



LIMPOPO

PROVINCIAL GOVERNMENT
REPUBLIC OF SOUTH AFRICA

DEPARTMENT OF
EDUCATION

MOGALAKWENA DISTRICT

GRADE 12

MATHEMATICS
TERM 2
PRE JUNE EXAM PAPER 1
MARKING GUIDELINE
MAY/JUNE 2023

MARKS: 150




INSTRUCTIONS:

NOTE:


- This memorandum consists of 19 pages including the cover page.
- If the candidate answers a question TWICE, only mark the FIRST attempt.
- Consistent accuracy applies in all aspects of the marking memorandum.






QUESTION 1

1.1.1	$x = 3 ; x = -\frac{2}{5}$ 	✓✓ each root	(2)
1.1.2	$3x^2 - 10x - 1 = 0$ $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ $= \frac{-(-10) \pm \sqrt{(-10)^2 - 4(3)(-1)}}{2(3)}$ $= \frac{10 \pm \sqrt{112}}{6}$ $x = 3,43$ of $x = -0,1$	✓ subst ✓✓ each root (-1, if incorrect rounding)	(3)
1.1.3	$x^2 - 4x \geq 21$ $x^2 - 4x - 21 \geq 0$ $(x - 7)(x + 3) \geq 0$ $\therefore x \leq -3 \text{ and } x \geq 7, x \in \mathbb{R}$	✓ std form ✓ factors ✓ critical pts ✓✓ conclusion	(5)
1.1.4	$2 \cdot 3^x + \frac{3^x}{2} = 7\frac{1}{2} (\times 2)$ $2 \cdot 2 \cdot 3^x + 3^x = \frac{15}{2} \times \frac{2}{1}$ $4 \cdot 3^x + 3^x = 15$ $3^x (4+1) = 15$ $3^x = 3$ $x = 1$	✓ simplify ✓ factorisation ✓ simplify ✓ $x = 1$	(4)




<p>1.2</p> <p> $x + 2y = 2 \dots\dots\dots(1)$ $x^2 + 8y = 8 \dots\dots\dots(2)$ $x = -2y + 2 \dots(3)$ Subst 3 in 2. $(-2y + 2)(-2y + 2) + 8y = 8$  $4y^2 - 8y + 4 + 8y - 8 = 0$ $4y^2 - 4 = 0$ $y^2 - 1 = 0$ $(y - 1)(y + 1) = 0$ $y = 1$ or $y = -1$ Subst $y = 1$ in 3 Subst $y = -1$ in 3 $x = -2(1) + 2$ $x = -2(-1) + 2$ $x = 0$ $x = 4$ Solution $(0 ; 1)$ $(4 ; -1)$ </p>	<p>✓substitution</p> <p>✓std form</p> <p>✓factorisation</p> <p>✓y-value</p> <p>✓✓x-values.</p> <p>(6)</p>
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<p>1.2.1</p> $2^{x+1} + 2^x = 3^{y+2} - 3^y$ $2^x(2^1 + 1) = 3^y(3^2 - 1)$ $2^x(3) = 3^y(8)$ $\therefore \frac{2^x}{8} = \frac{3^y}{3}$ $\therefore \frac{2^x}{2^3} = \frac{3^y}{3}$ $\therefore 2^{x-3} = 3^{y-1}$ $\therefore x-3=0 \quad \text{and} \quad y-1=0$ $\therefore x=3 \quad \text{and} \quad y=1$ <p>OR</p> $2^{x+1} + 2^x = 3^{y+2} - 3^y$ $2^x(2^1 + 1) = 3^y(3^2 - 1)$ $2^x(3) = 3^y(8)$ $2^x(3) = 3^y(2^3)$ $\therefore x=3 \quad \text{and} \quad y=1$	<p>✓ factorise</p> <p>✓ simplify equated bases</p> <p>✓ answers</p> <p>OR</p> <div style="display: flex; align-items: center;">  <p>✓ factorise</p> </div> <p>✓ simplified equated bases</p> <p>✓ answers</p> <p style="text-align: right;">(3)</p>
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<p>1.2.2</p>	<p> $\sqrt{x-2} + 3 = \frac{10}{\sqrt{x-2}}$  $\text{Let } k = \sqrt{x-2}$ $k + 3 = \frac{10}{k}$ $k^2 + 3k - 10 = 0$ $(k+5)(k-2) = 0$ $k = -5 \text{ or } k = 2$ $\sqrt{x-2} \neq -5 \text{ or } \sqrt{x-2} = 2$ $(\sqrt{x-2})^2 = (2)^2$ $\therefore x = 6$ <p style="text-align: center;">OR</p> $\sqrt{x-2} + 3 = \frac{10}{\sqrt{x-2}}$ $(\sqrt{x-2})(\sqrt{x-2}) + 3\sqrt{x-2} = 10$ $x-2 + 3\sqrt{x-2} = 10$ $3\sqrt{x-2} = 12-x$ $(3\sqrt{x-2})^2 = (12-x)^2$ $9x-18 = 144-24x+x^2$ $x^2-33x+162 = 0$ $(x-6)(x-27) = 0$ $x = 6 \text{ or } x \neq 27$ </p>	<p> ✓ standard form ✓ factors ✓ both answers for k ✓ selection ✓ answer ✓ simplified both sides ✓ squaring both sides ✓ standard form ✓ factors  selection </p>
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(5)

1.3.1	$\Delta = 16 - 4p = 16 - 4(4) = 0$ \therefore Roots equal rational	✓ equal & rational	(1)
1.3.2	 $\Delta = 16 - 4p < 0$ non-real $4p < -16$ $p > 4$	$\checkmark \Delta < 0$ $\checkmark p > 4$	(2)
1.4	$x^2 + rx + m = 0$ and $x^2 + mx + r = 0$ $x^2 + rx + m = 0$ For real and equal roots, $\Delta = 0$ $b^2 - 4ac = 0$ $(r)^2 - 4(1)(m) = 0$ $r^2 - 4m = 0$ $r^2 = 4m$ $m = \frac{r^2}{4} \quad \dots (1)$ $x^2 + mx + r = 0$ $b^2 - 4ac = 0$ $m^2 - 4(1)(r) = 0$ $m^2 - 4r = 0 \quad \dots (2)$ Substitute (1) in (2) $\left(\frac{r^2}{4}\right)^2 - 4r = 0$ $\frac{r^4}{16} - 4r = 0$ $r^4 - 64r = 0$ $r(r^3 - 64) = 0$	\checkmark substitute into $\Delta = 0$ \checkmark equation for m \checkmark equation 2 \checkmark substitute for m	

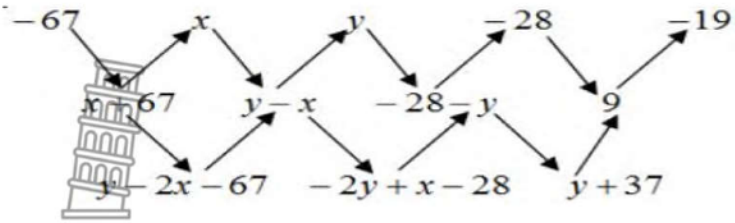



	$r(r-4)(r^2+4r+16)=0$ $\therefore r=4$ $m=\frac{r^2}{4}$ $m=\frac{4^2}{4}$ $\therefore m=4$  <p style="text-align: center;">OR</p> $x^2+rx+m=0 \text{ and } x^2+mx+r=0$ <p>For real and equal roots quadratic must be a perfect square.</p> $x^2+rx+m=0$ $(x+\sqrt{m})^2=0$ $x^2+2\sqrt{m}x+m=0$ $r=2\sqrt{m}$ $r^2=4m$ $\frac{r^2}{4}=m \quad \dots (1)$ $x^2+mx+r=0$ $(x+\sqrt{r})^2=0$ $x^2+2\sqrt{r}x+r=0$ $m=2\sqrt{r}$ $m^2=4r \quad \dots (2)$	<p>✓ value of r</p> <p>✓ value of m</p> <p>✓ $(x+\sqrt{m})^2=0$</p> <p>✓ equation for m</p> <p>✓ equation 2</p> <p>✓ substitute for m</p> 	
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	<p>Substitute (1) in (2)</p> $\left(\frac{r^2}{4}\right)^2 - 4r = 0$  $\frac{r^4}{16} - 4r = 0$ $r^4 - 64r = 0$ $r(r^3 - 64) = 0$ $r(r-4)(r^2 + 4r + 16) = 0$ $\therefore r = 4$ $m = \frac{r^2}{4}$ $m = \frac{4^2}{4}$ $\therefore m = 4$	<p>✓ value of r</p> <p>✓ value of m</p>	
			(6)
			[37]




QUESTION 2

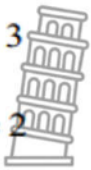
2.1	 $y - 2x - 67 = y + 37$ $-2x = 104$ $x = -52$ $y - 2x - 67 = -2y + x - 28$ $3y - 3x = 39$ $y - x = 13$ $y - (-52) = 13$ $y = -39$	<p>✓ first differences</p> <p>✓ second differences</p> <p>✓ equating second differences /</p> <p>✓ x-value</p> <p>✓ equating second differences /</p> <p>✓ y-value</p>	(6)
2.2	$\begin{array}{cccc} -67 & -52 & -39 & -28 \\ & 15 & 13 & 11 \\ & & -2 & -2 \end{array}$ $2a = -2$ $a = -1$ $3a + b = 15$ $3(-1) + b = 15$ $b = 18$ $a + b + c = -67$ $-1 + 18 + c = -67$ $c = -84$ $T_n = -n^2 + 18n - 84$	<p>✓ value of a</p> <p>✓ value of b</p>  <p>✓ value of c</p> <p>✓ answer</p>	(4)

2.3	$T_n > 0$ $-n^2 + 18n - 84 > 0$ $n^2 - 18n + 84 < 0$ no Solution \therefore the sequence will never contain a positive term	$\checkmark -n^2 + 18n - 84 > 0$ $\checkmark n^2 - 18n + 84 < 0$ \checkmark conclusion	(3)
			[13]

QUESTION 3


3.1	$T_4 = 24$ $T_9 = 768$ $\frac{T_9}{T_4} = r^{9-4}$ $\frac{768}{24} = r^5$ $32 = r^5$ $2^5 = r^5$ $r = 2$ $\frac{T_4}{T_1} = r^3$ or $\frac{T_9}{T_1} = r^8$ $\frac{24}{a} = 2^3$ $\frac{768}{a} = 2^8$ $a = \frac{24}{8}$ $a = \frac{768}{256}$ $a = 3$ $a = 3$ $T_n = ar^{n-1}$ $T_n = 3 \cdot 2^{n-1}$ 3; 6; 12; OR	$\checkmark \frac{768}{24} = r^5$ \checkmark simplification $\checkmark r$ $\checkmark a$ $\checkmark T_2 \& T_3$	(5)
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	$T_4 = 24$ $T_n = ar^{n-1}$ $T_4 = ar^3 = 24 \dots\dots\dots\text{equation1}$ $T_9 = ar^8 = 768 \dots\dots\dots\text{equation2}$ <div style="position: relative;"> equation2 equation1 <hr style="width: 50%; margin-left: auto; margin-right: 0;"/> </div> $ar^8 = 768$ $ar^3 = 24$ $r^5 = 32$ $r^5 = 2^5$ $r = 2$ $ar^3 = 24$ or $ar^8 = 768$ $a \cdot 2^3 = 24$ $a \cdot 2^8 = 768$ $a = 3$ $a = 3$ $T_n = ar^{n-1}$ $T_n = 3 \cdot 2^{n-1}$ 3; 6; 12; 	✓ equations	
		✓ $r^5 = 32$	
		✓ r	
		✓ a	
		✓ T ₂ & T ₃	
3.2.1	$T_{12} = S_{12} - S_{11}$ $= 324 - 275$ $= 49$	✓ $T_{12} = S_{12} - S_{11}$ ✓ substitution ✓ answer	(3)
3.2.2	$T_n = S_n - S_{n-1}$ $= 2n^2 + 3n - [2(n-1)^2 + 3(n-1)]$ $= 2n^2 + 3n - [2n^2 - 4n + 2 + 3n - 3]$ $= 2n^2 + 3n - 2n^2 + n + 1$ $= 4n + 1$	✓ $T_n = S_n - S_{n-1}$ ✓ substitution ✓ simplification ✓ answer	(4)
			

3.3	$\sum_{n=2}^{18} (2n - 1)$ $3 + 5 + 7 + \dots$ <div style="display: flex; justify-content: space-around; align-items: center;"> <div style="text-align: center;">  $a = 3$ $d = 2$ $n = 17$ </div> <div> $S_n = \frac{n}{2} [2a + (n-1)d]$ $S_{17} = \frac{17}{2} [2 \cdot 3 + (17-1)2]$ $= 323$ </div> </div>	<div style="text-align: center;">✓ a & d</div> <div style="text-align: center;">✓ $n = 17$</div> <div style="text-align: center;">✓ answer</div>	(3)
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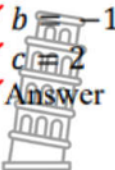
3.4	<div style="display: flex; justify-content: space-between;"> <div style="width: 45%;"> $S_{\infty} = \frac{40}{3}$ $S_{\infty} = \frac{a}{1-r} = \frac{40}{3}$ $a = \frac{40}{3}(1-r)$ $a = \frac{40}{3} \left(1 - \frac{5}{2a}\right)$ $a = \frac{40}{3} - \frac{100}{3a}$ $3a^2 = 40a - 100$ $3a^2 - 40a + 100 = 0$ $(3a - 10)(a - 10) = 0$ $a = \frac{10}{3} \text{ or } a = 10$ </div> <div style="width: 45%;"> $T_2 = \frac{5}{2}$ $T_n = ar^{n-1}$ $T_2 = ar = \frac{5}{2}$ $r = \frac{5}{2a}$ </div> </div>	<div style="text-align: center;">✓ substitution</div> <div style="text-align: center;">✓ standard form</div> <div style="text-align: center;">✓ factors</div> <div style="text-align: center;">✓ answer(both values of a)</div>	(3)
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3.5.1	$a = \frac{24}{x}$ $r = \frac{6x}{12} \text{ or } \frac{3x^2}{6x} \text{ or } \frac{x}{2}$  $S_{\infty} = \frac{x}{1 - \frac{x}{2}}$ $S_{\infty} = \frac{48}{2x - x^2}$	<p>✓ value of r</p> <p>✓ substitution in correct formula</p> <p>✓ correct numerator</p> <p>✓ correct denominator</p>	(4)
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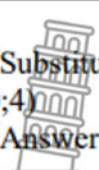
3.5.2	<p>It exists when:</p> $-1 < \frac{x}{2} < 1 \text{ i.e.: } -1 < r < 1$ $\therefore -2 < x < 2$ <p>ANSWER ONLY: Award full marks.</p>	<p>✓ $-1 < r < 1$</p> <p>✓ answer</p>	(2)
			[24]

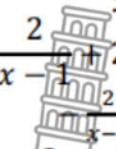
QUESTION 4

4.1	$f(x) = a(x - 2)^2 + 9$ $a(3 - 2)^2 + 9 = 8$ $a + 9 = 8$ $a = -1$ $\therefore f(x) = -1(x - 2)^2 + 9$ $f(x) = -1(x^2 - 4x + 4) + 9$ $f(x) = -x^2 + 4x - 4 + 9$ $f(x) = -x^2 + 4x + 5$	<p>✓ $T_n = 2^n$</p> <p>✓ $a = 1$</p> <p>✓ $b = 1$</p> <p>✓ $c = 2$</p> <p>✓ Answer</p> 	
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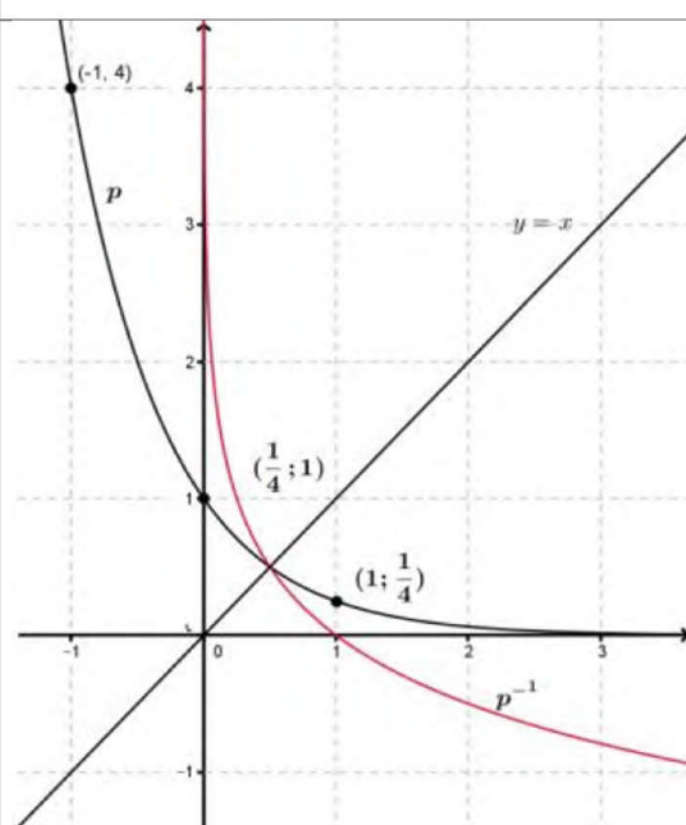

4.2	$x = 2$	✓ Answer	(1)
4.3	$g(x) = a^x$ $a^3 = 8$ $a^3 = 2^3$ $a = 2$ 	✓ Substitution ✓ Answer	(2)
4.4	(5; 0)	✓ Answer	(1)
4.5	$x < -1$ or $x > 5$	Mark per answer ✓ $x < -1$ ✓ $x > 5$	(2)
4.6	$g(x) = 2^x$ $g(2) = 2^2 = 4$ $AC = 9 - 4$ $AC = 5$ units	✓ $g(2) = 4$ ✓ Answer	(2)
4.7	Roots will be real and equal	✓ Real ✓ Equal	(2)
			[14]


QUESTION 5

5.1	x - intercept of the line: $-2x + 2 = 0 \therefore x = 1$ $p = -1$ y -intercept of line: $y = 2$ $q = 2$	✓ x int ✓ p ✓ q	(3)
5.2	$g(x) = \frac{a}{x-1} + 2$ $\frac{a}{0-1} + 2 = 4$ $\therefore a = -2$	 ✓ Substitute point (0;4) ✓ Answer	(2)


5.3	$g(x) = -\frac{2}{x-1} + 2$ $-\frac{2}{x-1} + 2 = 0$  $\frac{2}{x-1} = 2$ $-2x + 2 = -2$ $\therefore x = 2$ $\therefore A = (2; 0)$	<p>✓ Subst $y = 0$</p> <p>✓ Answer</p>	(2)
5.4	The graph moves up 1 unit	✓ Answer	(1)
			[8]


QUESTION 6

6.1	$x = \left(\frac{1}{4}\right)^y$ $y = \log_{\frac{1}{4}} x$	<p>✓ Interchange</p> <p>✓ Answer</p>	(2)
6.2		<p>Intercept of p ✓</p> <p>Intercept of p^{-1} ✓</p> <p>Line: $y = x$ ✓</p> <p>Shape of p and p^{-1} ✓✓</p> 	(5)


6.3	$0 < x \leq \frac{1}{4}$	✓ critical points ✓ answer	(2)
6.4	$h(x) = \left(\frac{1}{4}\right)^{x-2} + 3$ 	✓ -2 ✓ +3	(2)
			[11]

QUESTION 7

7.1	$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$ $= \lim_{h \rightarrow 0} \frac{5 - 2(x+h)^2 - (5 - 2x^2)}{h}$ $= \lim_{h \rightarrow 0} \frac{5 - 2x^2 - 4xh - 2h^2 - 5 + 2x^2}{h}$ $= \lim_{h \rightarrow 0} \frac{-4xh - 2h^2}{h}$ $= \lim_{h \rightarrow 0} \frac{h(-4x - 2h)}{h}$ $= \lim_{h \rightarrow 0} (-4x - 2h)$ $= -4x$	✓ substitution ✓ expansion ✓ simplification ✓ notation and $\lim_{h \rightarrow 0} (-4x - 2h)$ ✓ answer	(5)
7.2.1	$y = 7x^4 + \frac{2x^2}{\sqrt{x}}$ $= 7x^4 + 2x^{\frac{3}{2}}$ $\therefore \frac{dy}{dx} = 28x^3 + 3x^{\frac{1}{2}}$	✓ $2x^{\frac{3}{2}}$ ✓ $28x^3$ ✓ $3x^{\frac{1}{2}}$ 	(3)

7.2.2	$xy = 5$ $y = \frac{5}{x}$ $y = 5x^{-1}$ $\frac{dy}{dx} = -5x^{-2}$ 	$\checkmark y = 5x^{-1}$ \checkmark answer	(2)
7.2.3	$= D_x \left[\frac{3x^2 - 7x - 6}{x} \right]$ $= D_x [3x - 7 - 6x^{-1}]$ $= 3 + 6x^{-2}$	$\checkmark 3x - 7$ $\checkmark -6x^{-1}$ $\checkmark 3$ and differentiating constant $\checkmark +6x^{-2}$	(4)
			[14]

QUESTION 8

8.1	$x^3 - 6x^2 + 9x - 4 = 0$ $(x - 1)(x^2 - 5x + 4) = 0$ $(x - 1)(x - 4)(x - 1) = 0$ $x = 1; x = 4$ $B(4; 0)$ $\therefore f(3) = -4$ $D: (3; -4)$	$\checkmark (x - 1)$ $\checkmark (x^2 - 5x + 4)$ $\checkmark (x - 4)(x - 1)$ \checkmark coordinates	(4)
8.2	$(x) = x^3 - 6x^2 + 9x - 4$ $f'(x) = 3x^2 - 12x + 9 = 0$ $x^2 - 4x + 3 = 0$ $x = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(3)}}{2(1)}$ $x = 3; x = 1$ $\therefore f(3) = -4 \therefore D(3; -4)$	$\checkmark 3x^2 - 12x + 9$ $\checkmark = 0$ \checkmark subs. into formula $\checkmark x = 3$ $\checkmark y = -4$	(5)
8.3	$f'(x) = 3x^2 - 12x + 9$ $f''(x) = 6x - 12 = 0$ $x = 2$	$\checkmark 6x - 12 = 0$ $\checkmark x = 2$ 	(2)
8.4	$k < -4$ or/of $k > 0$	$\checkmark k < -4$ $\checkmark k > 0$	(2)

8.5	$x < 1$ or/of $x > 3$	✓ $x < 1$ ✓ or ✓ $x > 3$	(3)
8.6	0	✓	(1)
			[17]

QUESTION 9

9.1	<p>After t hours: $BF = 30t$ km and $CD = 40t$ km</p> <p>$\therefore BC = 100 - 40t$</p> $FC = \sqrt{(30t)^2 + (100 - 40t)^2}$ $= \sqrt{900t^2 + 10000 - 8000t + 1600t^2}$ $= \sqrt{2500t^2 - 8000t + 10000}$	<p>✓ $BF = 30t$</p> <p>✓ $BC = 100 - 40t$</p> <p>✓ Pythagoras</p> <p>✓ answer</p>	(4)
9.2	<p>FC is a minimum when FC^2 is a minimum.</p> $FC^2 = 2500t^2 - 8000t + 10000$ $\frac{dFC^2}{dt} = 5000t - 8000 = 0$ $t = \frac{8000}{5000} = 1.6 \text{ hrs (96 minutes)}$	<p>✓ $BF = 30t$</p> <p>✓ $BC = 100 - 40t$</p> <p>✓ Pythagoras</p> <p>✓ answer</p>	(4)
9.3	$FC = \sqrt{2500t^2 - 8000t + 10000}$ $= \sqrt{2500(1.6)^2 - 8000(1.6) + 10000}$ $= 60$ <p>They will be 60km apart.</p>	<p>✓ subs into equation</p> <p>✓ answer</p>	(2)
			[10]

