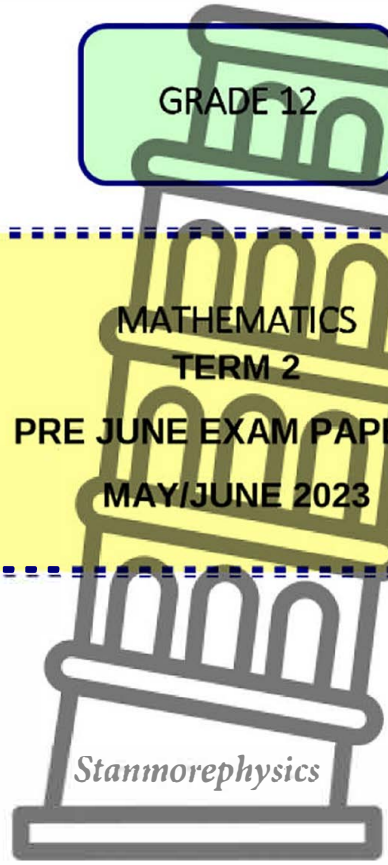
LIMPOPO
PROVINCIAL GOVERNMENT
REPUBLIC OF SOUTH AFRICA

DEPARTMENT OF
EDUCATION

MOGALAKWENA DISTRICT

GRADE 12

MATHEMATICS
TERM 2
PRE JUNE EXAM PAPER 1
MAY/JUNE 2023



MARKS: 150

TIME: 3Hours

INSTRUCTIONS AND INFORMATION:

Read the following instructions carefully before answering the questions.

1. This question paper consists of 9 questions.
2. Answer ALL questions.
3. Clearly show ALL calculations, diagrams, graphs, etc, that you have used in determining your answers.
4. Answers only will NOT necessarily be awarded full marks.
5. If necessary, round off answers correct to TWO decimal places, unless stated otherwise.
6. You may use an approved scientific calculator (no programmable and no graphical).
7. Diagrams not necessarily drawn to scale.
8. An information sheet with formulae is included at the end of the question paper.

QUESTION 1

1.1 Solve for x :

1.1.1 $(x - 3)(5x + 2) = 0$ (2)

1.1.2 $3x^2 - 10x - 1 = 0$ (Correct to two decimal places) (3)

1.1.3 $x(x - 4) \geq 21$ (5)

1.1.4 $2.3^x + \frac{3^x}{2} = 7\frac{1}{2}$ (4)

1.2. Solve for x and y simultaneously.

$$x + 2y = 2$$

$$x^2 + 8y = 8$$ (6)

1.2.1 (3)

Given: $2^{x+1} + 2^x = 3^{y+2} - 3^y$, where x and y are integers.
Determine the value of x and y.

1.2.2 (5)

$$\sqrt{x-2} + 3 = \frac{10}{\sqrt{x-2}}$$

1.3 Given:

$$x = \frac{4 \pm \sqrt{16 - 4p}}{2}$$

1.3.1 If $p = 4$, determine the nature of the roots. (1)

1.3.2 Determine the value(s) of p for which the roots are non-real. (2)

1.4 The equations $x^2 + rx + m = 0$ and $x^2 + mx + r = 0$ have real and EQUAL roots. (6)
Solve for the values of r and m if $r > 0$ and $m > 0$.

[37]

QUESTION 2

The sequence: $-67\ ;\ x\ ;\ y\ ;\ -28\ ;\ -19\ ;\ \ldots$ has a quadratic pattern.

2.1 Determine the value of x and y (6)

2.2 Find an expression for the nth term (4)

2.3 Prove that the sequence of numbers will never contain a positive term. (3)

[13]

QUESTION 3

3.1 The fourth term of a geometric progression is 24 and the ninth term is 768. Determine the first three terms of the Progression. (5)

3.2

$\text{If } S_n = 2n^2 + 3n \text{ then}$

3.2.1 Calculate T_{12} (3)

3.2.2 Find T_n in its simplest form. (4)

3.3 $\text{Calculate the value of } \sum_{n=2}^{18} (2n - 1)$ (3)

3.4 In a converging geometric series $S_\infty = \frac{40}{3}$ and $T_2 = \frac{5}{2}$; calculate the possible value(s) the first term of the series. (3)

3.5 Given the following geometric sequence:

$$\frac{24}{x} + 12 + 6x + 3x^2 + \ldots$$

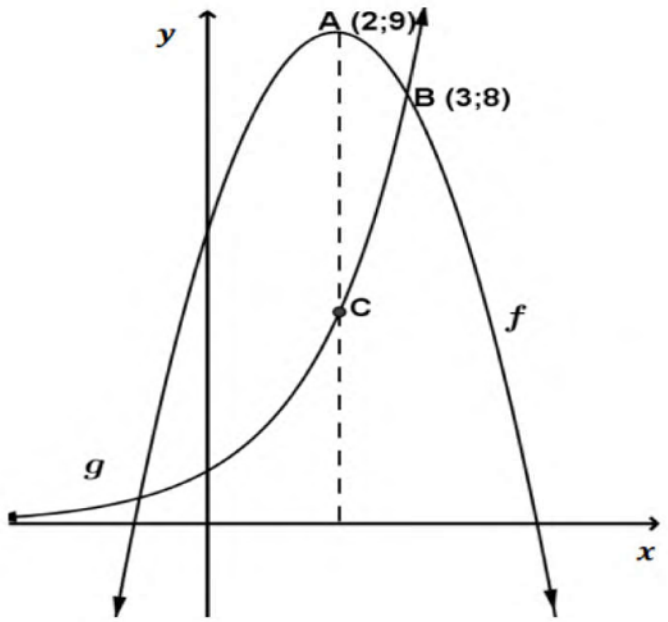
3.5.1 Calculate the sum to infinity of the series (4)

3.5.2 Write down the values of x for which this sequence converges. (2)

[24]

QUESTION 4

The graphs of $f(x)$ and $g(x)$ are shown in the diagram below.



The turning point of $f(x)$ is A (2 ; 9) and the graphs f and g intersects at B (3; 8). C is a point on $g(x)$ and is on the axis of symmetry of f.

4.1 Show that the function f can be defined by the equation: (4)

$$f(x) = -x^2 + 4x + 5$$

4.2 Write down the equation of the axis of symmetry of f. (1)

4.3 The graph $g(x)$, has the equation $y = a^x$. Determine the value of a. (2)

4.4 If it is given that $(-1; 0)$ is one root of f, write down the coordinates of the other root. (1)

4.5 For which value(s) of x will $f(x) < 0$? (2)

4.6 Determine the length of AC. (2)

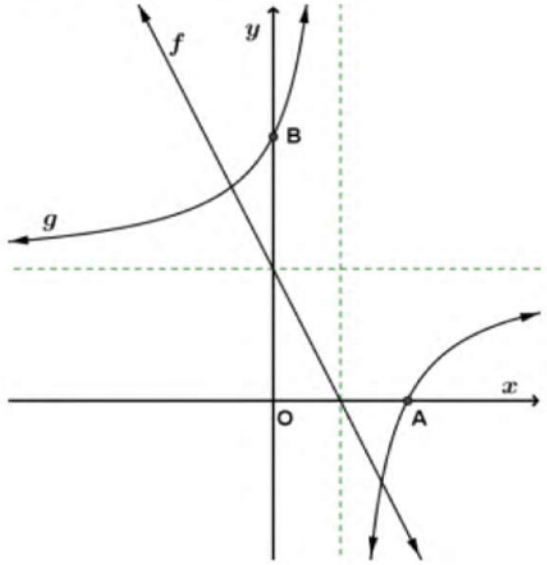
4.7 Discuss the nature of the roots of $h(x)$ if, $h(x) = f(x) - 9$. (2)

[14]

QUESTION 5

In the diagram below are the sketches of: $f(x) = -2x + 2$ and $g(x) = \frac{a}{x+p} + q$

The graph of g cuts the y-axis at $(0\,;4)$.



5.1 Calculate the values of p *and* q (3)

5.2 Show that $a = -2$ (2)

5.3 Determine the coordinates of point A the $x - intercept$ of $g(x)$ (2)

5.4 The value of q is increased by 1 unit. What effect will it have on the graph of g? (1)

5.5 Write the equations of the asymptotes. (2)

[10]

QUESTION 6

Given: $p(x) = \left(\frac{1}{4}\right)^x$

6.1 Write down the equation of p^{-1} in the form $y = \cdots$ (2)

6.2 Sketch the graphs of p and p^{-1} on the given system of axes on the diagram sheet. (5)
Show clearly all the intercepts with the axes as well as the line of symmetry.

6.3 Determine the value(s) of x for which $p^{-1}(x) \geq 1$ (2)

6.4 Write down the equation of the graph of $h(x)$, if $h(x)$ is formed by shifting the graph of $p(x)$ 2 units to the right and then 3 units up.

(2)

[11]

QUESTION 7

7.1 Determine $f'(x)$ from first principles if $f(x)=5-2x^2$. (5)

7.2 Determine $\frac{dy}{dx}$ if:

7.2.1 $y=7x^4+\frac{2x^2}{\sqrt{x}}$ (3)

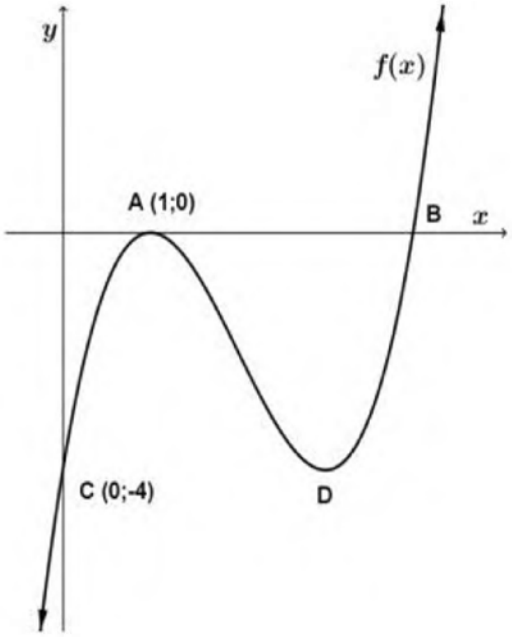
7.2.2 $xy=5$ (2)

7.2.3 $D_x\left[\frac{3x^2-7x-6}{x}\right]$ (4)

[14]

QUESTION 8

Given that $f(x) = x^3 - 6x^2 + 9x - 4$ with x-intercepts at A (1 ; 0) and B, y-intercept at C(0 ; -4).
A and D are the turning points of f.



8.1 Calculate the coordinates of B. (4)

8.2 Calculate the coordinates of the turning point D. (5)

8.3 Calculate the x-coordinate of the point of inflection of f. (2)

8.4 Write down the values of k for which $f(x) = k$ will have only ONE root. (2)

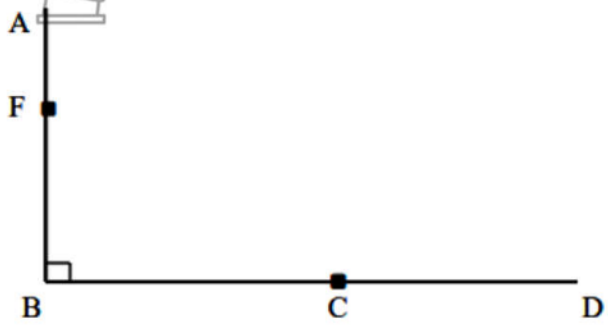
8.5 Determine the values of x where $f'(x) > 0$ (3)

8.6 Determine the gradient of the line DC. (1)

[17]

QUESTION 9

Two cyclists start to cycle at the same time. One starts at point B and is heading due north to point A, whilst the other starts at point D and is heading due west to point B. The cyclist starting from B cycles at 30 km/h while the cyclist starting from D cycles at 40 km/h. The distance between B and D is 100 km. After time t (measured in hours), they reach points F and C respectively.



9.1 Determine the distance between F and C in terms of t. **(4)**

9.2 After how long will the two cyclists be closest to each other? **(4)**

9.3 What will the distance between the cyclists be at the time determined in QUESTION 9.2? **(2)**

[10]