



education

Department of
Education
FREE STATE PROVINCE

**PREPARATORY EXAMINATION/
*VOORBEREIDENDE EKSAMEN***

GRADE/*GRAAD* 12

**MATHEMATICS P2/
*WISKUNDE V2***

SEPTEMBER 2022

MARKS/*PUNTE*: 150

**MARKING GUIDELINES/
*NASIENRIGLYNE***

**This marking guidelines consists of 14 pages./
*Hierdie nasienriglyne bestaan uit 14 bladsye.***

NOTE:

- If a candidate answers a question TWICE, only mark the FIRST attempt.
- If a candidate has crossed out an attempt of a question and not redone the question, mark the crossed out version.
- Consistent accuracy applies in ALL aspects of the marking memorandum. Stop marking at the second calculation error.
- Assuming answers/values in order to solve a problem is NOT acceptable.

NOTA:

- As 'n kandidaat 'n vraag TWEE KEER beantwoord, sien slegs die EERSTE poging na.
- As 'n kandidaat 'n antwoord van 'n vraag doodtrek en nie oordoen nie, sien die doodgetrekte poging na.
- Volgehoue akkuraatheid word in ALLE aspekte van die nasienriglyne toegepas. Hou op nasien by die tweede berekeningsfout.
- Om antwoorde/waardes te aanvaar om 'n probleem op te los, word NIE toegelaat NIE.

GEOMETRY/MEETKUNDE	
S	A mark for a correct statement (A statement mark is independent of a reason)
	<i>'n Punt vir 'n korrekte bewering</i> (<i>'n Punt vir 'n bewering is onafhanklik van die rede</i>)
R	A mark for the correct reason (A reason mark may only be awarded if the statement is correct)
	<i>'n Punt vir 'n korrekte rede</i> (<i>'n Punt word slegs vir die rede toegeken as die bewering korrek is</i>)
S/R	Award a mark if statement AND reason are both correct
	<i>Ken 'n punt toe as die bewering EN die rede beide korrek is</i>

QUESTION/VRAAG 1

1.1.1	Range= max–min =685–287 = 398	✓ 398 (1)
1.1.2	$x = \frac{287+328+374+\dots+619+685}{11}$ $x = \frac{5478}{11}$ = 498	sum & ✓ div by 11 ✓ 498 (2)
1.1.3	$\sigma = 119,4708028 \approx 119.47$	✓ 119.47 (1)
1.2	$(x - \sigma; x + \sigma)$ = (498 – 119.47; 498 + 119.47) = (378,53; 617,47) 287; 328; 374; 619; 685 ∴ 5 distances/afstande	✓ $(x - \sigma; x + \sigma)$ ✓ (378,53; 617,47) ✓ 5 distances (3)
1.3.1	119,47	✓ 119,47 (1)
1.3.2	$11y + 5478 = 5555$ $11y = 77$ ∴ $y = 7$ ∴ $Q_2 = 508$	✓ $11y = 77$ ✓✓ $Q_2 = 508$ (3)
		[11]

QUESTION/VRAAG 2

2.1	$y = a + bx$ $a = 57,86698281 \approx 57,87$ $b = 0,04935388263 \approx 0,05$ ∴ $y = 57,87 + 0,05x$	✓ value of a ✓ value of b ✓ correct equation (3)
2.2	$r = 0,9332540794$ $\approx 0,93$	✓ value of r (1)
2.3	$y = 57,87 + 0,05x$ = 57,87 + 0,05(465) = 81,12% ∴ 81% succesful average in the next tournament/ <i>sukcesvolle gemiddelde in die volgende toernooi</i>	✓ subst ✓ 81,12% (2)
2.4	Very strong positive correlation/ <i>Sterk positiewe korrelasie</i>	✓✓ very strong positive (2)
		[8]

QUESTION/VRAAG 3

3.1.1	$= G\left(\frac{x_E + x_D}{2}; \frac{y_E + y_D}{2}\right)$ $= G\left(\frac{-11 + (-8)}{2}; \frac{0 + 5}{2}\right)$ $= G\left(-9\frac{1}{2}; 2\frac{1}{2}\right)$	✓ x value ✓ y value (2)
3.1.2	$M_{FHG} = \frac{2\frac{1}{2} - 1}{-9\frac{1}{2} - (-8)}$ <p>G, H and F are collinear/ <i>G, H en F is kolloneêr</i></p> $= \frac{1\frac{1}{2}}{1\frac{1}{2}}$ $= -1$ $\therefore m_{FG} = -1$	✓ Substitute ✓ $m_{FG} = -1$ (2)
3.2	$y - y_1 = m(x - x_1)$ $y - 5 = -1(x + 8)$ $y = -x - 3$	✓ grad ✓ sub ✓ answer (3)
3.3.1	At A $y = 0$ A(-3; 0) OA = 3 units/eenhede $AE = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $AE = \sqrt{(-11 + 3)^2 + (0)^2}$ $= \sqrt{64}$ $= 8$	✓ sub dist formulae ✓ answer (2)
3.3.2	EF = 4 line drawn from midpoint / to 3 rd side/lyn getrek van middelpunt tot die 3 ^{de} sy $DA = 5\sqrt{2}$ $GF = \frac{5\sqrt{2}}{2}$ midpoint theorem/middelpunt stelling $m_{AD} = -1$ $\hat{D}AE = 45^\circ$ $\text{Area of/Oppvl van } \triangle AED = \frac{1}{2} AD \cdot AE \sin 45^\circ$ $= \frac{5\sqrt{2}}{2} \cdot 8 \sin 45^\circ$ $= 20$	✓ DA ✓ Area $\triangle AED = 20$

	$\begin{aligned}\text{Area of/ Oppvl van } \triangle GFE &= \frac{1}{2} \cdot GF \cdot EF \cdot \sin 45^\circ \\ &= \frac{1}{2} \cdot \frac{5\sqrt{2}}{2} \cdot 4 \sin 45^\circ \\ &= 5\end{aligned}$ $\begin{aligned}\text{Area of trapezium ADGF} &= \text{Area } \triangle AED - \text{Area } \triangle GFE \\ \text{Oppvl van trapesium ADGF} &= \text{Oppvl } \triangle AED - \text{Oppvl } \triangle GFE \\ &= 20 - 5 \\ &= 15 \text{ square units/vierkante}\end{aligned}$	<p>✓ Area $\triangle AED = 5$</p> <p>✓ answer</p> <p>(4)</p>
3.3.3	$\hat{C} + \hat{DAB} = 180^\circ$ opp angles of cyclic quad/ teenoorgest hoeke van 'n koordevierhoek $\begin{aligned}\hat{DAB} &= 180^\circ - 71,57^\circ \\ &= 108,43^\circ\end{aligned}$ $\begin{aligned}M_{DA} &= -1 \\ \therefore \hat{DAF} &= 45^\circ\end{aligned}$ $\hat{BAO} + \hat{DAE} + 108,43 = 180^\circ$ adj angles on str line/ aangr hoeke op 'n reguitlyn $\begin{aligned}\hat{BAO} + 45^\circ + 108,43^\circ &= 180^\circ \\ \hat{BAO} &= 26,57^\circ\end{aligned}$ $\begin{aligned}M_{AB} &= \tan \hat{BAO} \\ &= \tan 26,57^\circ \\ &= 0,5\end{aligned}$	<p>✓ S</p> <p>✓ S</p> <p>✓ S</p> <p>✓ S</p> <p>✓ S</p> <p>✓ answer</p> <p>(6)</p>
		[19]

QUESTION/VRAAG 4

4.1	M(3,4)	✓ x value ✓ y value (2)
4.2.1	$(x - 3)^2 + (y - 4)^2 = 25$ x-intercepts/x-afsnitte: $y=0$ $(x - 3)^2 + (0 - 4)^2 = 25$ $(x - 3)^2 + 16 = 25$ $(x - 3)^2 = 9$ $x - 3 = \pm 3$ $\therefore x = 6 \text{ or } x = 0$ K(6;0)	✓ sub $y = 0$ ✓ simplification ✓ x value s ✓ choose $x = 6$ (4)
4.2.2	$M_{KM} = \frac{0-4}{6-3}$ $= \frac{-4}{3}$	✓ subst ✓ $M_{KM} = \frac{-4}{3}$ (2)
4.3	Equation of/Vergelyking van PR MK perpendicular/loodreg op PR tan-rad $y = \frac{3}{4}x + c$ $0 = \frac{3}{4}(6) + c$ $\therefore c = \frac{-9}{2}$ $y = \frac{3}{4}x - \frac{9}{2}$	✓ sub point K (6;0) ✓ $c = \frac{-9}{2}$ ✓ answer (3)

4.4	$R\left(0; \frac{-9}{2}\right)$ $RT = RK (6,0)$ tangents from same point/ <i>raaklyne van dieselfde punt</i> $RT = \sqrt{(6-0)^2 + \left(0 + \frac{9}{2}\right)^2}$ $RT = \sqrt{36 + \frac{81}{4}}$ $RT = \frac{15}{2}$	✓ sub coordinates R ✓ simplification ✓ length of RT (3)
4.5.1	$MN = 10$ $MN = R_M + r_n$ $10 = 5 + r_n$ $r_n = 5$ $\therefore K$ is the midpoint of/is <i>die middelpunt van</i> MN $N(9; -4)$	✓ MN ✓ K is the midpoint ✓ N (9; -4) (3)
4.5.2	$(x-a)^2 + (y-b)^2 = r^2$ $(6-9)^2 + (0+4)^2 = r^2$ $9+16 = r^2$ $r^2 = 25$ $(x-9)^2 + (y+4)^2 = 25$ $x^2 - 18x + 81 + y^2 + 8y + 16 - 25 = 0$ $x^2 + y^2 - 18x + 8y + 72 = 0$	✓ sub K(6;0) ✓ 25 ✓ sub ✓ answer (4)
		[21]

QUESTION/VRAAG 5

5.1	$\frac{-\cos x \cdot (-\tan x) \cos^2 x}{\sin x} - 4\cos^2 x$ $\frac{-\cos x \frac{\sin x}{\cos x}}{\sin x} \cdot \cos^2 x - 4\cos^2 x$ $\cos^2 x - 4\cos^2 x$ $-3\cos^2 x$	<p>✓ $-\cos x$</p> <p>✓ $-\tan x$</p> <p>✓ $\cos^2 x$</p> <p>✓ $\sin x$</p> <p>✓ $\frac{\sin x}{\cos x}$</p> <p>✓ simplification</p> <p>✓ $-3\cos^2 x$</p> <p>(7)</p>
5.2.1	$\text{LHS} = \cos(A - B) - \cos(A + B)$ $= \cos A \cos B + \sin A \sin B - [\cos A \cos B - \sin A \sin B]$ $= \cos A \cos B + \sin A \sin B - \cos A \cos B + \sin A \sin B$ $= 2 \sin A \sin B$	<p>✓ correct exp</p> <p>✓ simplification</p> <p>(2)</p>
5.2.2	$\cos 15^\circ - \cos 75^\circ$ $= \cos(45^\circ - 30^\circ) - \cos(45^\circ + 30^\circ)$ $= 2 \sin 45^\circ \cdot \sin 30^\circ$ $= 2 \cdot \frac{2}{\sqrt{2}} \cdot \frac{1}{2}$ $= \frac{\sqrt{2}}{2}$	<p>✓ $\cos(45^\circ - 30^\circ)$</p> <p>✓ $\cos(45^\circ + 30^\circ)$</p> <p>✓ simplification</p> <p>✓ $\frac{\sqrt{2}}{2}$</p> <p>(4)</p>
5.3	$\frac{\cos 36^\circ}{\cos 12^\circ} - \frac{\sin 36^\circ}{\sin 12^\circ} = \frac{\cos 36^\circ \sin 12^\circ - \sin 36^\circ \cos 12^\circ}{\cos 12^\circ \sin 12^\circ}$ $\frac{-\sin(36^\circ - 12^\circ)}{\cos 12^\circ \sin 12^\circ}$ $\frac{-\sin 24^\circ}{\sin 24^\circ} \quad \text{but/maar } \sin 24^\circ = \sin 2(12)$ $\frac{-2 \sin 12 \cos 12}{\sin 12 \cos 12}$ -2	<p>✓ One fraction with common denominator</p> <p>✓ $\sin(36^\circ - 12^\circ)$</p> <p>✓ $\sin 24^\circ = \sin 2(12)$</p> <p>✓ Answer: -2</p> <p>(4)</p>

5.4.1	$\text{L.H.S} = \frac{2\sin^2 x + \sin 2x}{\cos 2x}$ $= \frac{2\sin^2 x + 2\sin x \cos x}{\cos^2 x - \sin^2 x}$ $= \frac{2\sin x(\sin x + \cos x)}{(\cos x - \sin x)(\cos x + \sin x)}$ $= \frac{2\sin x}{(\cos x - \sin x)}$ $\therefore \text{L.H.S} = \text{R.H.S}$	<p>✓ double angle sin</p> <p>✓ common factor</p> <p>✓ factor denominator</p> <p>✓ double angle cos</p> <p>(4)</p>
5.4.2	<p>Invalid for/Ongeldig vir:</p> <p>$\cos 2x = 0$ and/en $\cos x = \sin x$</p> <p>$2x = \pm \cos^{-1} 0 + 360^\circ \cdot k, k \in \mathbb{Z}$</p> <p>$2x = 90^\circ + 360 \cdot k$ or/of $2x = -90^\circ + 360 \cdot k$</p> <p>$x = 45^\circ + 180 \cdot k$ or of $x = -45^\circ + 180 \cdot k$</p> <p>$x \in \{-45^\circ; 45^\circ\}$</p>	<p>✓ $x = 45^\circ$</p> <p>✓ $x = -45^\circ$</p> <p>(2)</p>
5.5	$AB^2 = (x_B - x_A)^2 + (y_B - y_A)^2$ $(\sqrt{86})^2 = (6 - \cos \theta)^2 + (7 - \sin \theta)^2$ $86 = 36 - 12 \cos \theta + \cos^2 \theta + 49 - 14 \sin \theta + \sin^2 \theta$ $1 = -12 \cos \theta - 14 \sin \theta + \sin^2 \theta + \cos^2 \theta$ $1 = -12 \cos \theta - 14 \sin \theta + 1$ $14 \sin \theta = -12 \cos \theta$ $\therefore \frac{\sin \theta}{\cos \theta} = \frac{-12}{14}$ $\therefore \tan \theta = \frac{-12}{14} = \frac{-6}{7}$	<p>✓ sub into corr formulae</p> <p>✓ equating</p> <p>✓ simplification</p> <p>✓ $\frac{\sin \theta}{\cos \theta} = \frac{-12}{14}$</p> <p>✓ $\tan \theta = \frac{-12}{14}$</p> <p>(5)</p>
		[28]

QUESTION/VRAAG 6

6.1	$P\hat{S}Q = 90^\circ$ $Q\hat{S}R = \alpha$ $P\hat{S}R = 90^\circ + \alpha$ $R = 90^\circ - 2\alpha$	semi-circle/ <i>semi-sirkel</i> tan-chord/ <i>tankoord</i> sum of angles of triangle <i>som van hoeke van driehoek</i>	✓ answer (1)
6.2	In ΔQSR $\frac{QS}{\sin R} = \frac{QR}{\sin S}$ $QS = \frac{k \sin(90^\circ - 2\alpha)}{\sin \alpha}$ $= \frac{k \cos 2\alpha}{\sin \alpha}$	✓ sine rule ✓ $\frac{k \cos 2\alpha}{\sin \alpha}$	(2)
6.3	In ΔPRS $\frac{RS}{\sin \alpha} = \frac{4k}{\sin(90^\circ + \alpha)}$ $\frac{PS}{\sin(90^\circ - 2\alpha)} = \frac{4k}{\sin(90^\circ + \alpha)}$ $PS = \frac{4k \cos 2\alpha}{\cos \alpha}$	✓ sine rule ✓ $\cos \alpha$ ✓ $\cos 2\alpha$	(3)
6.4	$\tan \alpha = \frac{QS}{PS}$ $\sin \alpha = \frac{QS}{3k}$ $QS = 3k \sin \alpha$ $\tan \alpha = \frac{QS}{PS}$ $= \frac{3k \sin \alpha}{1} \times \frac{\cos \alpha}{4k \cos 2\alpha}$ $= 3 \cdot \frac{1}{2} \sin 2\alpha \div 4 \cos 2\alpha$ $= \frac{3 \sin 2\alpha}{8 \cos 2\alpha}$ $= \frac{3}{8} \tan 2\alpha$	✓ $QS = 3k \sin \alpha$ ✓ $\frac{1}{2} \sin 2\alpha$ ✓ $\frac{3 \sin 2\alpha}{8 \cos 2\alpha}$	(3)
			[9]

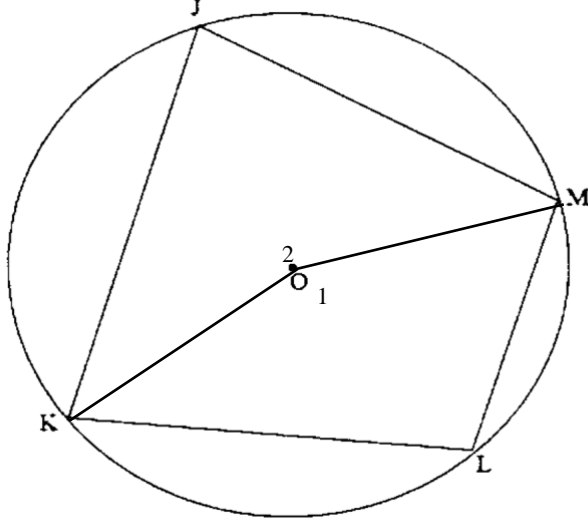
QUESTION/VRAAG 7

7.1.1	$A(-30^\circ; 0)$	✓ answer (1)
7.1.2	$f(180^\circ) = -\sin(210^\circ)$ $= 0,5$ $g(180^\circ) = 2 \cos(180^\circ)$ $= -2$ $\therefore CD = \frac{5}{2}$	✓✓ answer (2)
7.2	360°	✓ 360° (1)
7.3	$2 \cos x + \sin(x + 30^\circ) = 0$ $-\sin x \cos 30^\circ + \cos x \sin 30^\circ = 2 \cos x$ $-\frac{\sqrt{3}}{2} \sin x - \frac{1}{2} \cos x = 2 \cos x$ $-\frac{\sqrt{3}}{2} \sin x = \frac{5}{2} \cos x$ $\div -\frac{\sqrt{3}}{2} \cos x$ $\tan x = \frac{-5}{\sqrt{3}}$ $\therefore x = 109,11^\circ + 180.k \ (k \in \mathbb{Z})$	✓ expansion ✓ simplification ✓ equate ✓ $\div -\frac{\sqrt{3}}{2} \cos x$ ✓ $\tan x = \frac{-5}{\sqrt{3}}$ ✓ correct answer $k \in \mathbb{Z}$ (6)
7.4	For/Vir $g(x) > f(x)$: $-70,89^\circ < x < 109,11^\circ$ $-90,89^\circ < x < 89,11^\circ$	✓ ✓ $-90,89^\circ$ ✓ $89,11^\circ$ (3)
		[13]

QUESTION/VRAAG 8

8.1	$\hat{C}_1 = 48^\circ$ $\hat{C}_1 = G = 48^\circ$	given/gegee tan chord/tankoord	✓ S ✓ R (2)
8.2	$\hat{C}_2 = 90^\circ - 48^\circ$ $\hat{C}_2 = 42^\circ$	\angle between tan and diameter/ tussen tan en middellyn	✓ R ✓ S (3)
8.3	$\angle \hat{E}F = 90^\circ$ $\therefore \hat{D} = 90^\circ - 48^\circ = 42^\circ$	\angle in $\frac{1}{2}$ Ext/Verlengde \angle of/van Δ	✓ S ✓ R ✓ S/R (3)
			[8]

QUESTION/VRAAG 9

9.1	 <p>Construction/Konstruksie: Join KO and/en OM.</p> <p>$\hat{O}_1 = 2\hat{J}$ \angle at centre = 2 x \angle on circumference \angle by middelp = 2 x \angle op omtrek</p> <p>$\hat{O}_2 = 2\hat{L}$ \angle at centre = 2 x \angle on circumference \angle middelp = 2 x \angle op omtrek</p> <p>but/maar $\hat{O}_1 + \hat{O}_2 = 360^\circ$ revolution /revolusie</p> <p>$\therefore \hat{J} + \hat{L} = 180^\circ$</p>	<p>construction ✓</p> <p>S✓ R ✓</p> <p>S✓</p> <p>S✓</p> <p>(5)</p>
9.2.1	<p>$\hat{A}SB = \hat{A}TB$ angle sub by AB/hoek verv by AB</p> <p>$\hat{A}TB = \hat{B}DR$ ext angle of cyclic quad/buitehoek van koordevierkant</p> <p>$\therefore SCDB$ is a cyclic quad ext angle of quad = opp angle/ is 'n koordevierhoek, buitehoek van koordevierhoek = teenoorg hoek</p>	<p>✓✓ S/R</p> <p>✓✓ S/R</p> <p>✓ R</p> <p>(5)</p>
9.2.2	<p>$\hat{A} = \hat{B}_1$ angle in same segment/hoek in dieselfde segment</p> <p>$\hat{R}_1 = \hat{B}_2$ ext of cyclic quad/verl van koordevierkant</p> <p>$\therefore \hat{S}BD = \hat{B}_1 + \hat{B}_2$</p> <p>$= A + \hat{R}_1$</p>	<p>S✓</p> <p>S✓</p> <p>(2)</p>
		[12]

QUESTION/VRAAG 10

10.1.1	$\hat{S}_1 = \hat{V}_1$ But/ <i>Maar</i> $\hat{V}_1 = \hat{R}_1$ $\therefore \hat{S}_1 = \hat{R}_1$	alt angles/ <i>hoeke</i> SP// TV ext angle of cyclic quad/ <i>buitehoek van koordevierkant</i>	✓ S/R ✓ S✓R <
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QUESTION/VRAAG 11

11.1	<p>Let/Laat $BP = 4x$ and/en $PC = 3x$</p> <p>KMCP is a/n 11^m 2 pairs of opp sides $11/$ $2\text{ pare van teenoorg kante } 11$</p> <p>$\therefore KM = PC = 3x$ $\frac{KM}{BC} = \frac{3x}{7x} = \frac{3}{7}$</p>	<p>✓ S</p> <p>✓ S</p> <p>answer ✓ (3)</p>
11.2	<p>$\frac{AC}{AM} = \frac{AB}{AK}$ prop theorem/bewys, KM//BC</p> <p>$\frac{AB}{AK} = \frac{CB}{CP} = \frac{7}{3}$ prop theorem/bewys, KP//AC</p> <p>$\frac{AC}{AM} = \frac{7}{3}$</p>	<p>S ✓</p> <p>✓ S</p> <p>✓ $\frac{7}{3}$</p> <p>(3)</p>
11.3	<p>Area of/Oppvl van $\triangle KBP$ = $\frac{\frac{1}{2}KB \cdot BP \cdot \sin B}{\frac{1}{2}AB \cdot BC \cdot \sin B}$</p> <p>$= \frac{4}{7} \cdot \frac{4}{7}$</p> <p>$= \frac{16}{49}$</p>	<p>✓ areas</p> <p>✓ ratio</p> <p>✓ $\frac{4}{7} \cdot \frac{4}{7}$</p> <p>✓ answer (4)</p>
		[10]

TOTAL/TOTAAL: 150