GAUTENG PROVINCE

EDUCATION
REPUBLIC OF SOUTH AFRICA

JUNE EXAMINATION
GRADE 12

2023

MATHEMATICS

(PAPER 2)

TIME: 3 hours

MARKS: 130

MATHEMATICS P2

C2612E

11 pages, 1 information sheet and an answer book of 23 pages

X10

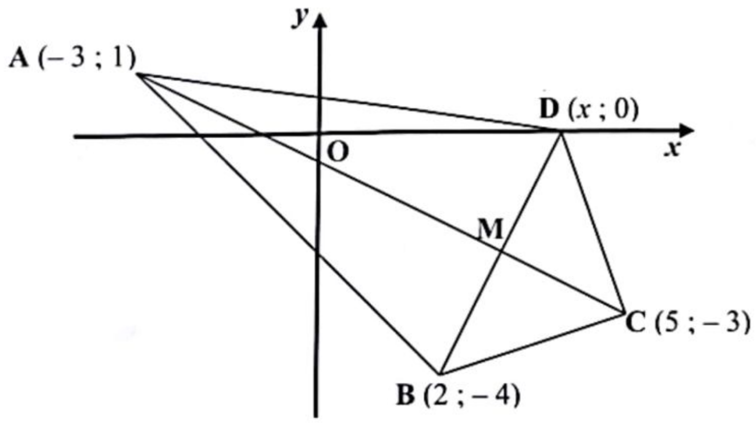
INSTRUCTIONS AND INFORMATION

Read the following instructions carefully before answering the questions.

1. This question paper consists of 8 questions.
2. Answer ALL the questions in the ANSWER BOOK provided.
3. Clearly show ALL calculations, diagrams, graphs, et cetera that you have used in determining your answers.
4. Answers only will NOT necessarily be awarded full marks.
5. You may use an approved scientific calculator (non-programmable and non-graphical), unless stated otherwise.
6. If necessary, round-off answers correct to TWO decimal places, unless stated otherwise.
7. Diagrams are NOT necessarily drawn to scale.
8. An information sheet with formulae is included at the end of the question paper.
9. Write neatly and legibly.

QUESTION 1

ABCD is a quadrilateral on the Cartesian plane with vertices, A(–3; 1), B(2 ; –4), C(5 ; –3) and D(x ; 0).



1.1 Determine the gradient of AC. (2)

1.2 Calculate the equation of AC. (3)

1.3 Calculate the angle of inclination of AC. (3)

1.4 Determine the coordinates of D if AC is the perpendicular bisector of DB. (5)

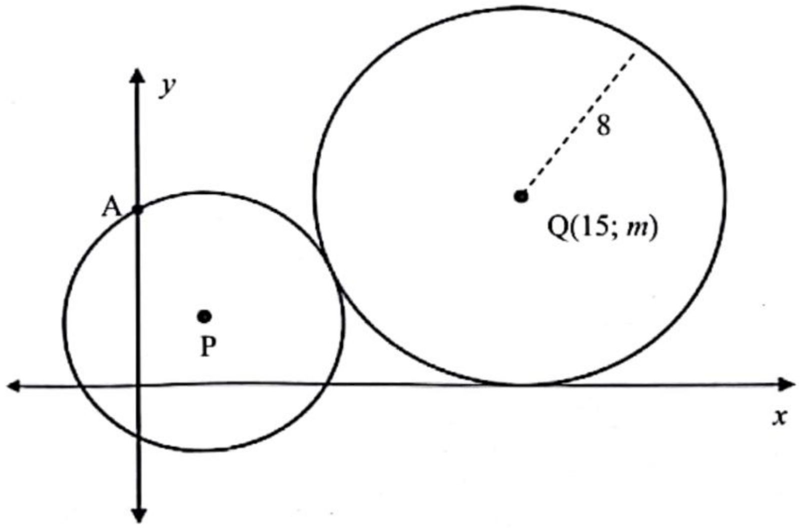
1.5 Hence, deduce that ABCD is a kite. (3)

1.6 Calculate the area of ΔABD. (5)

[21]

QUESTION 2

2.1 In the diagram below, Q(15; m) is the centre of the larger circle which touches both the x-axis and the circle centred at P. The circle with centre P, has the y-intercept at A and has equation $(x-3)^2 + (y-3)^2 = 25$. The radius of the circle, centre Q, is 8 units.



2.1.1 Determine the equation of the circle with centre Q in terms of x; y and m. (1)

2.1.2 Determine the value of m. (2)

2.1.3 Determine the length of PQ. (2)

2.1.4 Calculate the coordinates of A. (3)

2.1.5 Determine the equation of the tangent to the circle with centre P, at A. (4)

2.2 Two other circles are given:

- One has centre K, and equation $x^2 - 6x + y^2 + 4y - 12 = 0$.
- The other has centre T, and equation $(x-12)^2 + (y-10)^2 = 100$

2.2.1 Determine the centre and radius of the circle with centre K. (4)

2.2.2 Calculate the distance between the centres, K and T. (2)

2.2.3 At how many points do the two circles intersect? Motivate your answer. (2)

[20]

QUESTION 3

3.1 Simplify **without the use of a calculator:**

$$\sin^2(360^\circ - x) - \frac{\cos(x - 180^0).\tan(-x).\sin(90^\circ - x).\cos(360^\circ + x)}{\sin(180^\circ + x)}$$ (8)

3.2 If $\beta \in [-90^0; 270^0]$, determine β **without the use of a calculator,**

$$\cos(\beta + 80^0) = \frac{\sin(-300^0)\cos 45^0}{\cos 405^0}$$ (7)

3.3 Determine the value of the following **without the use of a calculator:**

3.3.1 $\frac{\sin 49^\circ}{\cos 41^\circ}$ (2)

3.3.2 $\sin 85^\circ \cos 65^\circ + \cos 85^\circ \sin 65^\circ$ (3)

3.3.3 $\frac{1}{2}(\cos 15^0 + \sqrt{3}\sin 15^0)$ (4)

[24]

QUESTION 4

4.1 Determine the general solution of the following equation:
$\cos(54^\circ - x) = \sin 2x$ (6)

4.2 If $13\sin x + 5 = 0$ and $x \in [0^\circ; 270^\circ]$, determine **without the use of a calculator**, the value of $\sin 2x$. (5)

4.3 Prove the following identity: $\frac{1 + \sin 2x}{\cos 2x} = \frac{\cos x + \sin x}{\cos x - \sin x}$ (7)

4.4 If $\sin 39^\circ = p$, determine the following in terms of p:

4.4.1 $\sin 129^\circ$ (3)

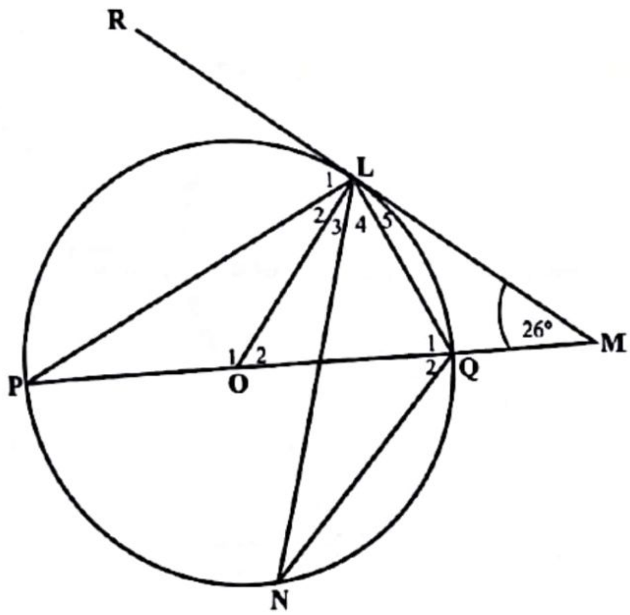
4.4.2 $\tan 321^\circ$ (2)

4.4.3 $\sin 78^\circ$ (2)

[25]

QUESTION 5

5.1 In the diagram below, O is the centre of the circle and L is a point on the circumference. The diameter, PQ makes an angle of 26° with the tangent RLM. N is a point on the lower part of the circle.



Determine, with reasons, the size of:

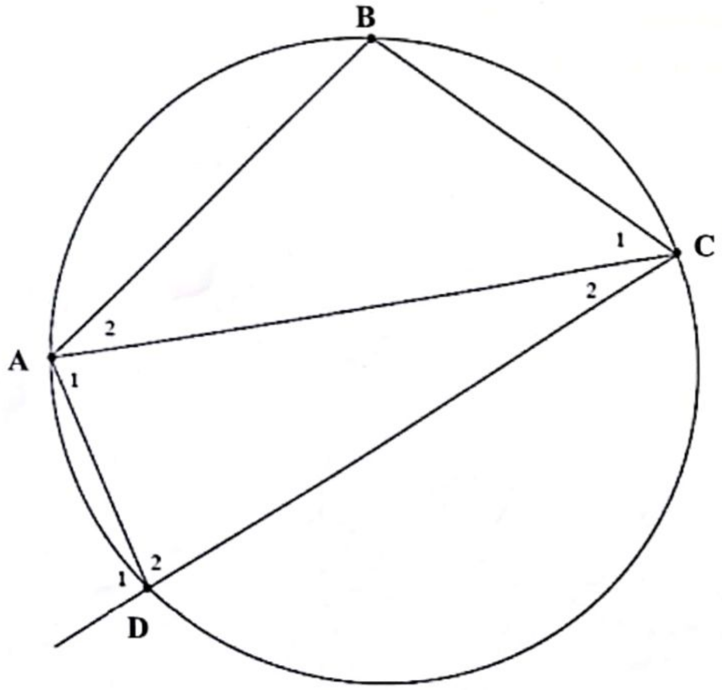
5.1.1 $O\hat{L}M$ (2)

5.1.2 $\hat{O}_2$ (1)

5.1.3 $\hat{P}$ (2)

5.1.4 Q_1 (3)

5.2 In the diagram below, points A, B, C and D lie on the circumference of a circle.

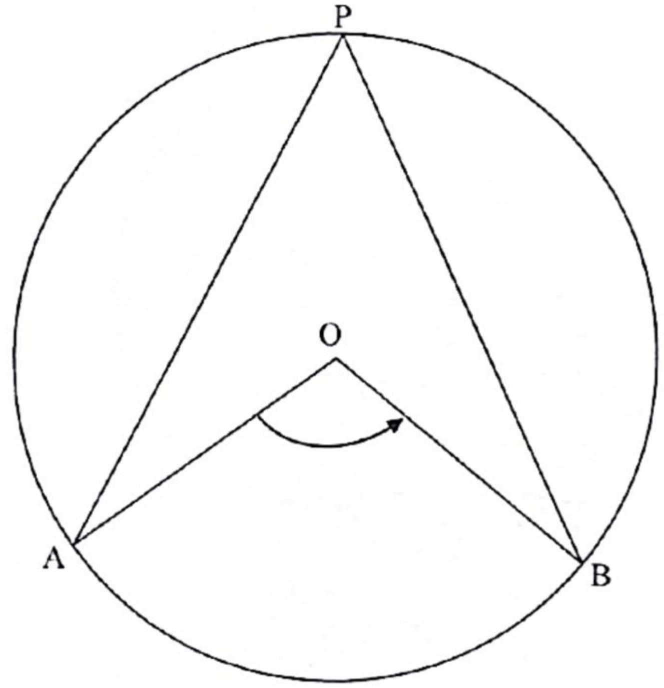


Prove that $\hat{C}_2 = \hat{B} - \hat{A}_1$ (5)

[13]

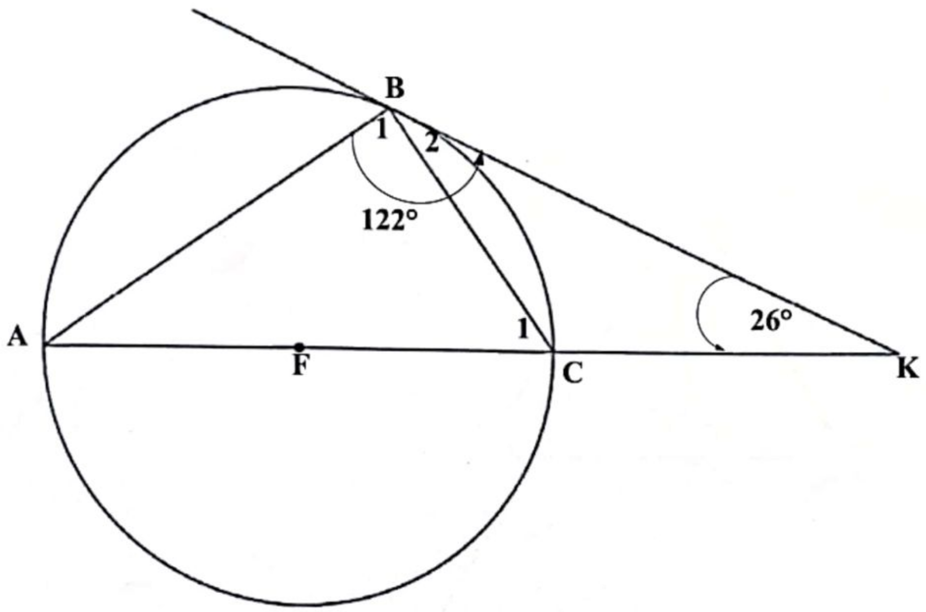
QUESTION 6

6.1 In the diagram below, O is the centre of the circle and A, P and B are points on the circumference of the circle. Arc AB subtends $A\hat{O}B$ at the centre of the circle and $A\hat{P}B$ at the circumference of the circle.



Use the diagram to prove the theorem which states that $A\hat{O}B = 2A\hat{P}B$. (5)

6.2 In the diagram below, F is the centre of the circle and AC is a diameter. AC is produced to K. $\hat{B}_1 + \hat{B}_2 = 122°$, and $\hat{K} = 26°$.



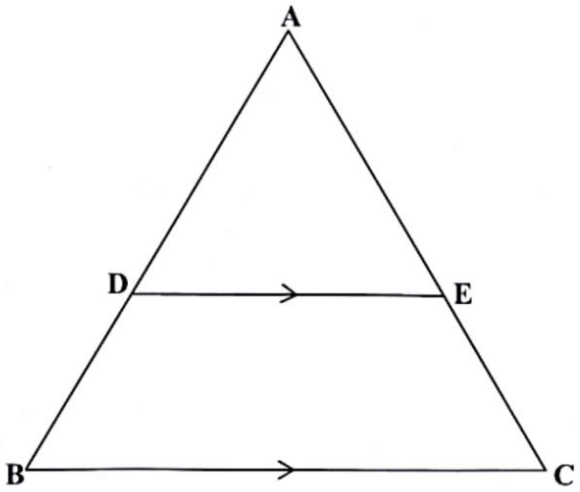
Show that BK is a tangent to the circle at B. (5)

[10]

QUESTION 7

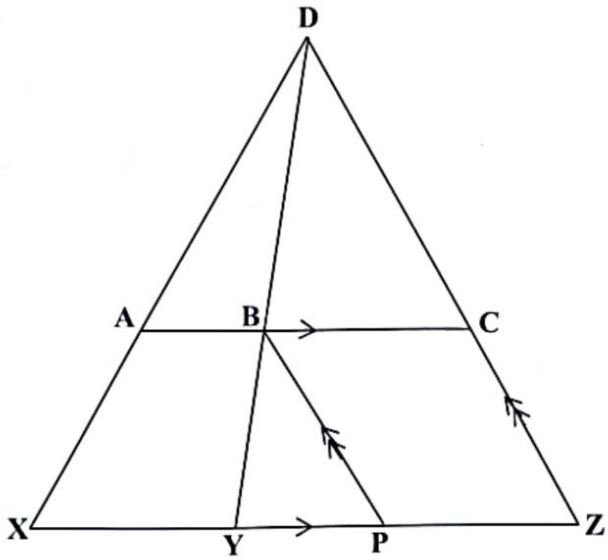
7.1 In ΔABC below, D and E are points on AB and AC respectively such that DE $\parallel$ BC.

Prove the theorem which states that $\frac{AD}{DB} = \frac{AE}{EC}$.



(6)

7.2 In ΔDXZ below, AC $\parallel$ XZ and BP $\parallel$ DZ. DY is drawn to intersect AC at B.

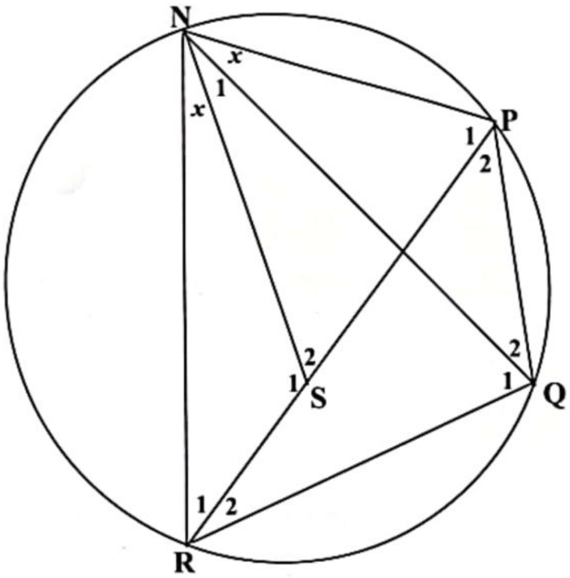


Prove that: $\frac{BC}{YZ} = \frac{DA}{DX}$

(5)
[11]

QUESTION 8

In the diagram below, NPQR is a cyclic quadrilateral with S a point on chord PR. N and S are joined and $R\hat{N}S = P\hat{N}Q = x$.



Prove that:

8.1 $\Delta NSR \parallel\!| \Delta NPQ$ (3)

8.2 $\Delta NQR \parallel\!| \Delta NPS$ (3)

[6]

TOTAL: 130

INFORMATION SHEET

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$A=P(1+ni)$ $\quad A=P(1-ni)$ $\quad A=P(1-i)^n$ $\quad A=P(1+i)^n$

$T_n=a+(n-1)d$ $\quad S_n=\frac{n}{2}[2a+(n-1)d]$

$T_n=ar^{n-1}$ $\quad S_n=\frac{a(r^n-1)}{r-1}\ ;\ r\neq 1$ $\quad S_\infty=\frac{a}{1-r}\ ;\ -1<r<1$

$F=\frac{x[(1+i)^n-1]}{i}$ $\quad P=\frac{x[1-(1+i)^{-n}]}{i}$

$$f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$

$d=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$ $\quad M\left(\frac{x_1+x_2}{2};\frac{y_1+y_2}{2}\right)$

$y=mx+c$ $\quad y-y_1=m(x-x_1)$ $\quad m=\frac{y_2-y_1}{x_2-x_1}$ $\quad m=\tan\theta$

$(x-a)^2+(y-b)^2=r^2$

In∆ABC: $\quad \frac{a}{\sin A}=\frac{b}{\sin B}=\frac{c}{\sin C}$

$a^2=b^2+c^2-2bc.\cos A$

$area\ \Delta ABC=\frac{1}{2}ab\sin C$

$\sin(\alpha+\beta)=\sin\alpha.\cos\beta+\cos\alpha.\sin\beta$ $\quad \sin(\alpha-\beta)=\sin\alpha.\cos\beta-\cos\alpha.\sin\beta$

$\cos(\alpha+\beta)=\cos\alpha.\cos\beta-\sin\alpha.\sin\beta$ $\quad \cos(\alpha-\beta)=\cos\alpha.\cos\beta+\sin\alpha.\sin\beta$

$$\cos 2\alpha=\begin{cases}\cos^2\alpha-\sin^2\alpha\\ 1-2\sin^2\alpha\\ 2\cos^2\alpha-1\end{cases}$$

$\sin 2\alpha=2\sin\alpha.\cos\alpha$

$\bar{x}=\frac{\sum x}{n}$ $\quad \sigma^2=\frac{\sum_{i=1}^{n}(x_i-\bar{x})^2}{n}$

$P(A)=\frac{n(A)}{n(S)}$ $\quad P(A\ or\ B)=P(A)+P(B)-P(A\ and\ B)$

$\hat{y}=a+bx$