GAUTENG PROVINCE

EDUCATION

REPUBLIC OF SOUTH AFRICA

JUNE EXAMINATION
GRADE 12

2023

MATHEMATICS

(PAPER 1)

MATHEMATICS P1

C2611E

TIME: 3 hours

MARKS: 120

7 pages and an information sheet

X05

INSTRUCTIONS AND INFORMATION

Read the following instructions carefully before answering the questions.

1. This question paper consists of 8 questions.

2. Answer ALL the questions.

3. Number the answers correctly according to the numbering system used in this question paper.

4. Clearly show ALL calculations, diagrams, graphs, etc. that you have used in determining your answers.

5. Answers only will NOT necessarily be awarded full marks.

6. You may use an approved scientific calculator (non-programmable and non-graphical), unless stated otherwise.

7. If necessary, round-off answers to TWO decimal places, unless stated otherwise.

8. Diagrams are NOT necessarily drawn to scale.

9. An information sheet with formulae is included at the end of the question paper.

10. Write neatly and legibly.

QUESTION 1

1.1 Given: $12x = x^2$

1.1.1 Solve for x. (3)

1.1.2 Hence, or otherwise, determine the value(s) of p if $\left(p^2-1\right)^2 = 12\left(p^2-1\right)$.
(Leave your answer in surd form, where necessary). (4)

1.2 Solve for x if $5x^2 + 7x - 2 = 0$. (Round-off the answer to TWO decimal places.) (4)

1.3 Solve for x if $\sqrt{x+6} = x$. (5)

1.4 Use the solution for x in QUESTION 1.3 to determine the value of y for which $\sqrt{y+1} = y-5$. (2)

1.5 A race requires an athlete to run 10 km and cycle 50 km. Tendani runs at a speed of x km/h and cycles at $(x+31)$ km/h. He takes $\frac{10}{x}$ hours for the 10 km run.

1.5.1 Express the time he takes for the 50 km cycle in terms of x. (1)

1.5.2 Calculate the speed (correct to TWO decimal places) at which he must run to complete the entire race in 2 hours. (6)

[25]

QUESTION 2

In a geometric series, the sum of the first n terms is given by $S_n = k\left(1-\left(\frac{1}{2}\right)^n\right)$ and the sum to infinity of this series is 10.

2.1 Calculate the value(s) of k. (4)

2.2 Calculate the second term of the series. (4)

[8]

QUESTION 3

3.1 Prove that in any arithmetic series in which the first term is a and whose constant difference is d, the sum of the first n terms is $S_n = \frac{n}{2}\left[2a + (n-1)d\right]$. (4)

3.2 Calculate the value of $\sum_{p=1}^{50}(100-3p)$. (4)

3.3 A quadratic sequence is defined with the following properties:

$T_2 - T_1 = 7$

$T_3 - T_2 = 13$

$T_4 - T_3 = 19$

3.3.1 Write down the values of:

(a) $T_5 - T_4$ (1)

(b) $T_{70} - T_{69}$ (3)

3.3.2 Calculate the value of T_{69} if $T_{69} = 23\ 594$. (5)

[17]

QUESTION 4

4.1 Given: $f(x) = x^2 - 2x - 3$ and $g(x) = x - 5$

4.1.1 Show that the turning point of f is $(1;-4)$. (3)

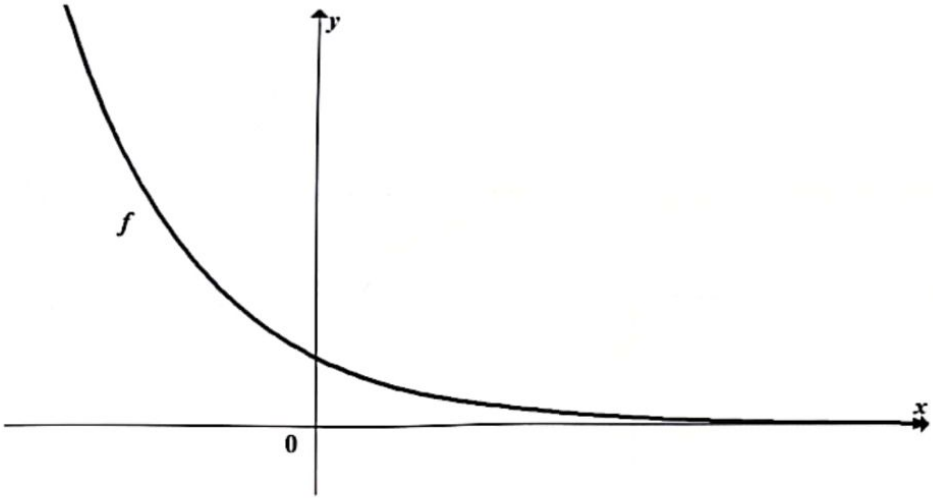
4.1.2 Determine the coordinates of the x- and y-intercepts of the graph of f. (3)

4.1.3 Determine the points of intersection of the graphs of f and g. (4)

4.1.4 Sketch neat graphs of f and g on the same system of axes. Clearly label the turning point and where the graphs of f and g intersect each other as well as the x- and y-intercepts of both graphs. (6)

4.1.5 Use the graph to determine the values of x where $f(x) \geq 0$. (2)

4.2 The graph of $f(x)=\left(\frac{1}{3}\right)^x$ is sketched below.



4.2.1 Write down the equation of the asymptote of f. (1)

4.2.2 Write down the equation of f^{-1} in the form $y = \ldots$ (2)

4.2.3 Sketch the graph of f^{-1} in your ANSWER BOOK.
Indicate the intercept and ONE other point on the graph. (3)

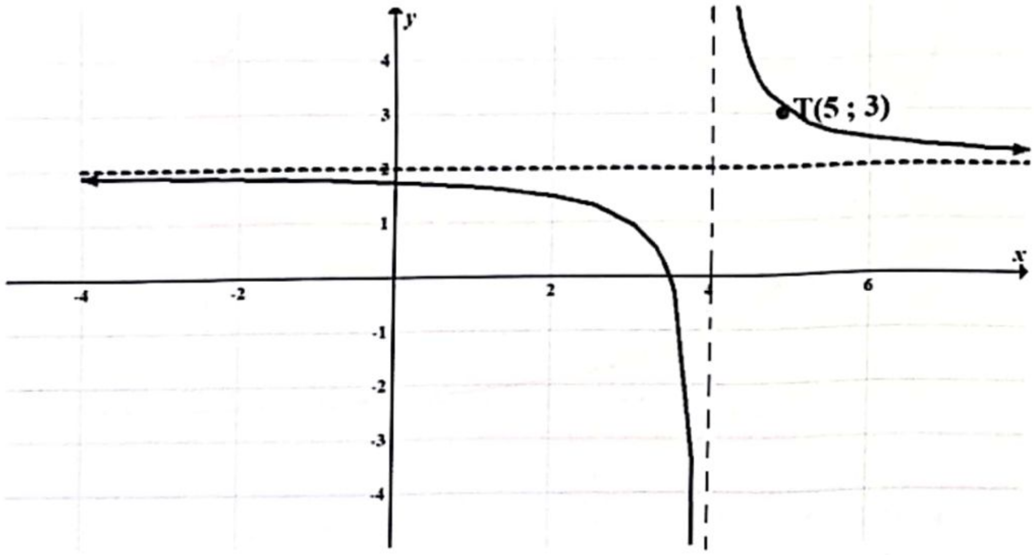
4.2.4 Write down the equation of the asymptote of $f^{-1}(x+2)$. (2)

4.2.5 Prove that: $\left[f(x)\right]^2-\left[f(-x)\right]^2=f(2x)-f(-2x)$ for all values of x. (3)

[29]

QUESTION 5

The diagram below represents the graph of $f(x)=\dfrac{a}{x-p}+q$. $T(5;3)$ is a point on f.



5.1 Determine the values of a, p and q. (4)

5.2 If the graph of f is reflected across the line having equation $y=-x+c$ and the new graph coincides with the graph of $y=f(x)$, determine the value of c. (2)

[6]

QUESTION 6

6.1 Given: $f(x)=3x-x^2$.

Use the definition (from first principles) of the derivative to calculate $f'(x)$. (4)

6.2 Determine $\dfrac{dy}{dx}$ if:

6.2.1 $y=\dfrac{x-3\sqrt{x}}{x^2}$ (4)

6.2.2 $\dfrac{y}{3x}=(1+x)^2$ (4)

6.3 A function h is given by $h(x)=ax^2+\dfrac{b}{x}$ and has a minimum value of 12 if $x=2$.

Calculate the values of a and b. (7)

[19]

QUESTION 7

The graph of the cubic function f has a turning point at $A(-1\ ;p)$ and $B(2\ ;q)$. The function f has the following properties:

$f'(x) > 0$ for $x < -1$ and $x > 2$

$f'(x) < 0$ for $-1 < x < 2$

$f(2) > 2$

7.1 Draw a neat sketch of f. Clearly label points A and B on the sketch. (It is NOT necessary to show x- and y-intercepts.) (4)

7.2 If $f(x) = x^3 + bx^2 + cx + d$, calculate the values of b and c. (6)

[10]

QUESTION 8

During an experiment, the temperature T (in degrees Celsius), varies with time t (in hours), according to the formula $T(t) = 30 + 4t - \frac{1}{2}t^2$, $t \in [0;10]$.

8.1 Determine an expression for the rate of change of temperature with time. (2)

8.2 During which time interval was the temperature decreasing? (4)

[6]

TOTAL: 120

INFORMATION SHEET

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$A=P(1+ni)$ $\quad A=P(1-ni)$ $\quad A=P(1-i)^n$ $\quad A=P(1+i)^n$

$T_n=a+(n-1)d$ $\quad S_n=\frac{n}{2}[2a+(n-1)d]$

$T_n=ar^{n-1}$ $\quad S_n=\frac{a(r^n-1)}{r-1}\ ;\ r\neq 1$ $\quad S_\infty=\frac{a}{1-r}\ ;\ -1<r<1$

$F=\frac{x[(1+i)^n-1]}{i}$ $\quad P=\frac{x[1-(1+i)^{-n}]}{i}$

$$f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$

$d=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$ $\quad \mathrm{M}\left(\frac{x_1+x_2}{2};\frac{y_1+y_2}{2}\right)$

$y=mx+c$ $\quad y-y_1=m(x-x_1)$ $\quad m=\frac{y_2-y_1}{x_2-x_1}$ $\quad m=\tan\theta$

$(x-a)^2+(y-b)^2=r^2$

In ΔABC: $\quad \frac{a}{\sin \mathrm{A}}=\frac{b}{\sin \mathrm{B}}=\frac{c}{\sin \mathrm{C}}$

$a^2=b^2+c^2-2bc.\cos A$

$area\ \Delta\mathrm{ABC}=\frac{1}{2}ab.\sin \mathrm{C}$

$\sin(\alpha+\beta)=\sin\alpha.\cos\beta+\cos\alpha.\sin\beta$ $\quad \sin(\alpha-\beta)=\sin\alpha.\cos\beta-\cos\alpha.\sin\beta$

$\cos(\alpha+\beta)=\cos\alpha.\cos\beta-\sin\alpha.\sin\beta$ $\quad \cos(\alpha-\beta)=\cos\alpha.\cos\beta+\sin\alpha.\sin\beta$

$$\cos 2\alpha=\begin{cases}\cos^2\alpha-\sin^2\alpha\\ 1-2\sin^2\alpha\\ 2\cos^2\alpha-1\end{cases}$$

$\sin 2\alpha=2\sin\alpha.\cos\alpha$

$\bar{x}=\frac{\sum x}{n}$ $\quad \sigma^2=\frac{\sum_{i=1}^{n}(x_i-\bar{x})^2}{n}$

$\mathrm{P(A)}=\frac{n(\mathrm{A})}{n(\mathrm{S})}$ $\quad$ P(A or B) = P(A) + P(B) – P(A and B)

$\hat{y}=a+bx$ $\quad b=\frac{\sum(x-\bar{x})(y-\bar{y})}{\sum(x-\bar{x})^2}$