Province of the
EASTERN CAPE
EDUCATION

NATIONAL SENIOR CERTIFICATE

GRADE 12

SEPTEMBER 2022

MATHEMATICS P2

MARKS: **150**

TIME: **3 hours**

This question paper consists of 14 pages, including a 1-page information sheet, and an answer book of 18 pages.

INSTRUCTIONS AND INFORMATION

Read the following instructions carefully before answering the questions.

1. This question paper consists of 10 questions.
2. Answer ALL the questions in the SPECIAL ANSWER BOOK provided.
3. Clearly show ALL calculations, diagrams, graphs, etc. which you have used in determining your answers.
4. Answers only will NOT necessarily be awarded full marks.
5. You may use an approved scientific calculator (non-programmable and non-graphical), unless stated otherwise.
6. If necessary, round off answers correct to TWO decimal places, unless stated otherwise.
7. Diagrams are NOT necessarily drawn to scale.
8. An information sheet with formulae is included at the end of the question paper.
9. Write neatly and legibly.

QUESTION 1

A hundred athletes took part in a long jump competition. The distance, in centimetres, of their best jumps is summarised in the table below.

Distance of Jumps (in cm)	**Number of athletes**
$420 < d \leq 460$	6
$460 < d \leq 500$	14
$500 < d \leq 540$	16
$540 < d \leq 580$	42
$580 < d \leq 620$	14
$620 < d \leq 660$	2
$660 < d \leq 700$	3
$700 < d \leq 740$	2
$740 < d \leq 780$	1

1.1 Complete the cumulative frequency column in your ANSWER BOOK. (2)

1.2 Draw an ogive (cumulative frequency curve) to represent the above information in your ANSWER BOOK. (4)

1.3 Use your graph to estimate the median jump of the competition. (2)

1.4 What percentage of athletes jumped over 560 cm? (2)

[10]

QUESTION 2

The following table shows a comparison of the distances (centimetres) jumped by 6 long jumpers and the hours spent practising their jumps in a week.

Long jumper	1	2	3	4	5	6
x: **Hours practised**	4,5	2	3,5	4	8	3
y: **Distance jumped (cm)**	650	420	580	490	780	525

2.1 Determine the equation for the least squares regression line for the data. (3)

2.2 Predict the distance jumped by a long jumper who practiced for 5,4 hours. (2)

2.3 Comment on the validity of your answer in QUESTION 2.2. Motivate your answer. (2)

2.4 At the end of the event, they found that the measuring tape used was broken and all distances were decreased by 13 cm. How will this influence the:

2.4.1 Mean jump of the event? (1)

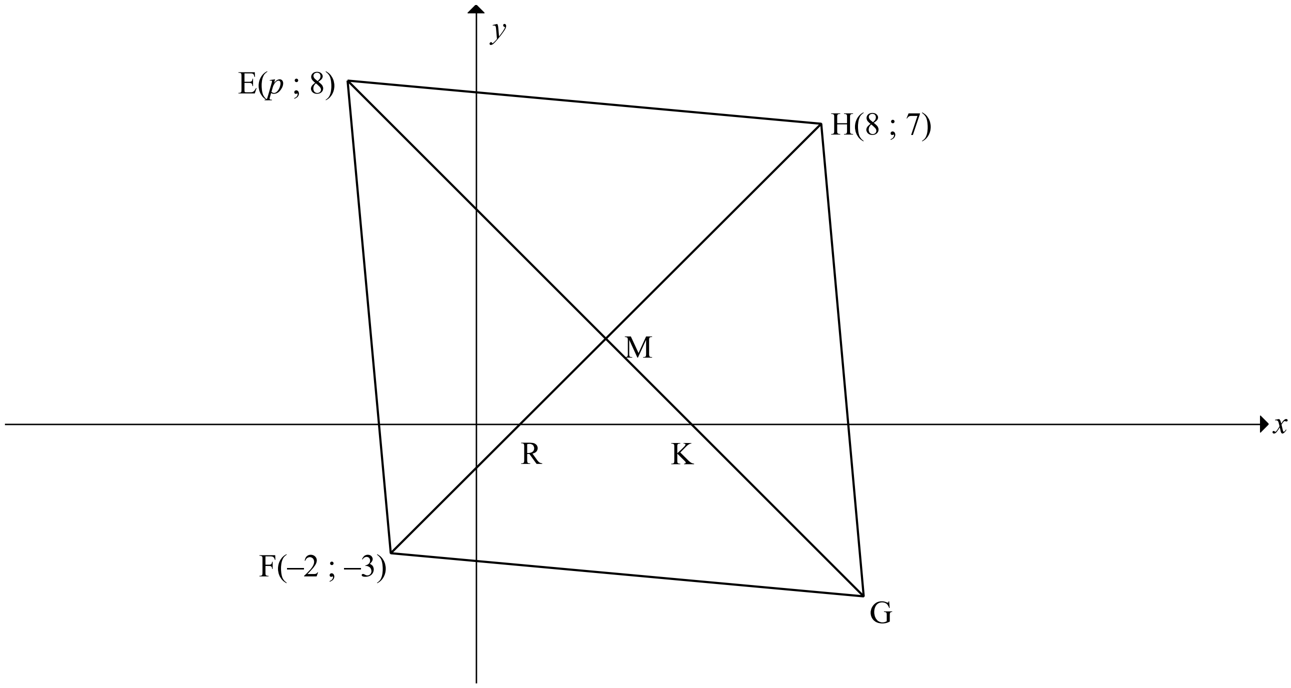
2.4.2 Range of the jumps during this event? (1)

2.4.3 Standard deviation? (1)

[10]

QUESTION 3

In the diagram below, E(p; 8), F(–2; –3), G and H(8; 7) are vertices of rhombus EFGH.
The diagonals EG and HF intersect at M and cut the x-axis at K and R respectively.



3.1 Calculate the:

3.1.1 Coordinates of M (2)

3.1.2 Gradient of FH (2)

3.1.3 Size of $M\hat{K}R$ (4)

3.2 Use the properties of a rhombus to calculate the value of p. (4)

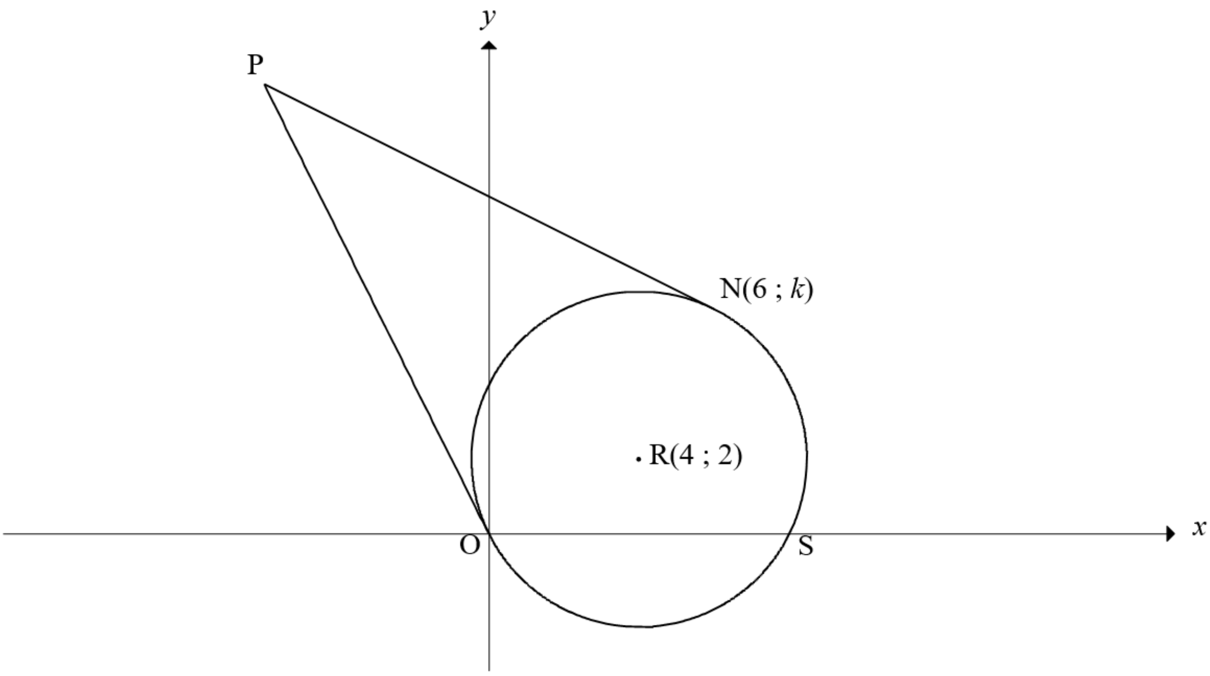
3.3 Calculate the coordinates of G. (2)

3.4 The rhombus is reflected about the line $x = -3$. N is the image of M after the reflection. Calculate the length of MN. (3)

[17]

QUESTION 4

In the diagram below, a circle centred at R(4; 2) passes through the origin O, S and N(6 ; y). From P, a point outside the circle, tangents are drawn to O and N.



4.1 Determine the equation of the circle in the form $(x-a)^2+(y-b)^2=r^2$. (3)

4.2 Calculate the value of k. (4)

4.3 Determine the equation of NP in the form $y=mx+c$. (5)

4.4 It is further given that the equation of OP is $y=-2x$.

Calculate the:

4.4.1 Coordinates of P (3)

4.4.2 Perimeter of PNRO (4)

4.5 Another circle, centred at T, is drawn to touch the circle, centred at R, at S externally. The radii of both circles are equal in length. Determine the coordinates of T. (4)

[23]

QUESTION 5

5.1 Given that: $\cos 26° = p$
Express each of the following in terms of *p*, **without using a calculator.**

5.1.1 $\sin 26°$ (2)

5.1.2 $\tan 154°$ (3)

5.1.3 $\sin 13°.\cos 13°$ (2)

5.2 Determine, **without using a calculator**, the value of the following expressions:

5.2.1 $\dfrac{\cos(-\theta).\tan(180° + \theta)}{2\cos(90° + \theta)}$ (5)

5.2.2 $1 + 2\cos 105°.\sin 15°$ (4)

5.3 Consider: $\dfrac{1 - \cos 2x - \sin x}{\sin 2x - \cos x} = \tan x$

5.3.1 Prove the identity. (4)

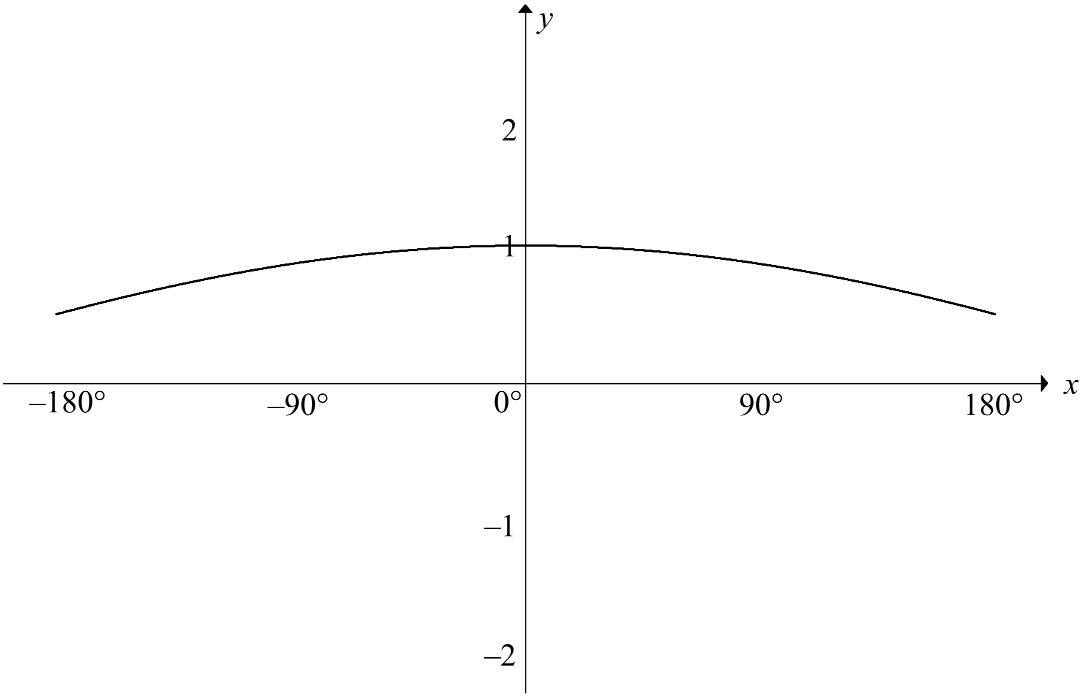
5.3.2 For which value(s) of *x*, in the interval $x \in [-180° ; 180°]$, is the identity not valid? (3)

5.4 Determine the general solution of: $\sin^2 x + 2\sin x \cos x = 3\cos^2 x$ (7)

[30]

QUESTION 6

Sketched below is the graph of $f(x) = \cos\left(\frac{x}{3}\right)$, in the interval $x \in [-180°; 180°]$.



6.1 On the grid given in the ANSWER BOOK, draw the graph of $g(x) = \sin x + 1$, clearly showing ALL intercepts with the axes as well as the coordinates of all turning points. (3)

6.2 Write down the:

6.2.1 Period of f (1)

6.2.2 Range of $g(x) - 3$ (2)

6.3 Determine the maximum distance of $g(x) - h(x)$, where h is the reflection of g in the x-axis, in the interval $x \in [-180°; 180°]$. (2)

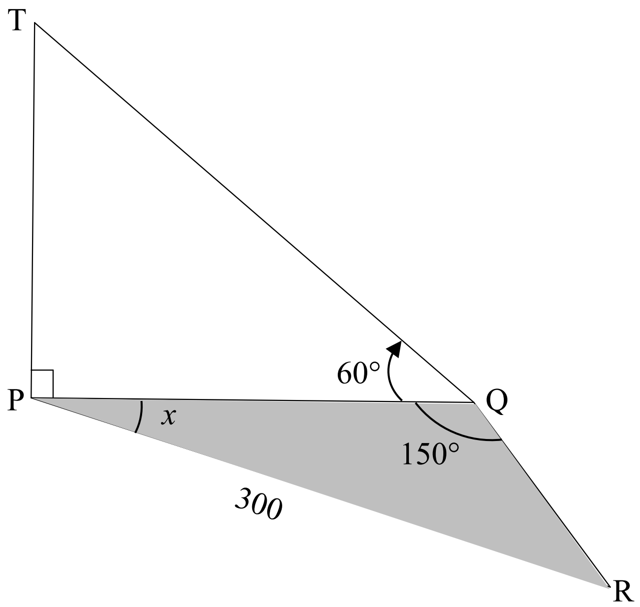
6.4 For which values of x in the interval $x \in [-180°; 180°]$ will $f(x).g'(x) > 0$? (2)

6.5 The graph of g undergoes a transformation to form a new graph $k(x) = \sin(x - 15°)$. Describe in words the transformation from g to k. (2)

[12]

QUESTION 7

In the diagram below, TP represents the height of a building. The foot of the building P and the points Q and R are in the same horizontal plane. From Q, the angle of elevation to the top of the building is 60°. $P\hat{Q}R = 150°$, $Q\hat{P}R = x$ and the distance between P and R is 300 metres.



7.1 Write down $\hat{R}$ in terms of x. (1)

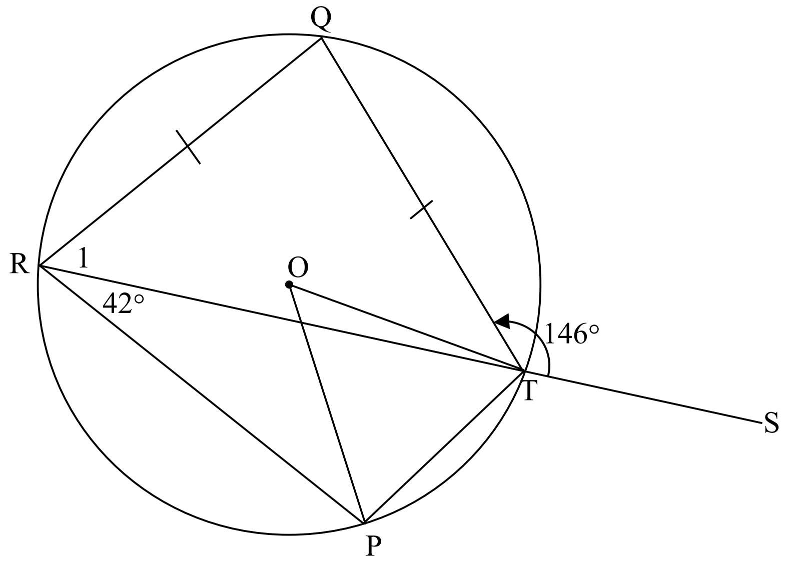
7.2 Determine the length of PQ in terms of x. (3)

7.3 Hence, show that: $TP = 300\sqrt{3}(\cos x - \sqrt{3}\sin x)$ (4)

[8]

QUESTION 8

In the diagram, PRQT is a cyclic quadrilateral in the circle with QR = QT. Chord RT is produced to S and radii OP and OT are drawn. $P\hat{R}T = 42°$ and $Q\hat{T}S = 146°$.



Determine, giving reasons, the size of the following angles:

8.1 $P\hat{O}T$ (2)

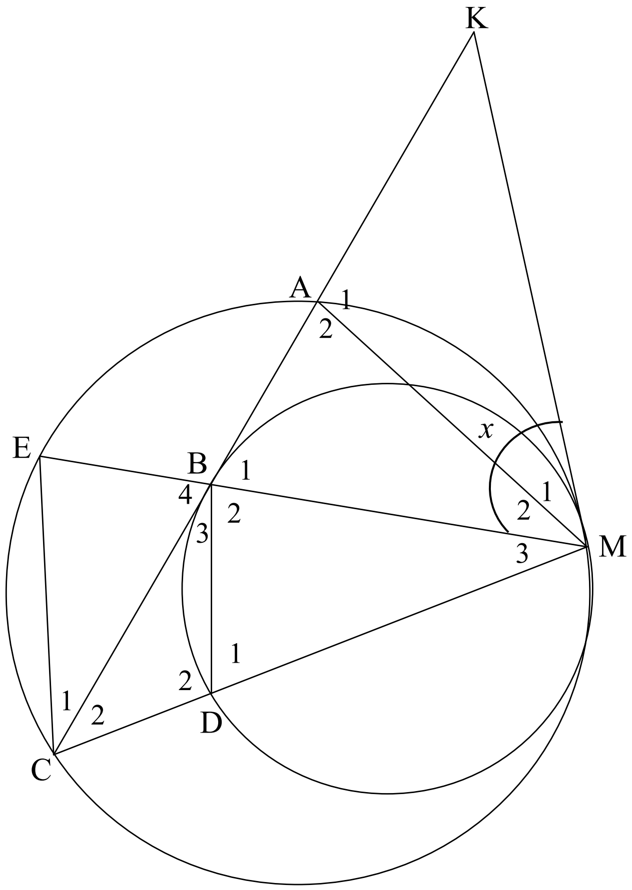
8.2 $\hat{R}_1$ (2)

8.3 $R\hat{P}T$ (3)

[7]

QUESTION 9

In the diagram, the two circles touch internally at M. MK is a common tangent to the circles. A, E and C are points on the larger circle and B and D are points on the smaller circle. Chord CA is produced to meet the tangent at K. ΔMEC is drawn. CA and EM meet at B. KB is a tangent to the smaller circle at B. D is a point on CM. AM and BD are drawn.
Let $K\widehat{M}B = x$.



9.1 Name, giving reasons, FOUR other angles each equal to x. (5)

9.2 Prove, giving reasons, that:

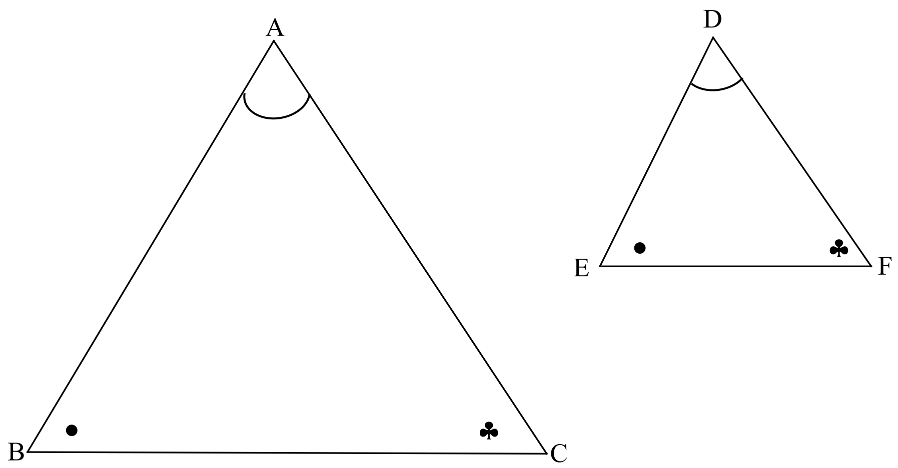
9.2.1 BD || EC (2)

9.2.2 $\widehat{A}_2 = \widehat{B}_2$ (3)

9.2.3 ME × MD = MC × MB (2)
[12]

QUESTION 10

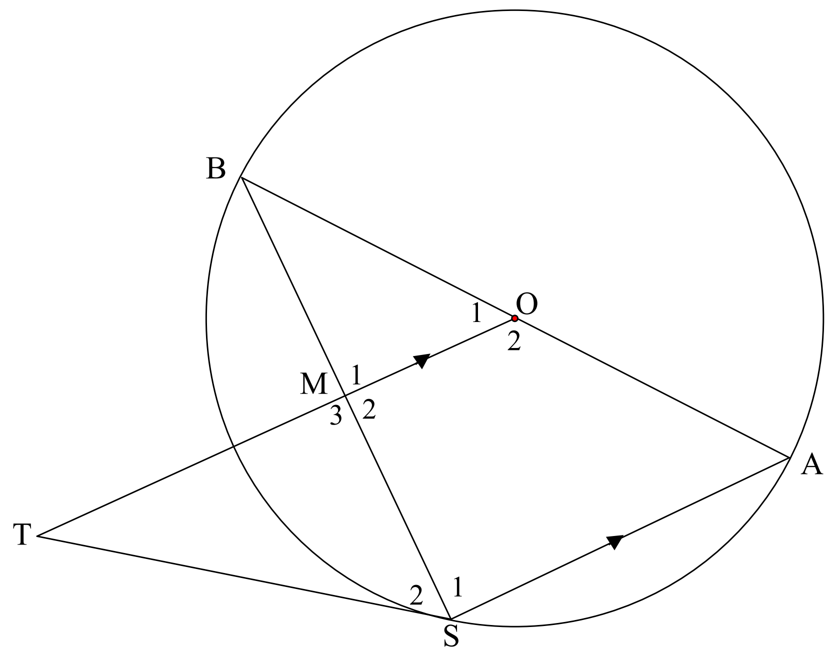
10.1 In the diagram below, ΔABC and ΔDEF are given such that $\hat{A} = \hat{D}$, $\hat{B} = \hat{E}$ and $\hat{C} = \hat{F}$.



Prove the theorem that states if two triangles are equiangular, then their sides are in proportion, i.e., prove that: $\dfrac{DE}{AB} = \dfrac{DF}{AC}$

(6)

10.2 In the diagram, AB is a diameter of the circle centred at O. ΔABS is drawn with S a point on the circle. M is a point on BS and OM is produced to T such that AS || OM. TS is drawn such that BOST is a cyclic quadrilateral.



Prove, giving reasons, that:

10.2.1 TS is a tangent to the circle at S (4)

10.2.2 TS is the diameter of a circle passing through points T, M and S (5)

10.2.3 $\Delta ABS \,|||\, \Delta STM$ (3)

10.2.4 $AS \cdot MT = 2SM^2$ (3)

[21]

TOTAL: 150

INFORMATION SHEET: MATHEMATICS

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$A = P(1+ni)$ $\quad$ $A = P(1-ni)$ $\quad$ $A = P(1-i)^n$ $\quad$ $A = P(1+i)^n$

$\sum_{i=1}^{n} 1 = n$ $\quad$ $\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$ $\quad$ $T_n = a + (n-1)d$ $\quad$ $S_n = \frac{n}{2}\left(2a + (n-1)d\right)$

$T_n = ar^{n-1}$ $\quad$ $S_n = \frac{a\left(r^n - 1\right)}{r-1}$; $r \neq 1$ $\quad$ $S_\infty = \frac{a}{1-r}$; $-1 < r < 1$

$F = \frac{x\left[(1+i)^n - 1\right]}{i}$ $\quad$ $P = \frac{x[1-(1+i)^{-n}]}{i}$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $\quad$ $\mathrm{M}\left(\frac{x_1 + x_2}{2}; \frac{y_1 + y_2}{2}\right)$

$y = mx + c$ $\quad$ $y - y_1 = m(x - x_1)$ $\quad$ $m = \frac{y_2 - y_1}{x_2 - x_1}$ $\quad$ $m = \tan\theta$

$$(x-a)^2 + (y-b)^2 = r^2$$

In ΔABC: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$ $\quad$ $a^2 = b^2 + c^2 - 2bc.\cos A$ $\quad$ $area\ \Delta ABC = \frac{1}{2}ab.\sin C$

$\sin(\alpha + \beta) = \sin\alpha.\cos\beta + \cos\alpha.\sin\beta$ $\quad$ $\sin(\alpha - \beta) = \sin\alpha.\cos\beta - \cos\alpha.\sin\beta$

$\cos(\alpha + \beta) = \cos\alpha.\cos\beta - \sin\alpha.\sin\beta$ $\quad$ $\cos(\alpha - \beta) = \cos\alpha.\cos\beta + \sin\alpha.\sin\beta$

$\cos 2\alpha = \begin{cases} \cos^2\alpha - \sin^2\alpha \\ 1 - 2\sin^2\alpha \\ 2\cos^2\alpha - 1 \end{cases}$ $\quad$ $\sin 2\alpha = 2\sin\alpha.\cos\alpha$

$\bar{x} = \frac{\sum x}{n}$ $\quad$ $\sigma^2 = \frac{\sum_{i=1}^{n}\left(x_i - \bar{x}\right)^2}{n}$

$P(A) = \frac{n(A)}{n(S)}$ $\quad$ $P(A\ or\ B) = P(A) + P(B) - P(A\ and\ B)$

$\hat{y} = a + bx$ $\quad$ $b = \frac{\sum (x - \bar{x})(y - \bar{y})}{\sum (x - \bar{x})^2}$