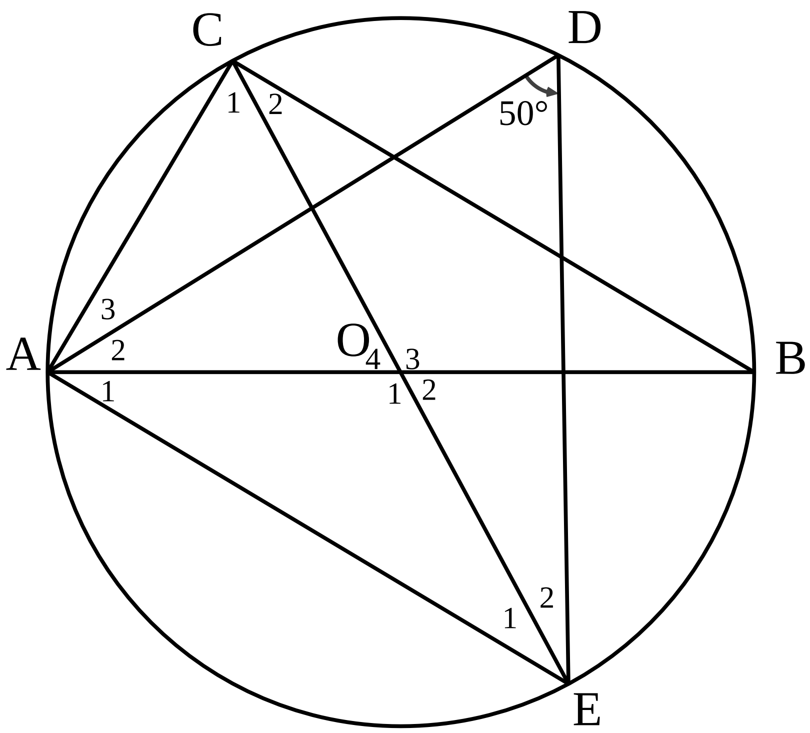
9.1 In the diagram below, O is the centre of the circle. P, A and T are points on the circumference of the circle. PA, TA, PO and TO are drawn.



Prove the theorem which states that $\hat{POT} = 2\hat{PAT}$. (5)

9.2 AOB and COB are diameters of circle ACDBE with centre O.

Chords AC , CB , AE , AD and DE are drawn. $\hat{D} = 50°$.



9.2.1 Calculate, with reasons, the size of the following angles:

(a) $\hat{O}_1$ (2)

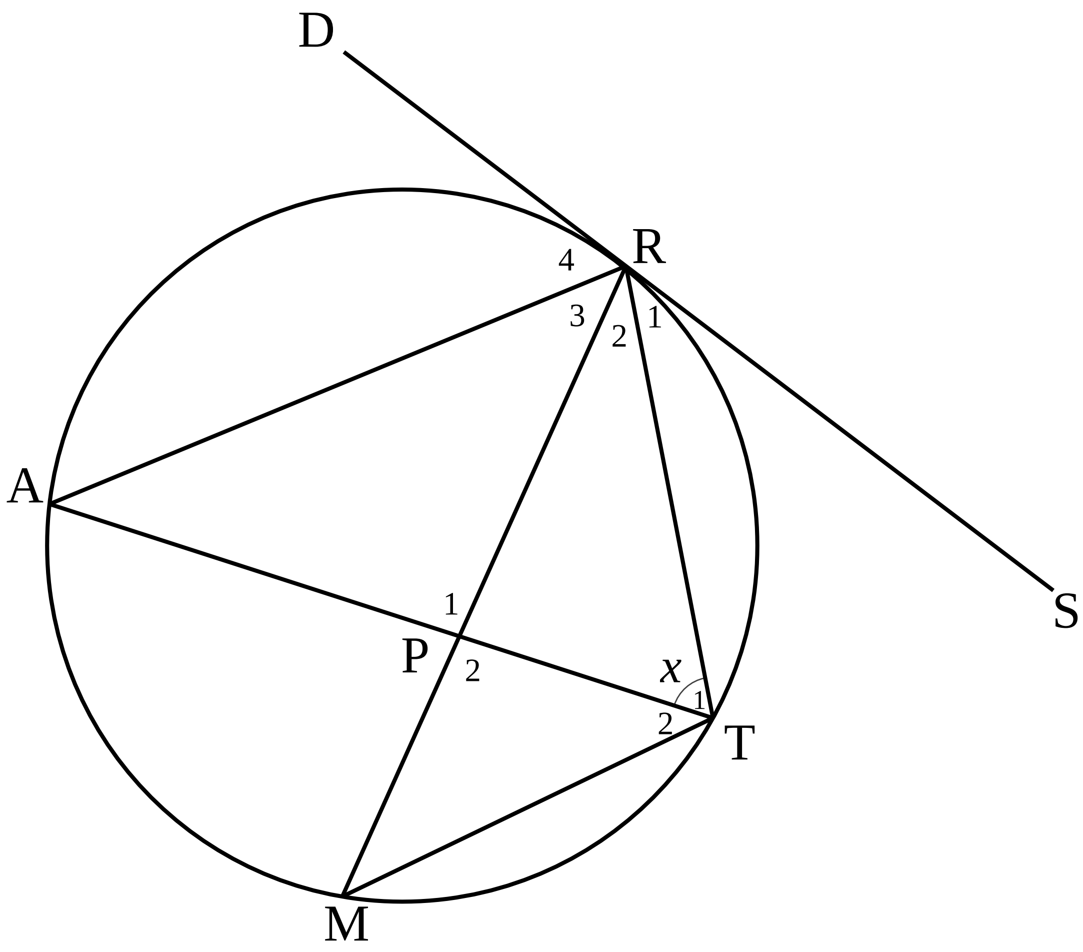
(b) $\hat{E}_1$ (3)

9.2.2 Prove that AE ∥ CB. (4)

[14]

QUESTION 10

In the diagram DRS is a tangent to the circle TMAR at R. AT bisects $M\hat{T}R$.
AT intersects MR at P. AR is drawn. $\hat{T}_1 = x$.



10.1 Prove, giving reasons, that:

10.1.1 $\hat{R}_3 = \hat{R}_4$. (4)

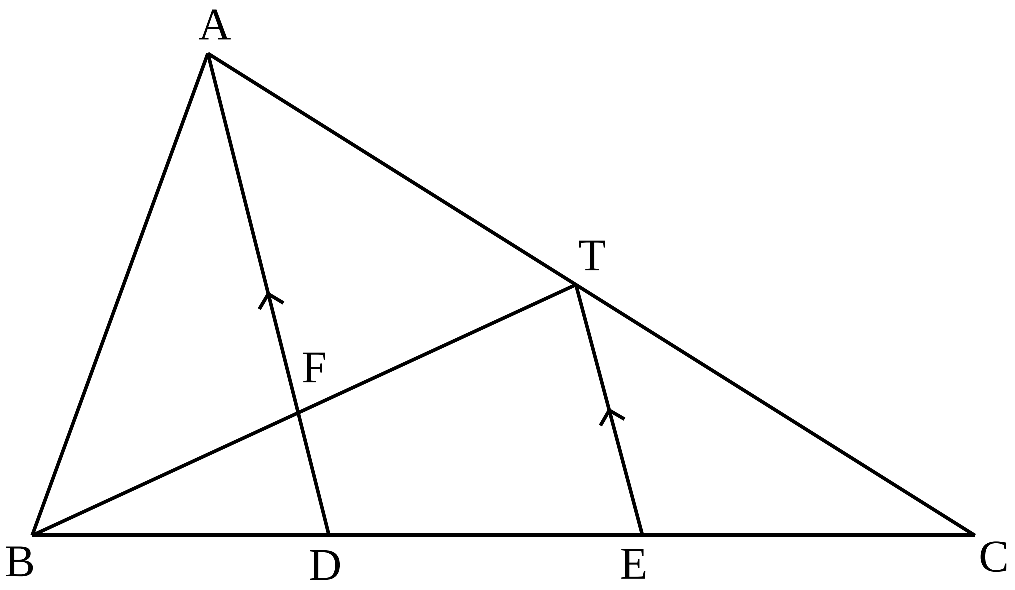
10.1.2 $\Delta APR \;|||\; \Delta MPT$. (3)

10.2 If $AR = \frac{3}{2}MT$, then calculate the value of $\frac{PT}{PR}$. (3)

[10]

QUESTION 11

In the diagram below, ΔABC has D and E on BC. BD = 6 cm and DC = 9 cm.
AT : TC = 2 : 1 and AD ∥ TE.



11.1	Write down the numerical value of $\frac{CE}{ED}$.	(1)
11.2	Show that D is the midpoint of BE.	(1)
11.3	If FD = 2 cm, calculate the length of TE.	(2)
11.4	Calculate the numerical value of :	
11.4.1	$\frac{\text{Area of } \Delta ADC}{\text{Area of } \Delta ABD}$	(2)
11.4.2	$\frac{\text{Area of } \Delta TEC}{\text{Area of } \Delta ABC}$	(3)
		[9]

TOTAL: 150

INFORMATION SHEET: MATHEMATICS

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$A = P(1+ni)$ $\quad$ $A = P(1-ni)$ $\quad$ $A = P(1-i)^n$ $\quad$ $A = P(1+i)^n$

$T_n = a + (n-1)d$ $\quad$ $S_n = \frac{n}{2}\{2a + (n-1)d\}$

$T_n = ar^{n-1}$ $\quad$ $S_n = \frac{a\left(r^n - 1\right)}{r-1}$; $r \neq 1$ $\quad$ $S_\infty = \frac{a}{1-r}$; $-1 < r < 1$

$F = \frac{x\left[\left(1+i\right)^n - 1\right]}{i}$ $\quad$ $P = \frac{x\left[1 - (1+i)^{-n}\right]}{i}$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $\quad$ $M\left(\frac{x_1 + x_2}{2}; \frac{y_1 + y_2}{2}\right)$ $\quad$ $m = \tan\theta$

$y = mx + c$ $\quad$ $y - y_1 = m(x - x_1)$ $\quad$ $m = \frac{y_2 - y_1}{x_2 - x_1}$

$(x-a)^2 + (y-b)^2 = r^2$

In ΔABC: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

$a^2 = b^2 + c^2 - 2bc.\cos A$

$\text{area } \Delta ABC = \frac{1}{2} ab.\sin C$

$\sin(\alpha + \beta) = \sin\alpha.\cos\beta + \cos\alpha.\sin\beta$ $\quad$ $\sin(\alpha - \beta) = \sin\alpha.\cos\beta - \cos\alpha.\sin\beta$

$\cos(\alpha + \beta) = \cos\alpha.\cos\beta - \sin\alpha.\sin\beta$ $\quad$ $\cos(\alpha - \beta) = \cos\alpha.\cos\beta + \sin\alpha.\sin\beta$

$$\cos 2\alpha = \begin{cases} \cos^2\alpha - \sin^2\alpha \\ 1 - 2\sin^2\alpha \\ 2\cos^2\alpha - 1 \end{cases}$$

$\sin 2\alpha = 2\sin\alpha.\cos\alpha$

$\bar{x} = \frac{\sum fx}{n}$ $\quad$ $\sigma^2 = \frac{\sum_{i=1}^{n}\left(x_i - \bar{x}\right)^2}{n}$

$P(A) = \frac{n(A)}{n(S)}$ $\quad$ $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$

$\hat{y} = a + bx$ $\quad$ $b = \frac{\sum (x - \bar{x})(y - \bar{y})}{\sum (x - \bar{x})^2}$

NAME and SURNAME:

ANSWER SHEET

Tear off and return

QUESTION 2.1

DISTANCE, d (in km)	FREQUENCY	CUMULATIVE FREQUENCY
$0 < d \leq 5$	8	
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QUESTION 2.2

CUMULATIVE FREQUENCY GRAPH (OGIVE)