



**education**

Department of  
Education  
**FREE STATE PROVINCE**

# **PREPARATORY EXAMINATION**

**GRADE 12**

**MATHEMATICS P1**

**SEPTEMBER 2022**

**TIME: 3 HOURS**

**MARKS: 150**

**This question paper consists of 9 pages and 1 information sheet.**

## INSTRUCTIONS AND INFORMATION

Read the following instructions carefully before answering the questions.

1. This question paper consists of 11 questions.
2. Answer ALL the questions.
3. Number the answers correctly according to the numbering system used in this question paper.
4. Clearly show ALL calculations, diagrams, graphs, et cetera that you have used in determining your answers.
5. Answers only will NOT necessarily be awarded full marks.
6. You may use an approved scientific calculator (non-programmable and non-graphical), unless stated otherwise.
7. If necessary, round off answers to TWO decimal places, unless stated otherwise.
8. Diagrams are NOT necessarily drawn to scale.
9. An information sheet with formulae is included at the end of this question paper.
10. Write neatly and legibly.

## QUESTION 1

1.1 Solve for  $x$ :

1.1.1  $3x^2 - 5x - 2 = 0$  (2)

1.1.2  $3x - 4 = \frac{2}{x} \quad (x \neq 0)$  (4)  
(correct to TWO decimal places)

1.1.3  $x^2 - 8x + 16 > 0$  (3)

1.1.4  $\sqrt{5x-1} = 2x-1$  (4)

1.1.5  $3^{x+1} + m \cdot 3^x = 2m + 6$  (correct to TWO decimal places) (4)

1.2 Solve simultaneously for  $x$  and  $y$  in:

$2x + y = -1$  and  $y^2 + 3xy + 2 = 0$  (6)

**[23]**

## QUESTION 2

2.1 Consider the arithmetic series:  $- 21 - 17 - 13 \dots + 91$

2.1.1 Determine the:

(a) smallest positive number of the series (2)

(b) the number of terms in the series (3)

2.1.2 Calculate the sum of the series. (2)

2.2 The  $n^{\text{th}}$  term of a quadratic pattern is  $T_n = an^2 + bn + c$  and the third term of the pattern is  $-1$ . The values  $-12 ; -8 ; -4 ; \dots$  are the first differences of this pattern.

2.2.1 Write down the values of the first two terms of the pattern. (2)

2.2.2 Determine the values of  $a$ ,  $b$  and  $c$ . (3)

2.2.3 Which term of the pattern is equal to 139? (4)

**[16]**



### QUESTION 3

3.1 Consider the following geometric series:

$$3 + m + \frac{m^2}{3} + \frac{m^3}{9} + \dots \quad (m \neq 0)$$

3.1.1 For which value(s) of  $m$  will the series converge? (3)

3.1.2 The sum to infinity of the series is  $\frac{27}{7}$ , calculate the value of  $m$ . (2)

3.2 Determine the value of  $p$  if :  $\sum_{x=2}^p 3(2)^{x-1} = 1530$  (4)  
[9]

### QUESTION 4

The equation of a parabola is given by  $f(x) = ax^2 + bx + c$ . The roots of  $f$  are  $(m-5)$  and  $(m+3)$ . The maximum value of  $f$  occurs at  $x = 2$ .

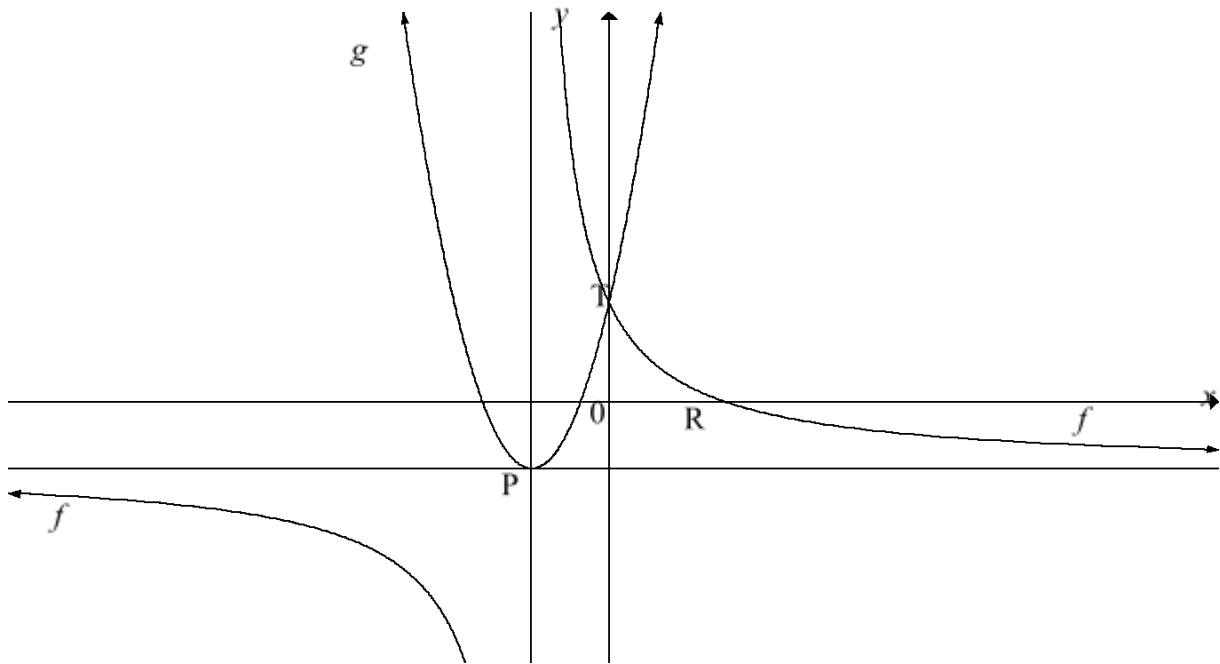
4.1 Determine the value of  $m$ . (2)

4.2 Determine the equation of  $f$  in the form  $f(x) = ax^2 + bx + c$  if it is also given that  $f(1) = 15$ . (4)

4.3 Determine the range of  $g$  if  $g(x) = f(x) - 4$ . (3)  
[9]

### QUESTION 5

The diagram below shows the graphs of  $f(x) = \frac{5}{x+p} + q$  and  $g(x) = 5x^2 + 10x + 3$ . The two graphs intersect at T, the y-intercepts of both graphs. R is the x-intercept of  $f$ . The asymptotes of  $f$  cut at P, the turning point of  $g$ .

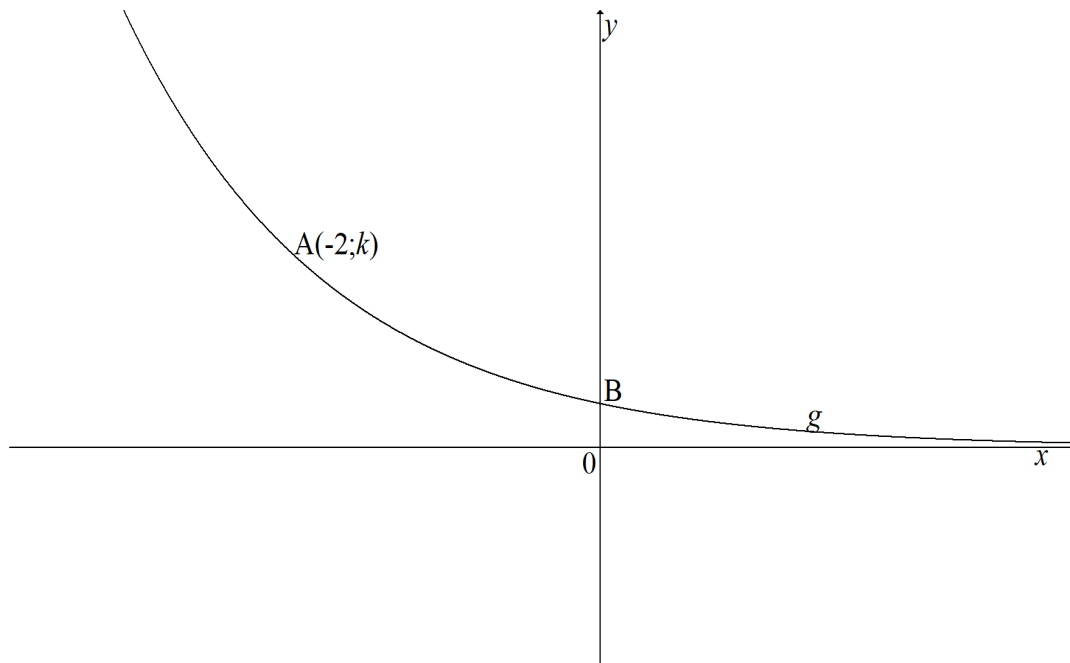


- 5.1 Write down the coordinates of T. (2)
- 5.2 Calculate the coordinates of P. (4)
- 5.3 Write down the equation of  $f$ . (2)
- 5.4 Determine the equation of the:
  - 5.4.1 Tangent to  $g$ , touching  $f$  at T. (3)
  - 5.4.2 Axis of symmetry of  $f$ , with a positive gradient. (3)
- 5.5 Determine for which values of  $x$  will  $g'(x) \times f(x) \leq 0$ . (4)

**[18]**

**QUESTION 6**

In the diagram, the graph of  $g(x) = \left(\frac{1}{3}\right)^x$  is drawn. B is the y-intercept of  $g$  and  $A(-2; k)$  is another point on  $g$ .



- 6.1 Calculate the value of  $k$ . (2)
- 6.2 Determine the equation of  $g^{-1}$ , the inverse of  $g$ , in the form  $y = \dots$  (2)
- 6.3 Draw the graph of  $g^{-1}$  in your ANSWER BOOK. Clearly show the intercepts with the axes as well as the coordinates of any other point. (3)
- 6.4 Consider the graph of  $h$ , where  $h(x) = g^{-1}(x+2)$  for which values of  $x$  is  $g^{-1}(x+2) \geq -2$ . (2)
- [9]**

### QUESTION 7

- 7.1 A motor car which costs R200 000 depreciates at 9% per annum on the reducing balance method. Calculate how long it will take for the car to depreciate to a value of R90 000. (3)
- 7.2 On 1 January 2017 Nelson deposited R3 500 into a savings account. On 1 January 2020, he deposited another R5 700 into the same account. The interest rate for the first two years is 7% per annum compounded quarterly, and the interest rate for the last three years is 8% per annum compounded monthly. Calculate the amount in the savings account after 5 years. (4)
- 7.3 Edward bought a house for R6 200 000. He paid 35% as a deposit and took out a bank loan for the balance at an interest rate of 15% per annum compounded monthly. Calculate the:
- 7.3.1 Monthly instalment if the loan was to be paid over 20 years and his first payment is made one month after the granting of the loan. (4)
- 7.3.2 Balance outstanding at the end of the sixth year. (3)
- [14]

### QUESTION 8

- 8.1 Given that:  $f(x) = -x^2 + 3$
- 8.1.1 Determine  $f'(x)$  from first principles. (5)
- 8.1.2  $y = 12x - 8$  is a tangent to the graph of  $f$  at A. Calculate the coordinates of A. (3)
- 8.2 Determine:
- 8.2.1  $D_x \left[ \sqrt[3]{x^2} - \frac{3}{x} + \pi \right]$  (3)
- 8.2.2  $\frac{dy}{dx}$ , if  $y = \frac{2x^2 - x - 6}{2x + 3}$  (2)
- [13]



### QUESTION 9

Given:  $f(x) = 2x^3 - 5x^2 - 4x + 3$

- 9.1 Solve for  $x$  if  $f(x) = 0$ . (4)
- 9.2 Calculate the coordinates of the turning points of the graph of  $f$ . (5)
- 9.3 Draw the graph of  $f$ , clearly indicate the intercepts with the axes and the turning points. (3)
- 9.4 For which values of  $x$  is the graph of  $f$  concave up? (3)
- 9.5 What constant must be added to  $f(x)$  to make it divisible by  $(x-2)$ ? (2)
- [17]

### QUESTION 10

A water tank with an inlet and an outlet is used to water the garden. The equation

$D(t) = 3 + \frac{1}{2}t^2 - \frac{1}{4}t^3$  gives the depth of water in metres where  $t$  is the time in hours that has elapsed since 07:30.

- 10.1 What is the depth of the water at 09:30? (2)
- 10.2 At what rate does the depth of the water change at 10:30? (3)
- 10.3 At what time will the inflow of the water be the same as the outflow of the water? (2)
- [7]

### QUESTION 11

- 11.1 A and B are two events in a sample space.  $P(\text{not } A) = 0,45$  and  $P(B) = 0,35$ .
- 11.1.1 Determine the  $P(A \text{ or } B)$  if A and B are mutually exclusive. (3)
- 11.1.2 Determine  $P(A \text{ and } B)$  if A and B are independent events. (2)
- 11.2 A classroom has 14 black chairs and 18 red chairs. During the day, two chairs are randomly removed for a meeting in another classroom.
- Determine the probability that:
- 11.2.1 Both taken chairs were red. (2)
- 11.2.2 A black and a red chair were taken. (2)
- 11.3 Five boys and four girls go to the movies. They are all seated next to each other in the same row.
- 11.3.1 In how many ways can the entire group be seated in the row? (1)
- 11.3.2 One boy and a girl are a couple and want to sit next to each other at any end of the row of friends. In how many different ways can the entire group be seated? (2)
- 11.3.3 If all friends are seated randomly, calculate the probability that all girls are seated next to each other. (3)

[15]

**TOTAL: 150**

# INFORMATION SHEET

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$A = P(1 + ni)$$

$$A = P(1 - ni)$$

$$A = P(1 - i)^n$$

$$A = P(1 + i)^n$$

$$T_n = a + (n - 1)d$$

$$S_n = \frac{n}{2}[2a + (n - 1)d]$$

$$T_n = ar^{n-1}$$

$$S_n = \frac{a(r^n - 1)}{r - 1} ; r \neq 1$$

$$S_\infty = \frac{a}{1 - r} ; -1 < r < 1$$

$$F = \frac{x[(1 + i)^n - 1]}{i}$$

$$P = \frac{x[1 - (1 + i)^{-n}]}{i}$$

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x + h) - f(x)}{h}$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \quad M\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

$$y = mx + c$$

$$y - y_1 = m(x - x_1)$$

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \tan \theta$$

$$(x - a)^2 + (y - b)^2 = r^2$$

$$\text{In } \triangle ABC: \quad \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cdot \cos A$$

$$\text{area } \triangle ABC = \frac{1}{2}ab \cdot \sin C$$

$$\sin(\alpha + \beta) = \sin \alpha \cdot \cos \beta + \cos \alpha \cdot \sin \beta$$

$$\sin(\alpha - \beta) = \sin \alpha \cdot \cos \beta - \cos \alpha \cdot \sin \beta$$

$$\cos(\alpha + \beta) = \cos \alpha \cdot \cos \beta - \sin \alpha \cdot \sin \beta$$

$$\cos(\alpha - \beta) = \cos \alpha \cdot \cos \beta + \sin \alpha \cdot \sin \beta$$

$$\cos 2\alpha = \begin{cases} \cos^2 \alpha - \sin^2 \alpha \\ 1 - 2\sin^2 \alpha \\ 2\cos^2 \alpha - 1 \end{cases}$$

$$\sin 2\alpha = 2\sin \alpha \cdot \cos \alpha$$

$$\bar{x} = \frac{\sum fx}{n}$$

$$\sigma^2 = \frac{\sum_{i=1}^n (x_i - \bar{x})^2}{n}$$

$$P(A) = \frac{n(A)}{n(S)}$$

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

$$\hat{y} = a + bx$$

$$b = \frac{\sum (x - \bar{x})(y - \bar{y})}{\sum (x - \bar{x})^2}$$