KWAZULU-NATAL PROVINCE

EDUCATION
REPUBLIC OF SOUTH AFRICA

NATIONAL SENIOR CERTIFICATE

GRADE 12

MATHEMATICS

COMMON TEST

MARCH 2023

MARKS: 100

TIME: 2 hours

N.B. This question paper consists of 10 pages including information sheet.

INSTRUCTIONS AND INFORMATION

Read the following instructions carefully before answering the questions.

1. This question paper consists of 10 questions.
2. Answer **ALL** questions.
3. Clearly show **ALL** calculations, diagrams, graphs, et cetera that you have used in determining your answers.
4. Answers only will not necessarily be awarded full marks.
5. An approved scientific calculator (non-programmable and non-graphical) may be used, unless stated otherwise.
6. If necessary, answers should be rounded off to TWO decimal places, unless stated otherwise.
7. Diagrams are NOT necessarily drawn to scale.
8. Number the answers correctly according to the numbering system used in this question paper. Write neatly and legibly.

QUESTION 1

Consider the quadratic sequence:

$2 \ ; \ 7 \ ; \ 14 \ ; \ 23 \ ; \ \ldots\ldots$

1.1 Write down the fifth term. (1)

1.2 Determine the general term T_n (4)

1.3 If 5 is added to each term of the quadratic sequence, between which two terms of the sequence is the first difference 57 ? (3)

[8]

QUESTION 2

2.1 $10 \ ; \ a \ ; \ 24 \ ; \ b \ ; \ 38 \ ; \ \ldots\ldots$ are the first five terms of an arithmetic progression.

2.1.1 Show that $a = 17$ and $b = 31$ (2)

2.1.2 Calculate the sum of the first 67 terms of the sequence (2)

2.1.3 If there are 67 terms in this arithmetic sequence, determine the sum of all the even terms of this sequence. (3)

2.2 Calculate the value of: $\sum_{r=2}^{\infty} 3.2^{1-r} + \sum_{r=2}^{12} 3.2^{1-r}$

(give your answer to 3 decimal places). (3)

[10]

QUESTION 3

3.1 $(x-2)^2+(x-2)^3+(x-2)^4+..........$ form a geometric series.

3.1.1 Write down the common ratio. (1)

3.1.2 Determine the value(s) of x for which the series will converge. (2)

3.2 Mr Peter gave his four sons R 8400 to share, such that their shares formed terms of a geometric sequence. The largest share was 27 times the smallest share.

Determine the amount each son received. (4)

[7]

QUESTION 4

Given: $f(x)=\dfrac{2}{x}$ and $g(x)=x-1$

4.1 Determine the coordinates of the point(s) where the two graphs intersect. (4)

4.2 On the same set of axes, draw the two functions. Indicate the coordinates of the point(s) of intersection of the two graphs. (3)

4.3 Use your graphs to determine the value(s) of x for which: (3)

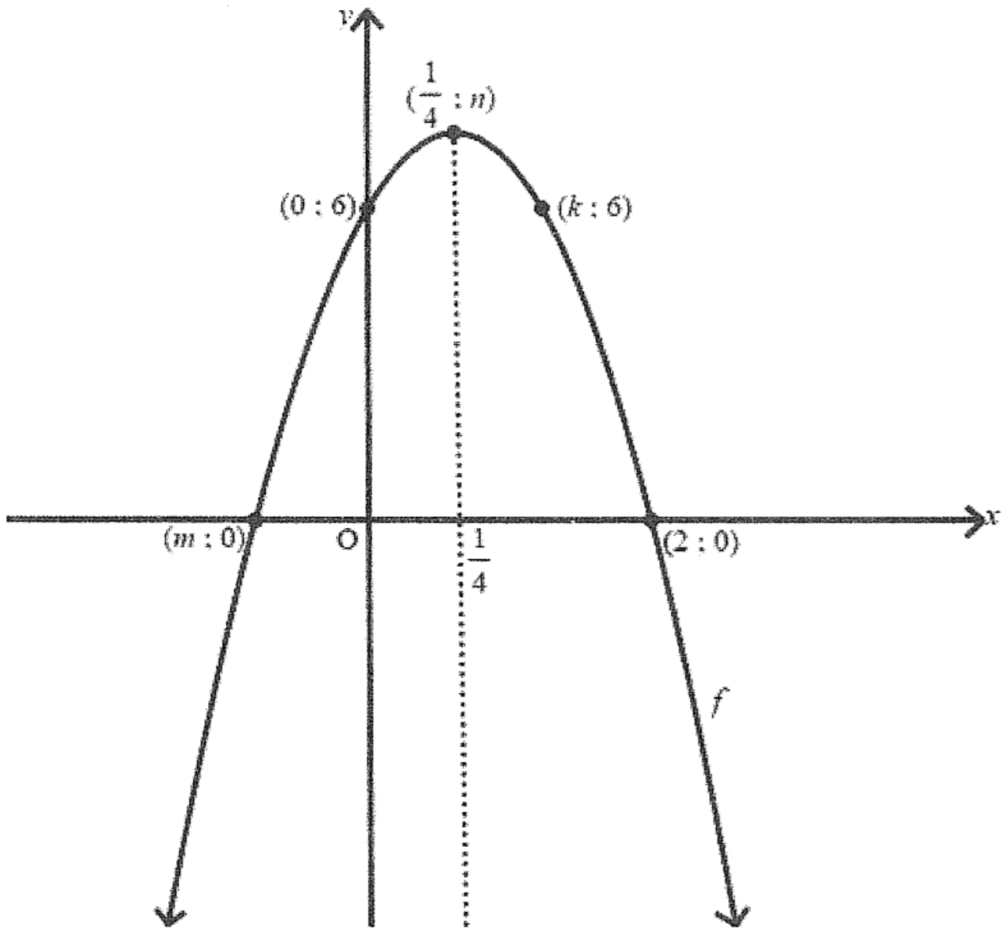
$\dfrac{2}{x}>x-1$

4.4 If $h(x)=\dfrac{2}{x+3}-4$, describe the transformation that takes $f(x)$ to $h(x)$. (2)

[12]

QUESTION 5

The diagram shows the graph of a parabola $f(x)$ which intersects the x-axis at $(m;0)$ and at $(2;0)$. It is further given that $(\frac{1}{4};n)$ is the turning point of the parabola while $(0;6)$ and also $(k;6)$ points on the curve of f.



Determine:

5.1 the value of k. (1)

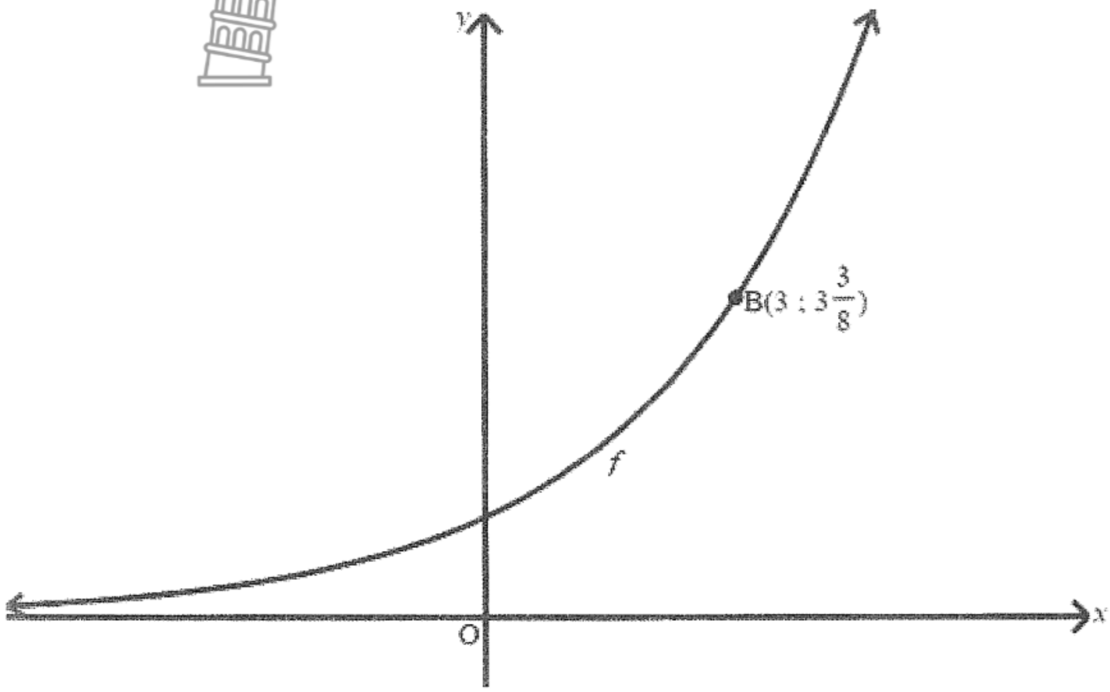
5.2 the value of m. (1)

5.3 the value of n (show all your working). (5)

[7]

QUESTION 6

The diagram shows the graph $f(x) = a^x$. Point B $(3;3\frac{3}{8})$, lies on f.



6.1 Show that $a = \frac{3}{2}$. (2)

6.2 Write down the domain of f. (1)

6.3 Draw the graph of g, the reflection of f in the line $y = x$ showing all the intercepts with the axes and the coordinates of another point on the graph. (3)

6.4 Write down the equation of g. (2)

6.5 Write down the values of x for which $g(x) < 3$ (1)

[9]

QUESTION 7

7.1 Given $\cos 40° = t$, **without using a calculator**, determine each of the following in terms of t:

7.1.1 $\tan 40°$ (2)

7.1.2 $\cos^2 130°$ (3)

7.1.3 $\cos 220°$ (3)

7.2 **Without using a calculator**, simplify the following expression:

$$\sin 237° \, . \, \cos 147° - \frac{\cos 213° \, . \, \cos 303^0}{\tan 237°}$$ (7)

[15]

QUESTION 8

8.1 Prove the identity:

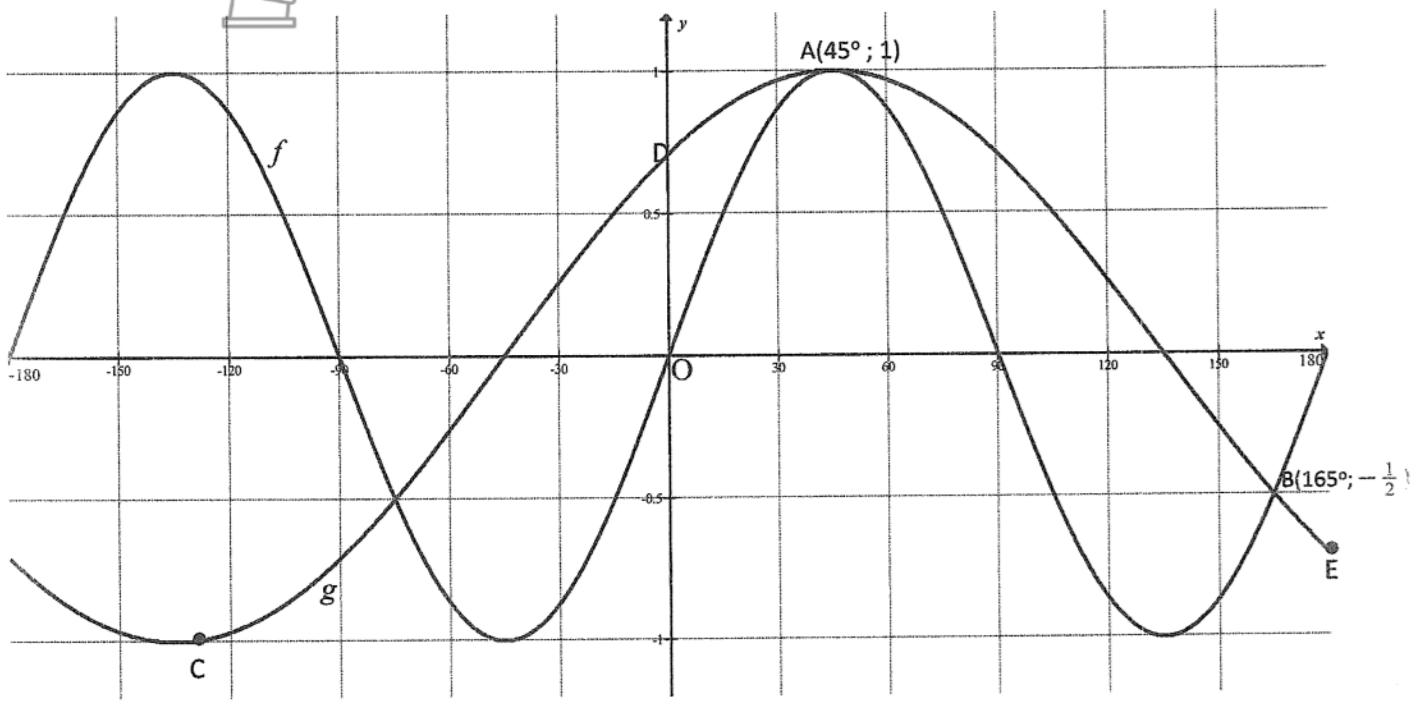
$$\tan x = \frac{1 - \cos 2x - \sin x}{\sin 2x - \cos x}$$ (5)

8.2 For which values of x in the interval $x \in [0°; 270°]$ is the identity not defined? (7)

[12]

QUESTION 9

In the diagram below, the graphs of $f(x) = \sin ax$ and $g(x) = \cos(x + b)$ are drawn for the Interval of $x \in [-180°; 180°]$



9.1 Determine the values of a and b. (2)

9.2 Write down the period of g. (1)

9.3 C is the turning point on g, determine the co-ordinates of C. (2)

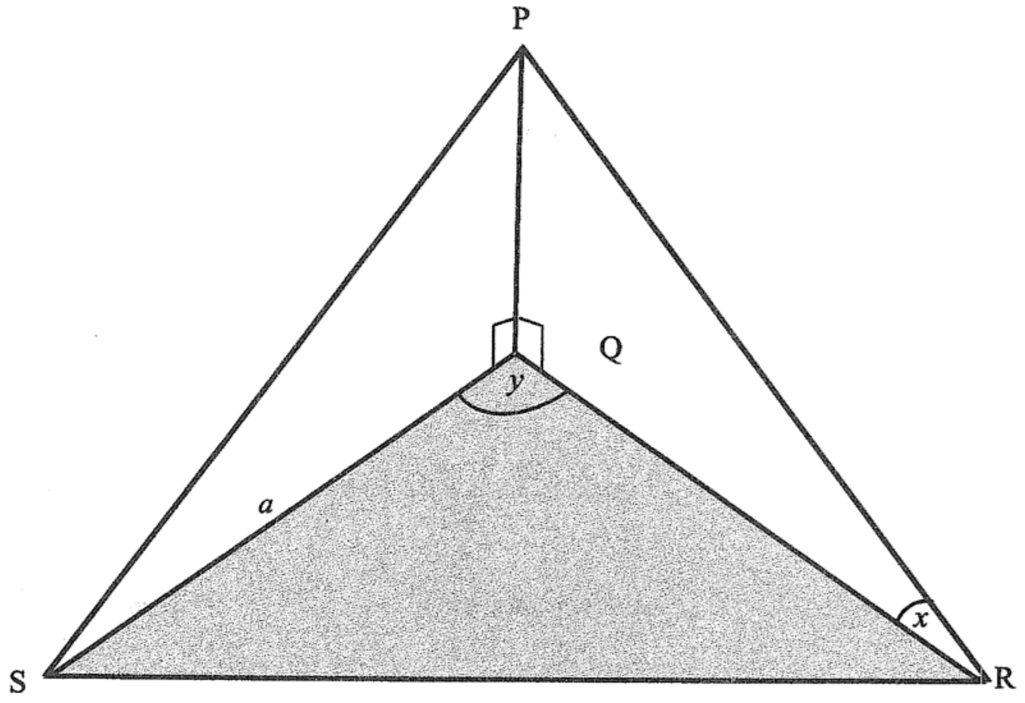
9.4 Determine the co-ordinates of D and E. (2)

9.5 Use the graphs to determine the values of x in the interval $x \in [0^0; 180°]$ for which $g(x) > f(x)$. (3)

[10]

QUESTION 10

In the diagram below, Q is the foot of a vertical tower PQ, while R and S are two points in the same horizontal plane as Q. The angle of elevation of P, as measured from R, is x.
$R\hat{Q}S = y$, QS = a metres and the area of a triangle RQS = Am^2.



10.1 Prove that $PQ = \dfrac{2A \cdot \tan x}{a \sin y}$ (5)

10.2 Calculate the area of Δ RQS, if:

$a = 89$m, PQ = 77m, $x = 46{,}5°$ and $y = 115°$. Round off the answer to two decimal digits. (5)

[10]

GRAND TOTAL: 100

INFORMATION SHEET: MATHEMATICS

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$A = P(1+ni)$ $\quad$ $A = P(1-ni)$ $\quad$ $A = P(1-i)^n$ $\quad$ $A = P(1+i)^n$

$T_n = a + (n-1)d$ $\quad$ $S_n = \frac{n}{2}\{2a + (n-1)d\}$

$T_n = ar^{n-1}$ $\quad$ $S_n = \frac{a\left(r^n - 1\right)}{r-1}$; $r \neq 1$ $\quad$ $S_\infty = \frac{a}{1-r}$; $-1 < r < 1$

$F = \frac{x\left[\left(1+i\right)^n - 1\right]}{i}$ $\quad$ $P = \frac{x\left[1-(1+i)^{-n}\right]}{i}$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $\quad$ $M\left(\frac{x_1 + x_2}{2}; \frac{y_1 + y_2}{2}\right)$ $\quad$ $m = \tan\theta$

$y = mx + c$ $\quad$ $y - y_1 = m(x - x_1)$ $\quad$ $m = \frac{y_2 - y_1}{x_2 - x_1}$

$$(x-a)^2 + (y-b)^2 = r^2$$

In ΔABC: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

$a^2 = b^2 + c^2 - 2bc.\cos A$

$\text{area } \Delta ABC = \frac{1}{2} ab.\sin C$

$\sin(\alpha + \beta) = \sin\alpha.\cos\beta + \cos\alpha.\sin\beta$ $\quad$ $\sin(\alpha - \beta) = \sin\alpha.\cos\beta - \cos\alpha.\sin\beta$

$\cos(\alpha + \beta) = \cos\alpha.\cos\beta - \sin\alpha.\sin\beta$ $\quad$ $\cos(\alpha - \beta) = \cos\alpha.\cos\beta + \sin\alpha.\sin\beta$

$$\cos 2\alpha = \begin{cases} \cos^2\alpha - \sin^2\alpha \\ 1 - 2\sin^2\alpha \\ 2\cos^2\alpha - 1 \end{cases}$$

$\sin 2\alpha = 2\sin\alpha.\cos\alpha$

$\bar{x} = \frac{\sum fx}{n}$ $\quad$ $\sigma^2 = \frac{\sum_{i=1}^{n}\left(x_i - \bar{x}\right)^2}{n}$

$P(A) = \frac{n(A)}{n(S)}$ $\quad$ $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$

$\hat{y} = a + bx$ $\quad$ $b = \frac{\sum(x - \bar{x})(y - \bar{y})}{\sum(x - \bar{x})^2}$