



**NATIONAL  
SENIOR CERTIFICATE  
*NASIONALE  
SENIORSERTIFIKAAT***

**GRADE/GRAAD 12**

**JUNE/JUNIE 2023**

**MATHEMATICS P2/WISKUNDE V2  
MARKING GUIDELINE/NASIENRIGLYN**

**MARKS/PUNTE: 150**

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This marking guideline consists of 12 pages.  
*Hierdie nasienriglyn bestaan uit 12 bladsye.*

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## QUESTION 1/VRAAG 1

1.1	16	✓✓ answer / antwoord	(2)
1.2	$sd = 5,83$	✓✓ answer / antwoord	(2)
1.3	$16 + 5,83 = 21,83 \therefore 2 \text{ above/bokant.}$	✓✓ answer / antwoord	(2)
1.4	$16 - 5,83 = 10,17$ $\frac{3}{11} \times 100 = 27,27\%$	✓✓	(2)
			[8]

## QUESTION 2/VRAAG 2

	<table><tr><th>Hours / Ure</th><th>Number of dads <i>Aantal pa's</i></th><th>Cumulative frequency <i>Kumulatiewe frekwensie</i></th></tr><tr><td><math>0 &lt; x \leq 5</math></td><td>1</td><td>1</td></tr><tr><td><math>5 &lt; x \leq 10</math></td><td>18</td><td>19</td></tr><tr><td><math>10 &lt; x \leq 15</math></td><td>24</td><td>43</td></tr><tr><td><math>15 &lt; x \leq 20</math></td><td>25</td><td>68</td></tr><tr><td><math>20 &lt; x \leq 25</math></td><td>18</td><td>86</td></tr><tr><td><math>25 &lt; x \leq 30</math></td><td>12</td><td>98</td></tr><tr><td><math>30 &lt; x \leq 35</math></td><td>1</td><td>99</td></tr><tr><td><math>35 &lt; x \leq 40</math></td><td>1</td><td>100</td></tr></table>	Hours / Ure	Number of dads <i>Aantal pa's</i>	Cumulative frequency <i>Kumulatiewe frekwensie</i>	$0 < x \leq 5$	1	1	$5 < x \leq 10$	18	19	$10 < x \leq 15$	24	43	$15 < x \leq 20$	25	68	$20 < x \leq 25$	18	86	$25 < x \leq 30$	12	98	$30 < x \leq 35$	1	99	$35 < x \leq 40$	1	100	
Hours / Ure	Number of dads <i>Aantal pa's</i>	Cumulative frequency <i>Kumulatiewe frekwensie</i>																											
$0 < x \leq 5$	1	1																											
$5 < x \leq 10$	18	19																											
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$30 < x \leq 35$	1	99																											
$35 < x \leq 40$	1	100																											
2.1	<div><p><b>Working dads help working moms.</b> <i>Werkende pa's help werkende ma's</i></p><p><b>CUMALITIVE FREQUENCY</b> <b>KUMULATIEWE FREKWENSIE</b></p><p><b>HOURS PER WEEK</b> <b>URE PER WEEK</b></p></div>	<div><p>✓ table / tabel</p><p>✓ anchor point / ankerpunt (0;0)</p><p>✓ (10; 19) (25; 86)</p><p>✓ (40; 100)</p></div>	(4)																										
2.2	$\pm 16$ (Accept values from 14 to 18) / (Aanvaar waardes vanaf 14 tot 18)	✓✓ answer antwoord	(2)																										
2.3	$15 < x \leq 20$	✓ answer / antwoord	(1)																										
2.4	$\bar{x} = 16,8$	✓✓ answer / antwoord	(2)																										
2.5	Mean $\approx$ Median and lies in the Modal class. Data is symmetrical / Normal / Data not skewed. <i>Gemiddelde <math>\approx</math> Mediaan en lê in die Modale klas</i> <i>Data is simmetries / Normaal / Data nie skeefnie.</i>	✓ ✓✓ reason / rede	(3)																										
			[12]																										

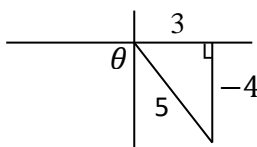
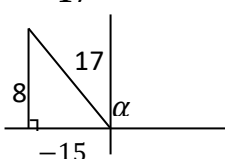
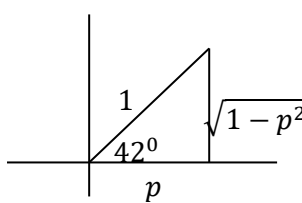
## QUESTION 3/VRAAG 3

3.1	$y = -\frac{1}{3}x + 10 \quad (m_1 \times m_2 = -1)$ $\frac{p}{2} = -\frac{1}{3}(4) + 10$ $p = \frac{52}{3}$	✓ equation of line / <i>verg. van lyn</i> ✓ substitution of point / <i>vervangings van punt</i>  ✓ answer / <i>antwoord</i>	(3)
3.2	O(0;0) and/en P(-2; p - 1) and/en OP = 2 units/eenhede. $OP^2 = (-2 - 0)^2 + (p - 1 - 0)^2$ $(2p)^2 = 4 + p^2 - 2p + 1$ $4p^2 = p^2 - 2p + 5$ $3p^2 + 2p - 5 = 0$ $(3p + 5)(p - 1) = 0$ $p = -\frac{5}{3}$ or $p = 1$ (slegs) $p = 1$ only	✓ substitution / <i>vervangings</i> ✓ simplification / <i>vereenvoudiging</i> ✓ standard form / <i>standaardvorm</i> ✓ factors / <i>faktore</i>  ✓ answer / <i>antwoord</i>	(5)
3.3.1	$m_{BD} = 2$	✓✓ answer / <i>antwoord</i>	(2)
3.3.2	Midpoint of BD : <i>Middelpunt van BD</i> (0; -1) $y = -\frac{1}{2}x - 1$	✓✓ midpoint / <i>middelpunt</i>  ✓ answer / <i>antwoord</i>	(3)
3.3.3	$x^2 + y^2 = 25$	✓✓ answer / <i>antwoord</i>	(2)
3.3.4	$x^2 + \left(-\frac{1}{2}x - 1\right)^2 = 25$ $x^2 + \frac{1}{4}x^2 + x + 1 = 25$ $4x^2 + x^2 + 4x + 4 = 100$ $5x^2 + 4x - 96 = 0$ $(5x + 24)(x - 4) = 0$ $x = -\frac{24}{5}$ or/of $x = 4$ $\therefore y = \frac{7}{5}$	✓ substitution / <i>vervangings</i>  ✓ simplification / <i>vereenvoudiging</i>  ✓ standard form / <i>standaardvorm</i> ✓ factorisation / <i>faktorisering</i>  ✓ x-values / <i>x-waardes</i>  ✓ answer / <i>antwoord</i>	(6)
			[21]

## QUESTION 4 / VRAAG 4

4.1	$x^2 + y^2 = 16$ $x^2 + (4 - 2x)^2 = 16$ $x^2 + 16 - 16x + 4x^2 = 16$ $5x^2 - 16x = 0$ $x(5x - 16) = 0$ $x = 0$ or/of $x = \frac{16}{5} = 3,2$ $y = 4 - 2(0)$ $y = 4 - 2(3,2)$ $y = 4$ $y = -2,4$	$y = 4 - 2x$ $\checkmark$ substitution / <i>vervanging</i> $\checkmark$ standard form / <i>standaardvorm</i> $\checkmark$ factors / <i>faktore</i>  $\checkmark$ x-values / <i>x-waardes</i> $\checkmark$ substitution / <i>vervanging</i> $\checkmark$ y-values / <i>y-waardes</i>	(7)
4.2	$S(-3,2 ; 2,4)$	$\checkmark\checkmark$ answer / <i>antwoord</i>	(2)
4.3	$y = 4 - 2x$ $0 = 4 - 2x$ $2x = 4$ $x = 2$ $R(0 ; 2)$ $\therefore$ radius = 2 units / <i>eenhede</i> $(x - 2)^2 + y^2 = 4$	$\checkmark$ equating to 0 / <i>gelyk stel aan 0</i> $\checkmark$ $x = 2$ $\checkmark$ radius = 2 units/ <i>eenhede</i>  $\checkmark$ answer / <i>antwoord</i>	(4)
4.4	$(x - y)^2 + y^2 - y = 12$ $(x - 6)^2 + y^2 - y + \frac{1}{4} = 12 + \frac{1}{4}$ $(x - 6)^2 + \left(y - \frac{1}{2}\right)^2 = \frac{49}{4}$ Centre/Middelpunt $\left(6 ; \frac{1}{2}\right)$ and/en $O(0 ; 0)$  $d = \sqrt{36 + \frac{1}{4}}$ $d \approx 6,02 / \frac{\sqrt{145}}{2}$	$\checkmark$ completing the square <i>vierkantsvoltooiing</i>  $\checkmark$ simplification / <i>vereenvoudiging</i>  $\checkmark$ coordinates of midpoint <i>koördinate van middelpunt</i>  $\checkmark$ substitution in distance formula <i>vervanging in afstand formule</i> $\checkmark$ answer / <i>antwoord</i>	(5)
			<b>[18]</b>

## QUESTION 5/VRAAG 5

5.1	$5 \cos \theta - 3 = 0$ $\cos \theta = \frac{3}{5}$ 	$17 \sin \alpha = 8$ $\sin \alpha = \frac{8}{17}$ 	$\checkmark \cos \theta = \frac{3}{5}$ $\checkmark \sin \alpha = \frac{8}{17}$ $\checkmark -4$ in correct quadrant in korrekte kwadrant $\checkmark -15$ in correct quadrant in korrekte kwadrant $\checkmark$ correct values / korrekte waardes $\checkmark$ answer / antwoord	(6)
	$\tan \alpha + \tan \theta$ $= -\frac{8}{15} + \left(-\frac{4}{3}\right)$ $= -\frac{28}{15}$			
5.2	$\cos 42^\circ = p$ 			
5.2.1	$\sin 48^\circ = p$	$\checkmark \checkmark$ answer / antwoord	(2)	
5.2.2	$\sin(-2202^\circ)$ $= \sin(-42^\circ)$ $= -\sin 42^\circ$ $= -\sqrt{1-p^2}$	$\checkmark -\sin(42^\circ)$ $\checkmark$ answer / antwoord	(2)	
5.2.3	$\cos 84^\circ$ $= \cos 2(42^\circ)$ $= 2 \cos^2 42^\circ - 1$ $= 2p^2 - 1$	$\checkmark \cos 2(42^\circ)$ $\checkmark$ answer / antwoord	(2)	
5.3	$\frac{\tan 300^\circ + \cos(90^\circ + x)}{\sin(180^\circ - x) + 2 \cos(-30^\circ)}$ $= \frac{-\tan 60^\circ - \sin x}{\sin x + 2\left(\frac{\sqrt{3}}{2}\right)}$ $= \frac{-\sqrt{3} - \sin x}{\sin x + \sqrt{3}}$ $= \frac{-(\sin x + \sqrt{3})}{(\sin x + \sqrt{3})}$ $= -1$		$\checkmark -\tan 60^\circ$ $\checkmark -\sin x$ $\checkmark \sin x$ $\checkmark \frac{\sqrt{3}}{2}$ $\checkmark$ taking out of negative sign. uithaal van negatiewe teken $\checkmark$ answer / antwoord	(6)

5.4	$\frac{1 - \sin 2x}{\cos 2x} = \frac{\cos x - \sin x}{\cos x + \sin x}$ $LHS = \frac{1 - 2 \sin x \cos x}{\cos^2 x - \sin^2 x}$ $LHS = \frac{\cos^2 x - 2 \sin x \cos x + \sin^2 x}{\cos^2 x - \sin^2 x}$ $LHS = \frac{(\cos x - \sin x)(\cos x + \sin x)}{(\cos x - \sin x)(\cos x + \sin x)}$ $LHS = \frac{\cos x - \sin x}{\cos x + \sin x}$	$\checkmark 2 \sin x \cos x$ $\checkmark \cos^2 x - \sin^2 x$ $\checkmark 1 = \cos^2 x + \sin^2 x$ $\checkmark$ factorising / <i>faktorisering</i> $\checkmark$ factorising / <i>faktorisering</i>	(5)
5.5	$\cos x - \sin x = \sqrt{2}$ $\frac{1}{\sqrt{2}} \cos x - \frac{1}{\sqrt{2}} \sin x = 1$ $\cos 45^\circ \cos x - \sin 45^\circ \sin x = 1$ $\cos(45^\circ + x) = 1$ $45^\circ + x = 0^\circ + 360^\circ \cdot k$ $x = -45^\circ + 360^\circ \cdot k$	$\checkmark$ division by / <i>deling deur</i> $\sqrt{2}$ $\checkmark \frac{1}{\sqrt{2}} = \cos 45^\circ / \sin 45^\circ$ $\checkmark$ expansion rule / <i>reël</i> $\checkmark 45^\circ + x = 0^\circ + 360^\circ \cdot k$ $\checkmark$ answer / <i>antwoord</i>	(5)
			[28]

## QUESTION 6/VRAAG 6

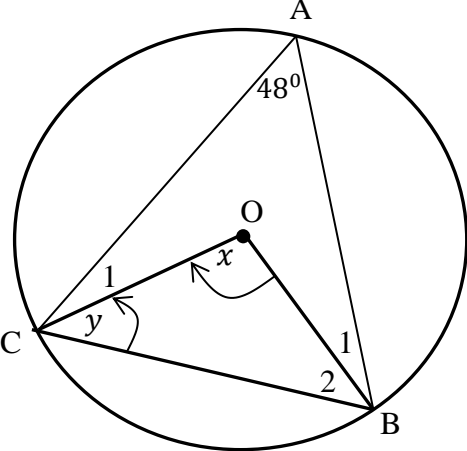
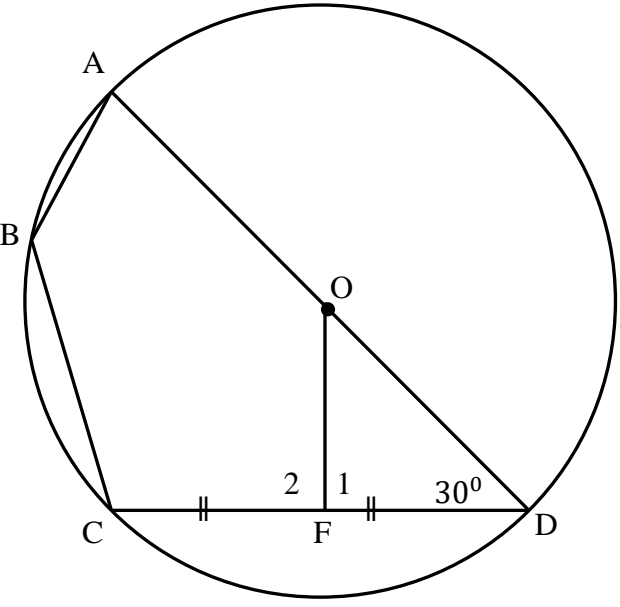
6.1		✓ shape / vorm ✓ intercepts / afsnitte ✓ starting and end points / begin en eindpunte	(3)
6.2	Period / Periode = $180^0$	✓ answer / antwoord	(1)
6.3	$h(x) = \tan x + 2$	✓ answer / antwoord	(1)
6.4	$-135^0 \leq x < -90^0$	✓ $-135^0$ ✓ $-90^0$ ✓ answer / antwoord	(3)
6.5	$\cos B + 1 = \tan \frac{1}{2}B$ Let/Laat $B = 2x$ $\cos 2x + 1 = \tan \frac{1}{2}(2x)$ $\cos 2x = \tan x - 1$ $x = -135^0$ and/en $x = 45^0$ $\therefore B = -270^0$ and/en $B = 90^0$	✓ $\cos 2x + 1 = \tan \frac{1}{2}(2x)$ ✓ $\cos 2x = \tan x - 1$ ✓ both $x$ -values <i>beide <math>x</math>-waardes</i> ✓ both $B$ values / <i>beide <math>B</math> waardes</i>	(4)
			[12]

## QUESTION 7/VRAAG 7

7.1	<p>In <math>\triangle OAK</math>  <math>\sin x = \frac{AK}{2}</math></p> <p><math>AK = 2 \sin x</math></p> <p><math>2 \sin x = 2 KT \sin x</math>  <math>KT = 1</math></p>	<p>In <math>\triangle KAT</math>  <math>\frac{AK}{\sin 2x} = \frac{KT}{\sin(90^\circ + x)}</math></p> <p><math>AK = \frac{KT 2 \sin x \cos x}{\cos x}</math></p> <p><math>AK = 2KT \sin x</math></p>	<p>✓ <math>\sin x = \frac{AK}{2}</math>          ✓ <math>AK = 2 \sin x</math></p> <p>✓ use of sine rule /  <i>gebruik van sinusreël</i>          ✓ <math>AK = 2KT \sin x</math>          ✓ <math>KT = 1</math></p>	(5)
7.2	<p>In <math>\triangle KAT</math>  <math>T\hat{K}A = 90^\circ - 3x</math>  <math>\frac{AT}{\sin(90^\circ - 3x)} = \frac{1}{\sin(90^\circ + x)}</math>  <math>AT = \frac{\cos 3x}{\cos x}</math></p>		<p>✓ <math>T\hat{K}A = 90^\circ - 3x</math></p> <p>✓ use of sine rule  <i>gebruik van sinusreël</i></p>	(2)
7.3	<p><math>AT = \frac{\cos 3x}{\cos x}</math></p> <p><math>AT = \frac{\cos(2x + x)}{\cos x}</math>  <math>AT = \frac{\cos 2x \cos x - \sin 2x \sin x}{\cos x}</math>  <math>AT = \frac{\cos 2x \cos x - 2 \sin x \cos x \sin x}{\cos x}</math>  <math>AT = \frac{\cos x (\cos 2x - 2 \sin^2 x)}{\cos x}</math>  <math>AT = 1 - 2 \sin^2 x - 2 \sin^2 x</math>  <math>AT = 1 - 4 \sin^2 x</math></p>		<p>✓ splitting of <math>\cos 3x</math> and expansion  <i>opbreek van <math>\cos 3x</math> en uitbreiding</i></p> <p>✓ common factor / <i>gemene faktor</i></p> <p>✓ expansion of <math>\cos 2x</math>  <i>uitbreiding van <math>\cos 2x</math></i>          ✓ answer / <i>antwoord</i></p>	(4)
				[11]

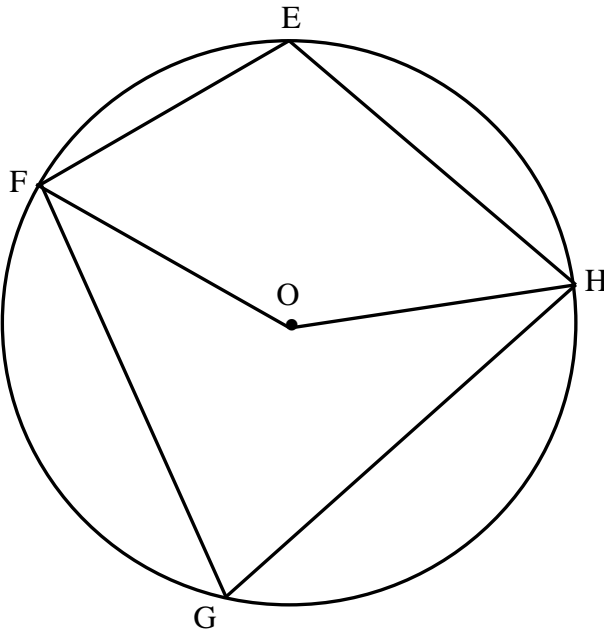


## QUESTION 8/VRAAG 8

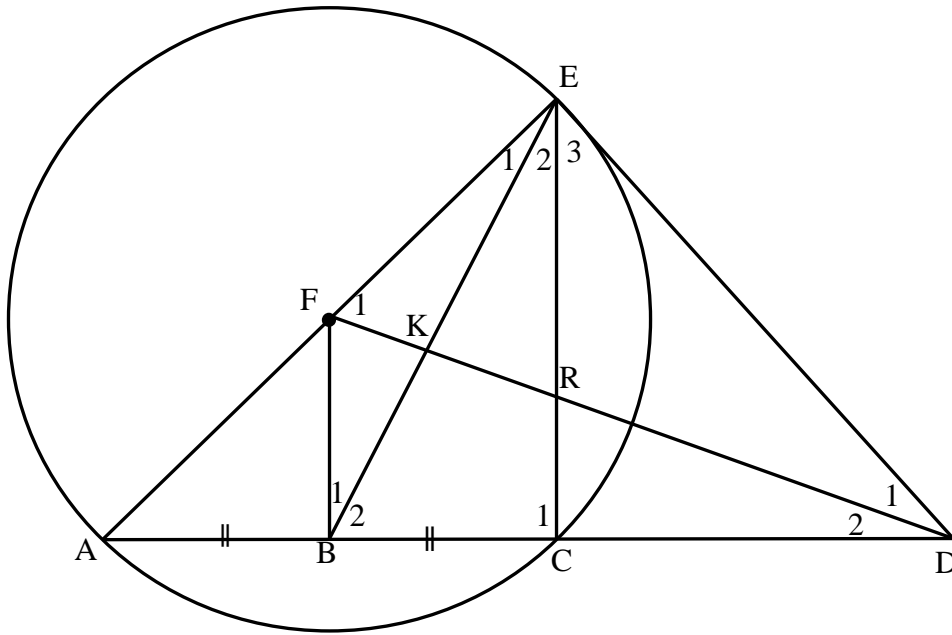
8.1			
8.1.1	$x = 96^{\circ}$ [∠ at centre = $2 \times$ ∠ at circumference] [middelpunte ∠ = $2 \times$ omtrekshoek]	✓ answer / antwoord ✓R	(2)
8.1.2	$\hat{B}_2 = y$ [angles opp = sides] [hoeke teenoor = sye] $2y + 96^{\circ} = 180^{\circ}$ [sum of angles of Δ] [som van die hoeke van Δ] $y = 42^{\circ}$	✓ S/R  ✓ answer / antwoord	(2)
8.2			
8.2.1	$\hat{F}_1 = 90^{\circ}$ [line from centre bisects chord] [lyn vanaf middelpunt halveer koord]	✓ S ✓R	(2)
8.2.2	$\hat{A}\hat{B}\hat{C} = 150^{\circ}$ [opp angles of cyclic quad] [teenoorst. hoeke van koordevierhoek]	✓ S ✓R	(2)

8.3				
8.3.1	$\hat{B}_1 = x$ [angles opp equal sides] [hoëke teenoor gelyke sye]	✓S ✓R	(2)	
8.3.2	$\hat{K}_2 + \hat{C} = x$ $\hat{C} = \hat{K}_2$ $\hat{C} = \frac{x}{2}$ [ext angle of $\Delta$ ] / [buitehoek van $\Delta$ ] [angles opp equal sides] [hoëke teenoor gelyke sye]	✓S ✓R ✓S/R	(3)	
8.3.3	$\hat{K}_1 = 180^\circ - 2x$ [sum of angles of $\Delta$ ] [som van die hoëke van $\Delta$ ] $108^\circ + 180^\circ - 2x + \frac{x}{2} = 180^\circ$ [adj angles on str line] [aangrensende hoëke op reguitlyn] $x = 72^\circ$	✓S/R ✓S ✓R ✓answer / antwoord	(4)	
				[17]

## QUESTION 9 / VRAAG 9

		
	<p>Construct radii OF and OH.</p> $\widehat{FOH} = 2 \times \widehat{G}$ <p>[angle at centre = <math>2 \times</math> angle at circum]</p> $\text{Reflex } \widehat{FOH} = 2 \times \widehat{E}$ <p>[angle at centre = <math>2 \times</math> angle at circum]</p> $\widehat{FOH} + \widehat{F\hat{O}H} = 2\widehat{G} + 2\widehat{E}$ <p>but <math>\widehat{FOH} + \widehat{F\hat{O}H} = 360^\circ</math> [angles around a point]</p> $2\widehat{G} + 2\widehat{E} = 360^\circ$ $\widehat{G} + \widehat{E} = 180^\circ$	<p>✓S</p> <p>✓S/R</p> <p>✓S/R</p> <p>✓S</p> <p>✓S/R</p> <p>✓S</p>
	<p>Trek radiusse OF en OH.</p> $\widehat{FOH} = 2 \times \widehat{G}$ <p>[middelpunte <math>\angle = 2 \times</math> omtrekshoek]</p> $\text{Omwenteling } \widehat{FOH} = 2 \times \widehat{E}$ <p>[middelpunte <math>\angle = 2 \times</math> omtrekshoek]</p> $\widehat{FOH} + \widehat{F\hat{O}H} = 2\widehat{G} + 2\widehat{E}$ <p>maar <math>\widehat{FOH} + \widehat{F\hat{O}H} = 360^\circ</math> [hoeke rondom 'n punt]</p> $2\widehat{G} + 2\widehat{E} = 360^\circ$ $\widehat{G} + \widehat{E} = 180^\circ$	<p>✓S</p> <p>✓S/R</p> <p>✓S/R</p> <p>✓S</p> <p>✓S/R</p> <p>✓S</p>
		[6]

## QUESTION 10/VRAAG 10



10.1	$FE \perp DE$ [radius $\perp$ tang] / [radius $\perp$ raaklyn] $\hat{FED} = 90^\circ$ $\hat{FBA} = 90^\circ$ [line from centre bisects chord [lyn vanaf middelpunt halveer koord]] $\therefore EFBD = \text{cyclic quad}$ [opp angles = $180^\circ$ ] $EFBD$ is 'n koordevierhoek [teenoorst. hoeke = $180^\circ$ ]	$\checkmark$ S/R $\checkmark$ S $\checkmark$ S/R $\checkmark$ S/R	(4)
10.2	$\hat{C}_1 = 90^\circ$ [angle in semi-circle] / [hoek in semi-sirkel] In $\triangle BCE$ and /en $\triangle FED$ $\hat{C}_1 = \hat{FED}$ [ $90^\circ$ ] $\hat{B}_2 = \hat{F}_1$ [angles in same segment [hoeke in dieselfde segment]] $\hat{E}_2 = \hat{D}_1$ [3rd angle of $\Delta$ ] / [3de hoek van $\Delta$ ] $\triangle BCE \parallel \triangle FED$ [ $\angle$ ; $\angle$ ; $\angle$ ]	$\checkmark$ S/R $\checkmark$ S/R $\checkmark$ S $\checkmark$ R $\checkmark$ S $\checkmark$ R	(6)
10.3	$\frac{BC}{FE} = \frac{CE}{ED}$ [similar triangles] / [gelykvormige $\Delta$ e] $BC = \frac{FA \cdot CE}{ED}$	$\checkmark$ S $\checkmark$ R $\checkmark$ answer antwoord	(3)
10.4	$\hat{FBA} = \hat{C}_1$ [proven] / [bewys] $\therefore FB \parallel CE$ [corresponding angles equal [ooreenkomstige hoeke gelyk]] $\frac{AC}{BC} = \frac{AE}{FE}$ [line parallel to one side of triangle [lyn ewewydig aan een sy van driehoek]] $BC = \frac{AC \cdot FE}{AE}$	$\checkmark$ S/R $\checkmark$ S/R $\checkmark$ S $\checkmark$ R	(4)
			[17]
TOTAL/TOTAAL:			150