FINAL

KWAZULU-NATAL PROVINCE

EDUCATION
REPUBLIC OF SOUTH AFRICA

NATIONAL SENIOR CERTIFICATE

GRADE 12

MATHEMATICS P2

PREPARATORY EXAMINATION

SEPTEMBER 2022

MARKING GUIDELINE

MARKS: 150

TIME: 3 hours

These marking guideline consists of 15 pages.

NOTE:

- If a candidate answered a QUESTION TWICE, mark only the FIRST attempt.
- If a candidate crossed out an answer and did not redo it, mark the crossed-out answer.
- Consistent accuracy applies to ALL aspects of the marking guidelines.
- Assuming values/answer in order to solve a problem is unacceptable.

GEOMETRY	
S	**A mark for a correct statement (A statement mark is independent of a reason.)**
R	**A mark for a correct reason (A reason mark may only be awarded if the statement is correct.)**
S/R	**Award a mark if the statement AND reason are both correct.**

QUESTION 1

40	47	48	51	53	57	58	58	59	59
60	60	60	60	61	62	63	64	66	69

1.1.1	$\bar{x}=\frac{1155}{20}$ $\bar{x}=57,75\ kg$ Answer only: full marks	A✓ 1155 CA✓ answer **Penalty 1 mark for rounding here for the entire paper**	(2)
1.1.2	$\sigma=6,73702\approx 6,74\ kg$	A✓ answer	(1)
1.2	$(\bar{x}-\sigma\ ;\ \bar{x}+\sigma\ )$ $(57,75-6,74\ ;\ 57,75+6,74)$ limit $=(51,01\ ;\ 64,49)$ $\therefore$ 14 boys	CA✓ interval CA✓ answer	(2)
1.3.1	22	A✓ answer	(1)
1.3.2	$\bar{x}=\frac{1320}{22}$ $\bar{x}=60\ kg$	CA✓ based on 1.3.1	(1)
1.4	$\bar{x}=\frac{5x+1155}{25}=60$ $5x+1155=1500$ $5x=345$ $x=69\ kg$	CA✓ $\frac{5x+1155}{25}$ CA✓ equation CA✓ simplification CA✓ answer	(4)
			[11]

QUESTION 2

2.1

DISTANCE, d (in km)	FREQUENCY	CUMULATIVE FREQUENCY
$0 < d \leq 5$	8	**8**
$5 < d \leq 10$	41	**49**
$10 < d \leq 15$	63	**112**
$15 < d \leq 20$	52	**164**
$20 < d \leq 25$	41	**205**
$25 < d \leq 30$	38	**243**
$30 < d \leq 35$	7	**250**
TOTAL	**250**	

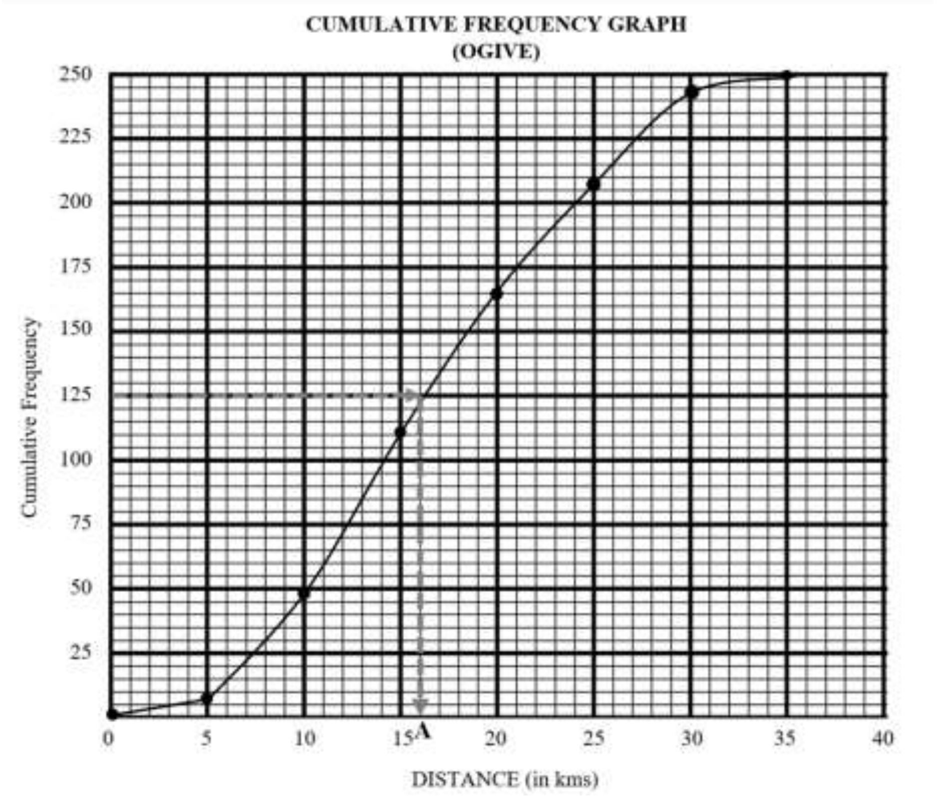
A✓ 8 and 49

CA✓ 112 and 164

CA✓ 205 , 243 and 250

(3)

2.2



A✓ grounded at (0 ; 0)

CA✓ cumulative frequencies for y-coords

CA✓ 5 other points correct

A✓ smooth shape

(4)

2.3

See ogive for dotted lines and point marked A.
Median = 16 km

OR

Answer only: Full marks (accept 15,16,17)

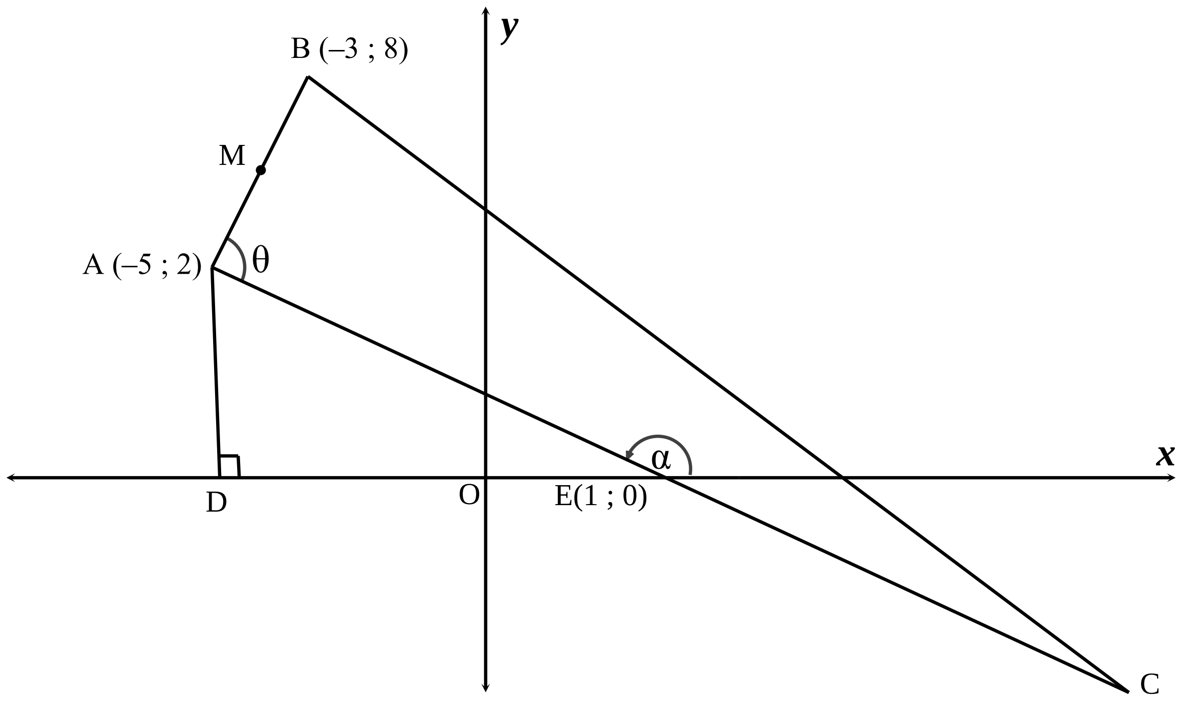
CA✓ indication on graph

CA✓ approx. median

(2)

[9]

QUESTION 3

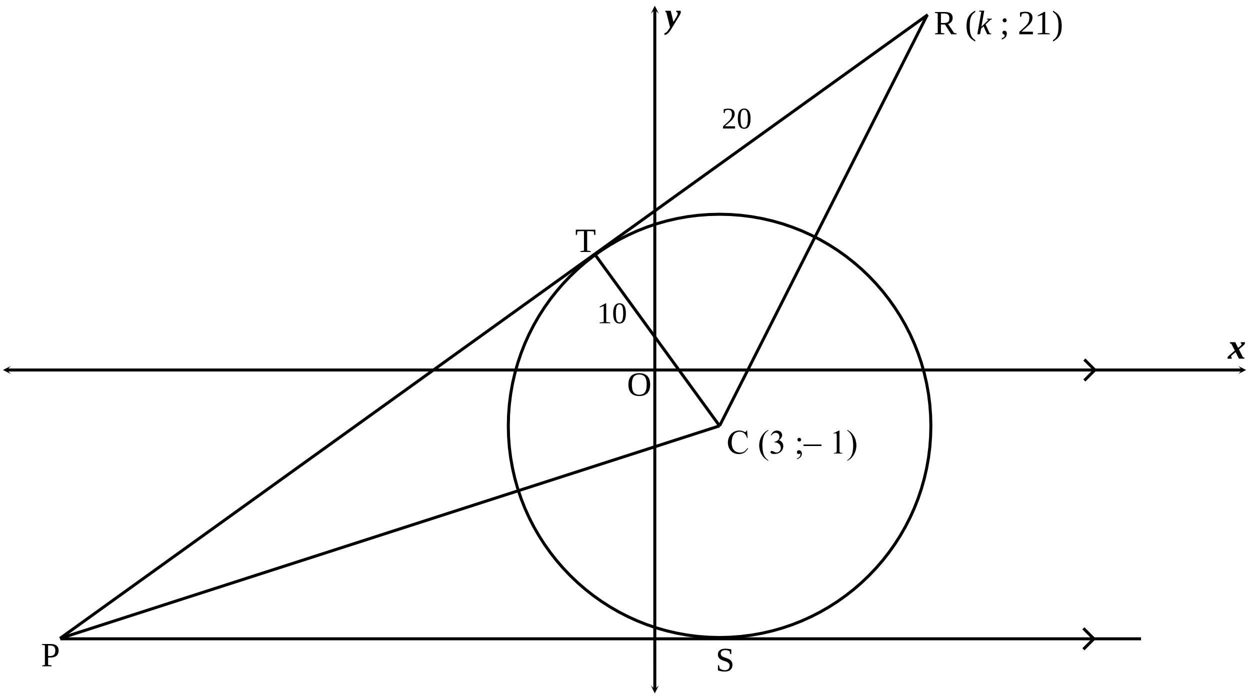


3.1	$M\left(\frac{x_2+x_1}{2};\frac{y_2+y_1}{2}\right)$ $M\left(\frac{-5-3}{2};\frac{2+8}{2}\right)$ $M(-4;5)$ Answer only: full marks	A✓ Substitution of A and B into midpoint formula CA✓ answer	(2)
3.2	$D(-5;0)$	A✓ answer	(1)
3.3	$\frac{-5+x_C}{2}=1$ $\quad$ $\frac{2+y_C}{2}=0$ $\therefore x_C=7$ $\quad$ $y_C=-2$ $C(7;-2)$ **OR** C(1 + 6 ; 0 – 2) Using transformations C(7 ; –2)	A✓ midpoint ITO *x* A✓ midpoint ITO *y* (2) A✓ *x*-value A✓ *y*-value	(2) (2)
3.4	$AC=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$ $AC=\sqrt{(-5-7)^2+(2+2)^2}$ $AC=4\sqrt{10}$	A✓ substitute A and C into distance formula CA✓ answer	(2)

3.5	Method: translation A → B: $(x\,;y) \rightarrow (x+2;\,y+6)$ $\therefore$ by symmetry: C → F: C(7 ; –2) → F(7 + 2 ; –2 + 6) $\therefore$ F(9 ; 4) **OR** Midpoint of intersection of diagonals = $T(2;3)$ Let coordinates of F be $(a\,;b)$ $\frac{a-5}{2}=2 \quad and \quad \frac{b+2}{2}=3$ $a=9 \quad and\ b=4$ $\therefore$ F(9 ; 4)	A✓ x-coordinate A✓ y-coordinate **OR** A✓ x-coordinate A✓ y-coordinate	(2) (2)
3.6	$m\text{AB}=\frac{8-2}{-3+5}=3$ $\therefore \text{gradient of perpendicular bisector}: -\frac{1}{3}$ $\therefore y=-\frac{1}{3}x+c$ $sub\ (-4;5):\quad 5=-\frac{1}{3}(-4)+c$ $c=\frac{11}{3}$ $\therefore \text{equation of perpendicular bisector}:\ y=-\frac{1}{3}x+\frac{11}{3}$	A✓ $m\text{AB}=3$ CA✓ $m_{\text{perp bisector}}=-\frac{1}{3}$ CA✓ substitution CA✓ equation	(4)
3.7	$m\text{AC}=\frac{2+2}{-5-7}=-\frac{1}{3}$ $\tan\alpha=-\frac{1}{3}$ $\alpha=\tan^{-1}\left(-\frac{1}{3}\right)+180°$ $\alpha=161,57°$ $\therefore \alpha\approx162°$ OR Accept method using Cosine Rule	A✓ mAC CA✓ $\tan\alpha=-\frac{1}{3}$ CA✓ answer	(3)
3.8	$m\text{AB}=3$ $\therefore$ gradient of new line $=3$ (// lines = gradients) $y=3x+c$ sub $E(1;0):\ 0=3(1)+c$ $=-3$ $y=3x-3$	CA✓ equal gradients CA✓ answer	(2)

3.9	$mAB \times mAC = 3 \times -\frac{1}{3}$ $\therefore mAB \times mAC = -1$ $\therefore AB \perp AC$ $\therefore \theta = 90°$ Accept methods using Cosine Rule & Trig ratios	A✓ $mAB \times mAC = -1$ A✓ answer	(2)
3.10	$AC = 4\sqrt{10}$ units $AB = \sqrt{(-3+5)^2 + (8-2)^2} = 2\sqrt{10}$ units $\therefore$ Area of $\Delta ABC = \frac{1}{2} AC.AB$ $\therefore$ Area of $\Delta ABC = \frac{1}{2}\left(4\sqrt{10}\right)\left(2\sqrt{10}\right)$ $\therefore$ Area of $\Delta ABC = 40$ units^2	A✓ substitute A and B into distance formula CA✓ length of AB CA✓ substitution CA✓ answer	(4)
			[24]

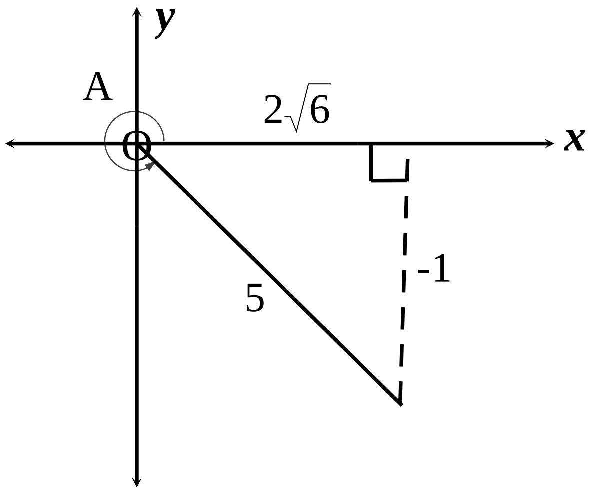
QUESTION 4

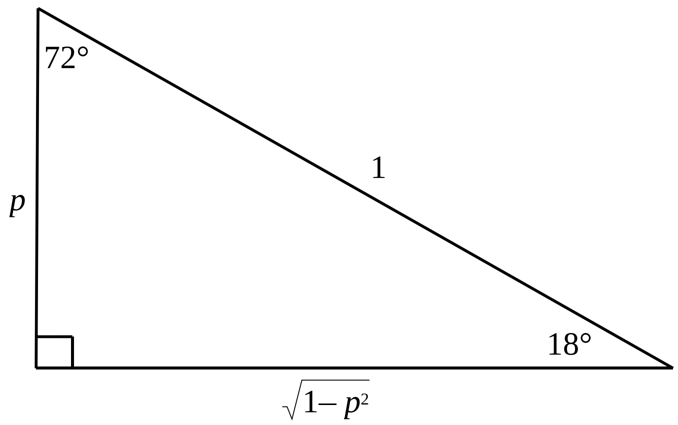


4.1	Radius is perpendicular to tangent	A✓ reason	(1)
4.2	$\text{CR} = \sqrt{10^2 + 20^2} = \sqrt{500}$ (pythag) $\text{CR} = \sqrt{(k-3)^2 + (21+1)^2} = \sqrt{(k-3)^2 + 484}$ $\therefore \sqrt{(k-3)^2 + 484} = \sqrt{500}$ $\therefore (k-3)^2 + 484 = 500$ $\therefore (k-3)^2 = 16$ $\therefore k-3 = \pm 4$ $\therefore k = -1$ or $k = 7$ but $k \neq -1$ (given that R is the first quadrant) $k = 7$ only	A✓ 500 A✓ equate CA✓ simplification CA✓ both values and rejection of $k = -1$	(4)
4.3	$(x-3)^2 + (y+1)^2 = 100$	A✓ $(x-3)^2 + (y+1)^2$ A✓ 100	(2)
4.4	$\text{S}(3;-11)$ $\therefore y = -11$	A✓ answer	(1)
4.5.1	$y = -11$ $Eq\ 1$ $3y = 4x + 35$ $Eq\ 2$ Sub $Eq\ 1$ into $Eq\ 2$: $3(-11) = 4x + 35$ $4x = -68$ $x = -17$ $\therefore \text{P}(-17;-11)$	CA✓ substitution CA✓ x – value	(2)

4.5.2	$PC=\sqrt{(3+17)^2+(-1+11)^2}=10\sqrt{5}$ $TC=10$ $PT^2=PC^2-TC^2$ (pythag) $PT=\sqrt{\left(10\sqrt{5}\right)^2-(10)^2}$ $PT=20$ units **OR** $PT=PS$ (tangents from common point) $PS=3-(-17)=20$ units $\therefore PT=20$ units	CA✓ $PC^2=500$ $PC=10\sqrt{5}$ CA✓ Pythagoras CA✓ answer A✓ S/R CA✓ Substitution CA✓ answer	(3) (3)
			[13]

QUESTION 5

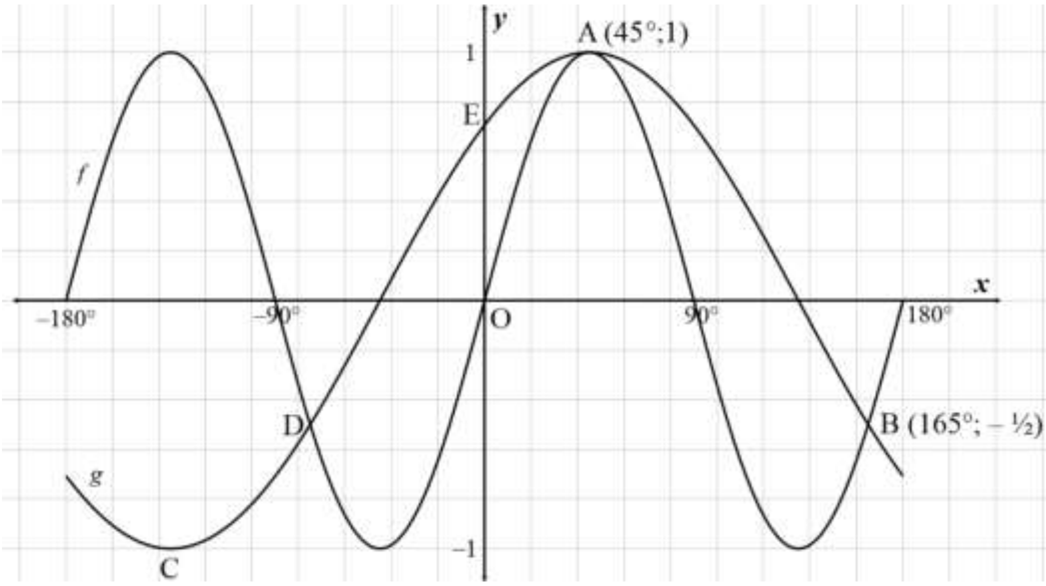
5.1.1	 $y=\sqrt{r^2-x^2}$ $y=\sqrt{(5)^2-\left(2\sqrt{6}\right)^2}=1$ $\therefore y=-1$ (quadrant 4) $\therefore$ $-\sqrt{6}.\tan A$ $=-\sqrt{6}\times\left(\frac{-1}{2\sqrt{6}}\right)$ $=\frac{1}{2}$	A✓ diagram A✓ $y=-1$ CA✓ $\tan A=\frac{2\sqrt{6}}{-1}$ CA✓ answer	(4)

5.1.2	$\sin 2A = 2\sin A.\cos A$ $= 2\left(\frac{-1}{5}\right)\left(\frac{2\sqrt{6}}{5}\right)$ $= -\frac{4\sqrt{6}}{25}$	A✓ double angle identity CA✓ $\sin A = \frac{-1}{5}$ CA✓ $\cos A = \frac{2\sqrt{6}}{5}$ CA✓ answer	(4)
5.2			
5.2.1	$\cos 18° = \sqrt{1-p^2}$	A✓ diagram A✓ answer Answer only full marks	(2)
5.2.2	$\cos 48° = \cos(30°+18°)$ $= \cos 30° \cos 18° - \sin 30° \sin 18°$ $= \left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{1-p^2}}{1}\right) - \left(\frac{1}{2}\right)\left(\frac{p}{1}\right)$ $= \frac{\sqrt{3}.\sqrt{1-p^2} - p}{2}$ $= \frac{\sqrt{3-3p^2} - p}{2}$	A✓ $(30°+18°)$ A✓ expansion A✓ special angle substitution CACA✓✓substitution of $\sin 18°$ and $\cos 18°$	(5)
5.2.3	$\cos 18° = 1-2\sin^2 9°$ $-2\sin^2 9° = \cos 18° - 1$ $\sin^2 9° = \frac{\cos 18° - 1}{-2}$ $\sin 9° = \sqrt{\frac{\cos 18° - 1}{-2}}$ $\sin 9° = \sqrt{\frac{\sqrt{1-p^2}-1}{-2}} = \sqrt{\frac{1-\sqrt{1-p^2}}{2}}$	A✓ double angle expansion A✓ making $\sin 9°$ the subject CA✓ answer	(3)
			[18]

QUESTION 6

6.1	$\dfrac{\sin(180°-x).\tan(x-180°).\cos(360°+x)}{\sin^2(180°+x)+\sin^2(90°-x)}$ $=\dfrac{\sin x.\tan x.\cos x}{\sin^2 x+\cos^2 x}$ $=\sin x.\dfrac{\sin x}{\cos x}.\cos x$ $=\sin^2 x$	A✓ $\sin x$ A✓ $\tan x$ A✓ $\sin^2 x$ A✓ $\cos^2 x$ A✓ $\tan x=\dfrac{\sin x}{\cos x}$ CA✓ answer	(6)
6.2	$\dfrac{\cos 330°.\tan 150°.\sin 12°}{\tan 675°.\cos 258°}$ $=\dfrac{(\cos 30°).(-\tan 30°).(\sin 12°)}{(-\tan 45°).(-\cos 78°)}$ $=-\dfrac{(\cos 30°).(\tan 30°).(\sin 12°)}{(\tan 45°).(\sin 12°)}$ $=-\dfrac{\left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{3}}{3}\right)}{1}$ $=-\dfrac{1}{2}$	A✓ $\cos 30°$ A✓ $-\tan 30°$ A✓ co ratio A✓ $-\cos 78°$ A✓ $\tan 45°$ A✓ special angles CA✓ answer	(7)
6.3.1	LHS: $\dfrac{\cos\alpha+\cos 2\alpha}{\sin 2\alpha-\sin\alpha}$ $=\dfrac{\cos\alpha+(2\cos^2\alpha-1)}{(2\sin\alpha\cos\alpha)-\sin\alpha}$ $=\dfrac{2\cos^2\alpha+\cos\alpha-1}{2\sin\alpha\cos\alpha-\sin\alpha}$ $=\dfrac{(2\cos\alpha-1)(\cos\alpha+1)}{\sin\alpha(2\cos\alpha-1)}$ $=\dfrac{(\cos\alpha+1)}{\sin\alpha}$ = RHS	A✓ cos double angle A✓ sin double angle A✓ numerator factors A✓ denominator factors	(4)
6.3.2	$\sin 2\alpha-\sin\alpha=0$ $2\sin\alpha\cos\alpha-\sin\alpha=0$ $\sin\alpha\,(2\cos\alpha-1)=0$ $\therefore \sin\alpha=0$ or $2\cos\alpha-1=0$ $\cos\alpha=\frac{1}{2}$ $\alpha=0°+k.180; k\in Z$ $\alpha=\pm 60°+k.360; k\in Z$ N.B. If $\sin\alpha=0$ is solved only – max. 3/5 marks	A✓ equating to 0 A✓ factors CA✓ $0°+k.180$ CA✓ $\pm 60°+k.360$ A✓ $k\in Z$	(5)
			[22]

QUESTION 7



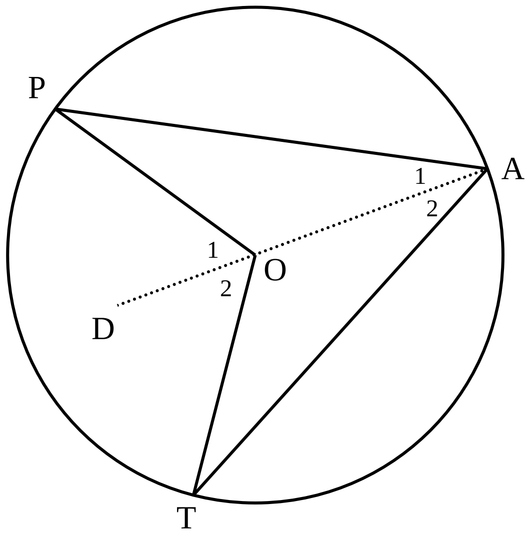
7.1	$a = -45°$	A✓	answer	(1)
7.2	$180°$	A✓	answer	(1)
7.3	$C(-135°;-1)$ E: $y=\cos(0°+45°)$ $y=\frac{\sqrt{2}}{2}$ $E\left(0;\frac{\sqrt{2}}{2}\right)$	A✓ A✓ A✓	$C(-135°;-1)$ solving for y $E\left(0;\frac{\sqrt{2}}{2}\right)$	(3)
7.4	Amplitude = 3	A✓	answer	(1)
7.5.1	$x \in [0°;165°); x \neq 45°$ OR $x \in [0°;45°) \cup (45°;165°)$ OR $0° \leq x < 165°; x \neq 45°$	A✓ A✓ A✓ A✓ A✓ A✓	$[0°;165°)$ $x \neq 45°$ critical values notation $0° \leq x < 165°$ $x \neq 45°$	(2)
7.5.2	$0° \leq x \leq 135°$	A✓ A✓	critical values notation	(2)
7.6	$\sqrt{2}\sin 2x = \cos x + \sin x$ $\sin 2x = \frac{\cos x}{\sqrt{2}} + \frac{\sin x}{\sqrt{2}}$ $\sin 2x = \cos x\,\frac{1}{\sqrt{2}} + \sin x\,\frac{1}{\sqrt{2}}$ $\sin 2x = \cos x.\cos 45° + \sin x.\sin 45°$ $\sin 2x = \cos(x-45°)$	A✓ A✓	division of $\sqrt{2}$ identifying compound $\angle$ $\cos(x-45°)$	

	Equating the 2 functions gives the points of intersection of the 2 graphs.	A✓ conclusion	(3)
			[13]

QUESTION 8

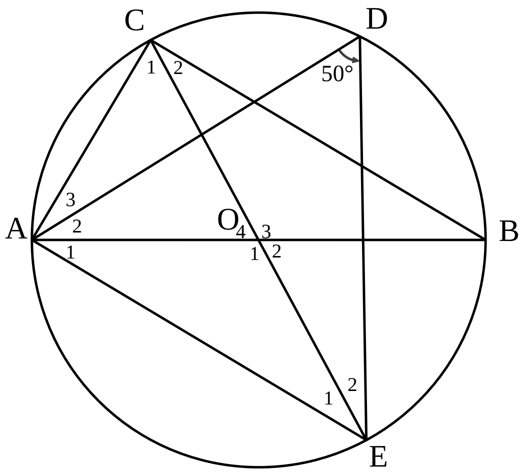
8.1	bisects the chord.	A✓ answer	(1)
8.2.1	$AM = 4\text{cm}$ (line from centre $\perp$ chord)	A✓ S A✓R	(2)
8.2.2	$OM = (r-2)$ $\therefore \quad r^2 = (r-2)^2 + 4^2$ (pythag) $r^2 = r^2 - 4r + 4 + 16$ $4r = 20$ $r = 5$ cm	A✓ $OM = (r-2)$ CA✓ $r^2 = (r-2)^2 + 4^2$ CA✓ simplification CA✓ answer	(4)
			[7]

QUESTION 9



9.1	Constr: Draw line AO and extend to D. Proof: OP = OT (radii) $\therefore \hat{A}_1 = \hat{P}$ ($\angle$s opp $=$ sides) but $\hat{O}_1 = \hat{A}_1 + \hat{P}$ (ext $\angle$ of Δ) $\therefore \hat{O}_1 = 2\hat{A}_1$ *Similarly* $\hat{O}_2 = 2\hat{A}_2$ $\therefore \hat{O}_1 + \hat{O}_2 = 2\left(\hat{A}_1 + \hat{A}_2\right)$ $\therefore P\hat{O}T = 2P\hat{A}T$	A✓ construction A✓ S/R A✓ S/R A✓ S A✓ S	(5)

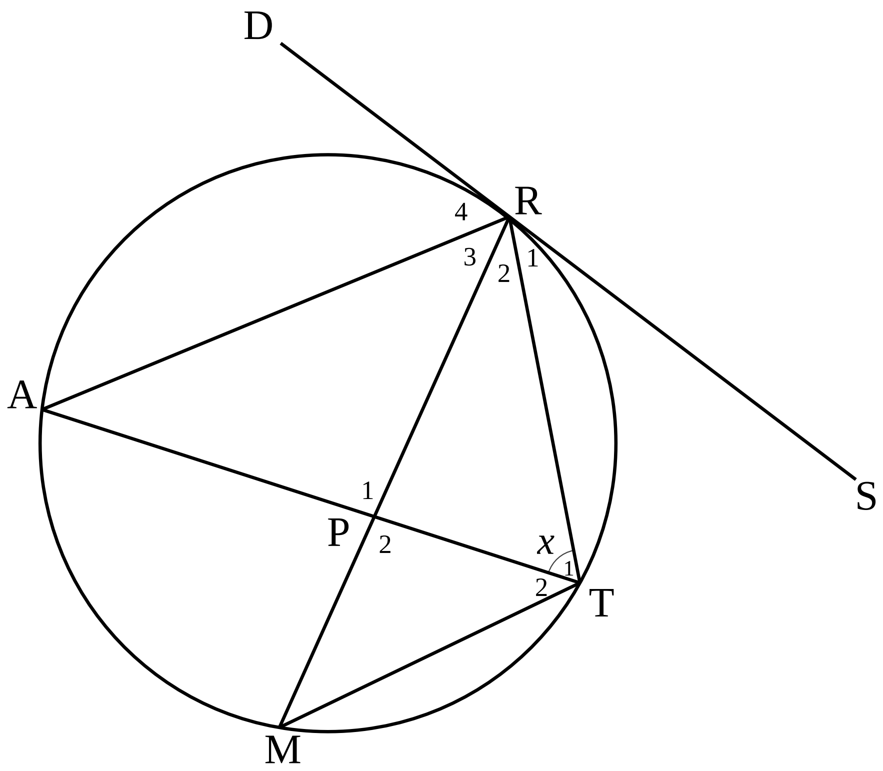
9.2 **Do not mark this question!!! (Maximum for Paper 2 – 141)**



9.2.1a	$\hat{O}_1 = 100°$ ($\angle$ at centre $= 2\times\angle$ at circ)	A✓ S A✓R	(2)
9.2.1b	$\hat{O}_1 = 100°$ $\hat{A}_1 = \hat{E}_1$ ($\angle$s opp = radii) $\therefore \hat{E}_1 = \frac{180° - 100°}{2}$ (sum $\angle$s Δ) $\therefore \hat{E}_1 = 40°$	A✓ S A✓R CA✓ S/R	(3)
9.2.2	$\hat{A}_1 = \hat{E}_1$ ($\angle s$ opp = radii) but $\hat{A}_1 = \hat{C}_2$ ($\angle s$ in same segment) $\therefore \hat{E}_1 = \hat{C}_2$ $\therefore AE \parallel CB$ (alt $\angle s$ =) OR $\hat{A}_1 = \hat{E}_1$ ($\angle s$ opp = radii) but $\hat{E}_1 = \hat{B}$ ($\angle s$ in same segment) $\therefore \hat{A}_1 = \hat{B}$ $\therefore AE \parallel CB$ (alt $\angle s$ =) OR	A✓ S/R A✓ S/R A✓ S A✓ R A✓ S/R A✓ S/R A✓ S A✓ R A✓ S/R A✓ S/R	(4) (4)

	$C\hat{A}E = 90°$ $(\angle s \text{ in semi} \odot)$ $A\hat{C}B = 90°$ $(\angle s \text{ in semi} \odot)$ $C\hat{A}E + A\hat{C}B = 180°$ $\therefore AE \parallel CB$ $(\text{co - int } \angle s)$	A✓ S A✓ R	(4)
			[14]

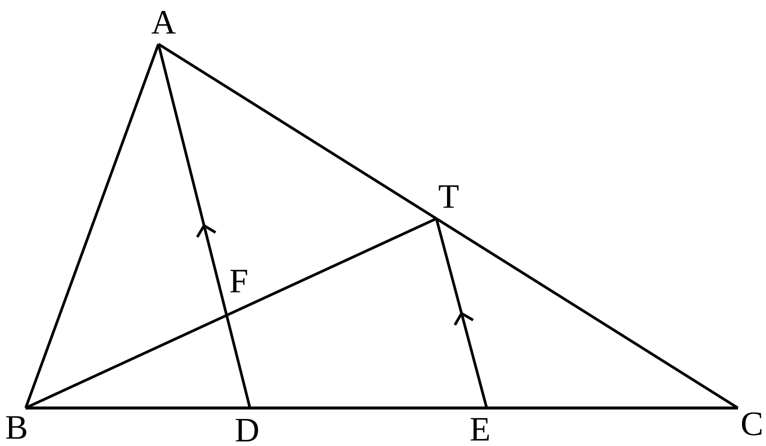
QUESTION 10



10.1.1	$\hat{R}_3 = \hat{T}_2$ $(\angle s \text{ in same segment})$ $\hat{R}_4 = \hat{T}_1$ (tan chord theorem) but $\hat{T}_1 = \hat{T}_2$ (given AT bisects $M\hat{T}R$) $\therefore \hat{R}_3 = \hat{R}_4$	A✓ S A✓R A✓ S A✓R	(4)
10.1.2	In ΔAPR and ΔMPT 1. $\hat{A} = \hat{M}$ $(\angle s \text{ in same segment})$ 2. $\hat{P}_1 = \hat{P}_2$ (vert opp $\angle s$) 3. $\hat{R}_3 = \hat{T}_2$ (remaining $\angle s$ Δ) $\therefore$ $\Delta APR \,\|\|\, \Delta MPT$ $(\angle\angle\angle)$	A✓ S/R A✓ S/R A✓ R	(3)

10.2	$\frac{\text{AR}}{\text{MT}}=\frac{\text{PR}}{\text{PT}}$ (||| Δs) $\text{AR}=\frac{3}{2}\text{MT}$ (*given*) $\therefore \frac{\text{AR}}{\text{MT}}=\frac{3}{2}$ $\therefore \frac{\text{PT}}{\text{PR}}=\frac{2}{3}$	A✓ S A✓R A✓ S	(3)
			[10]

QUESTION 11



11.1	$\frac{\text{CE}}{\text{ED}}=\frac{1}{2}$	A✓ S	(1)
11.2	$\text{DE}=\frac{2}{3}(9\,\text{cm})=6\,\text{cm}$	A✓ S	(1)
11.3	$\frac{\text{TE}}{2}=\frac{\text{BE}}{\text{BD}}=\frac{2}{1}$ (/// triangles) $\therefore \text{TE}=4\text{cm}$ **Accept solution using the Midpoint theorem**	A✓ S/R A✓ S	(2)
11.4.1	$\frac{\text{Area of }\Delta\text{ADC}}{\text{Area of }\Delta\text{ABD}}=\frac{\frac{1}{2}\text{DC}\ \times \square}{\frac{1}{2}\text{BD}\ \times \square}=\frac{6cm}{6cm}=1$	A✓Areas A✓ Answer	(2)

11.4.2	$\dfrac{\text{Area of } \Delta TEC}{\text{Area of } \Delta ABC}$ $=\dfrac{\text{Area of } \Delta TEC}{\text{Area } \Delta TBC}\times\dfrac{\text{Area of } \Delta TBC}{\text{Area } \Delta ABC}$ $=\dfrac{EC}{BC}\times\dfrac{TC}{AC}$ $=\dfrac{1}{4}\times\dfrac{1}{3}$ $=\dfrac{1}{12}$	A✓ S A✓ S A✓Answer	(3)
	OR $\text{Area of } \Delta TEC=\dfrac{1}{4}(\text{Area of } \Delta TBC)$ (common vertex=altit.) $=\dfrac{1}{4}\left(\dfrac{1}{3}\text{Area of } \Delta ABC\right)$ (common vertex=altit.) $\dfrac{\text{Area of } \Delta TEC}{\text{Area of } \Delta ABC}=\dfrac{1}{12}$ OR **Accept area rule with the use of the angle of $\hat{C}$.**	A✓ S A✓ S A✓ S	(3)
			[9]
		TOTAL:	**150**