



education
MPUMALANGA PROVINCE
REPUBLIC OF SOUTH AFRICA

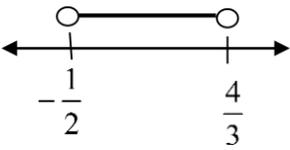
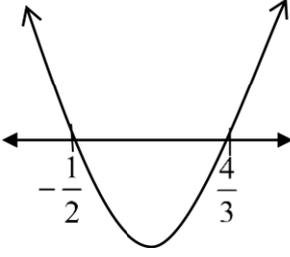
**NATIONAL
SENIOR CERTIFICATE**

GRADE 12

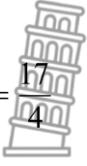
**MATHEMATICS PAPER 1
SEPTEMBER 2023
MARKING GUIDELINES**

MARKS: 150

This marking guideline consists of 13 pages.

QUESTION 1 / VRAAG 1		
1.1.1	$(3-x)(2-x) = 0$ $x = 3$ or $x = 2$	✓ ✓ answers (2)
1.1.2	$2x^2 + 7x - 2 = 0$ $2x^2 + 7x - 2 = 0$ $x = \frac{-(-7) \pm \sqrt{7^2 - 4(2)(-2)}}{2(2)}$ $x = \frac{-7 \pm \sqrt{65}}{4}$ $x = 0,27$ or $x = -3,77$ <div style="border: 1px solid black; padding: 2px; display: inline-block; margin-top: 10px;"> Penalise 1 mark for incorrect rounding </div>	✓ standard form ✓ substitution into correct formula ✓ ✓ answers (4)
1.1.3	$4 + 5x > 6x^2$ $6x^2 - 5x - 4 < 0$ $(3x - 4)(2x + 1) < 0$  OR  $-\frac{1}{2} < x < \frac{4}{3}$ OR $x \in \left(-\frac{1}{2}; \frac{4}{3}\right)$	✓ factors ✓ critical values ✓ ✓ answer (4)
1.1.4	$9^x + 9 = 10 \cdot 3^x$ $3^{2x} - 10 \cdot 3^x + 9 = 0$ $(3^x - 9)(3^x - 1) = 0$ $3^x = 3^2$ or $3^x = 3^0$ $x = 2$ or $x = 0$	✓ 3^{2x} ✓ factors ✓ exponential law ✓ answer  (4)

<p>1.2</p> $y - 2x = -1$ $y = 2x - 1 \dots (1)$ $y^2 - 3xy = -2 \dots (2)$ $(2x - 1)^2 - 3x(2x - 1) = -2$ $4x^2 - 4x + 1 - 6x^2 + 3x = -2$ $-2x^2 - x + 3 = 0$ $2x^2 + x - 3 = 0$ $(2x + 3)(x - 1) = 0$ $x = -\frac{3}{2} \text{ or } x = 1$ $y = 2\left(-\frac{3}{2}\right) - 1 \text{ or } y = 2(1) - 1$ $y = -4 \text{ or } y = 1$ <p style="text-align: center;">OR</p> $y - 2x = -1$ $x = \frac{y + 1}{2} \dots (1)$ $y^2 - 3xy = -2 \dots (2)$ $y^2 - 3\left(\frac{y + 1}{2}\right)y = -2$ $2y^2 - 3y^2 - 3y + 4 = 0$ $-y^2 - 3y + 4 = 0$ $y^2 + 3y - 4 = 0$ $(y + 4)(y - 1) = 0$ $y = -4 \text{ or } y = 1$ $x = -\frac{3}{2} \text{ or } x = 1$	<p>✓ equation 1</p> <p>✓ substitution</p> <p>✓ standard form</p> <p>✓ factors</p> <p>✓ x-values</p> <p>✓ y-values (6)</p> <p>✓ equation 1</p> <p>✓ substitution</p> <p>✓ standard form</p> <p>✓ factors</p> <p>✓ y-values</p> <p>✓ x-values (6)</p>
<p>1.3</p> <p>Multiply every term by LCD $4xy$</p> $4x^2 + 4y^2 = 17xy$ $4x^2 - 17xy + 4y^2 = 0$ $(4x - y)(x - 4y) = 0$ $4x - y = 0$ $\frac{x}{y} = \frac{1}{4}$ $x - 4y = 0$ $\frac{x}{y} = 4$	<p>✓ multiply by LCD</p> <p>✓ standard form</p> <p>✓ factors</p> <p>✓ answer</p> <p>✓ answer (5)</p>

	<p>OR</p> $\frac{x}{y} + \frac{y}{x} = \frac{17}{4}$  $\frac{x}{y} + \frac{1}{\frac{x}{y}} = \frac{17}{4}$ $\frac{x^2}{y^2} - \frac{17x}{4y} + 1 = 0$ $\frac{x}{y} = \frac{\frac{17}{4} \pm \sqrt{\left(\frac{17}{4}\right)^2 - 4(1)(1)}}{2(1)}$ $= \frac{\frac{17}{4} \pm \sqrt{\frac{225}{16}}}{2}$ $\frac{x}{y} = 4 \text{ or } \frac{x}{y} = \frac{1}{4}$	<p>✓ fraction</p> <p>✓ standard form</p> <p>✓ substitution into correct formula</p> <p>✓ simplification</p> <p>✓ answers (5)</p>
[25]		

Stanmorephysics

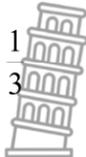
QUESTION 2 / VRAAG 2

2.1	319 ; 280 ; 243 ; 208 ; 175 ; 144 ;	
2.1.1	$ \begin{array}{cccc} 319 & 280 & 243 & 208 \\ & \swarrow & \searrow & \swarrow & \searrow \\ & -39 & -37 & -35 & \\ & & \swarrow & \searrow & \\ & & 2 & 2 & \end{array} $ $ \begin{array}{lll} 2a = 2 & 3a + b = -39 & a + b + c = 319 \\ a = 1 & 3(1) + b = -39 & 1 - 42 + c = 319 \\ & b = -42 & c = 360 \end{array} $ $T_n = n^2 - 42n + 360$	<p>✓ First differences</p>  <p>✓ $2a = 2$</p> <p>✓ $3(1) + b = -39$</p>

		✓ $1 - 42 + c = 319$ (4)
2.1.2	$n^2 - 42n + 360 = 0$ $(n - 12)(n - 30) = 0$ $n = 12$ or $n = 30$	✓ $T_n = 0$ ✓ factors ✓ answer (3)
2.1.3	$2n - 42 = 0$ OR $n = \frac{-(-42)}{2(1)}$ $n = 21$ $n = 21$ T ₂₁ is lowest	✓ derivative = 0 ✓ answer (2) OR ✓ substitution into $x = \frac{-b}{2a}$ ✓ answer (2)
2.2	$3t ; 4t - 1 ; 23$	
2.2.1	$3t ; 4t - 1 ; 23$ $4t - 1 - 3t = 23 - (4t - 1)$ $4t - 1 - 3t = 23 - 4t + 1$ $5t = 25$ $t = 5$	✓ subtraction ✓ simplification (2)
2.2.2	$15 ; 19 ; 23$ $a = 15 \quad d = 4$ $S_{50} = \frac{50}{2} [2(15) + (49)4]$ $= 5650$	✓ $a = 15$ ✓ value of d ✓ substitution into correct formula ✓ answer (4)
		[15]



QUESTION 3

3.1	$x = 4 + 12 + 36 + \dots$ to 15 terms	
3.1.1	$T_n = ar^{n-1}$ $T_n = 4 \cdot 3^{n-1}$ $\sum_{n=1}^{15} (4 \cdot 3^{n-1})$ 	✓ $T_n = 4 \cdot 3^{n-1}$ ✓ answer (2)
3.1.2	$S_n = \frac{a(r^n - 1)}{r - 1}$ $S_n = \frac{4(3^{15} - 1)}{3 - 1}$ $= 28697812$	✓ substitution into the correct formula ✓ answer (2)
3.1.3	$r = 3$ and not between -1 and 1 \therefore not converging	✓ $r = 3$ not in $-1 < r < 1$ ✓ answer (2)
3.2.1	$S_\infty = \frac{a}{1 - r} = 13,5$ $a = 13,5(1 - r)$ $S_\infty = \frac{ar^2}{1 - r} = 1,5$ $ar^2 = 1,5(1 - r)$ $a = \frac{1,5(1 - r)}{r^2}$ $13,5(1 - r) = \frac{1,5(1 - r)}{r^2}$ $r^2 = \frac{1,5}{13,5}$ $= \frac{1}{9}$ $r = \pm \frac{1}{3}$ But $r > 0$ $\therefore r = \frac{1}{3}$	✓ substitution into correct formula ✓ $a = 13,5(1 - r)$ ✓ equating ✓ $r = \frac{1}{3}$  (4)

3.2.2	$\frac{a}{1 - \frac{1}{3}} = 13,5$ $a = 9$ $9 + 3 + 1 + \dots$ 	✓ value of a ✓ answer (2)
QUESTION 4		[12]
4.1	$0 = \frac{a}{-2+1} + 2$ $a = 2$	✓ substitution ✓ answer (2)
4.2	Average gradient = $\frac{h(-2) - h(-4)}{-2 - (-4)}$ $= \frac{\frac{2}{-2+1} + 2 - \left(\frac{2}{-4+1} + 2\right)}{2}$ $= -\frac{2}{3}$	✓ $h(-4)$ and $h(-2)$ ✓ substitution ✓ answer (3)
4.3	$-1 < x < 0$	✓ critical values ✓ answers (2)
4.4	$y = x + 1 + 2$ $y = x + 3$ OR $y = x + c$ $2 = -1 + c$ $c = 3$ $y = x + 3$	✓ gradient ✓ answer (2) OR ✓ value of c ✓ answer (2)
4.5	$g(x) = \frac{-2}{x+1} - 2$ $x = -1$ $y = -2$	✓ horizontal asymptote ✓ vertical asymptotes (2)
		[11]

QUESTION 5		
	$h(x) = q^x \quad f(x) = -\frac{1}{2}x^2 + \frac{1}{2}x + k \quad g(x) = mx + c$	
5.1	$h(x) = q^0$ $= 1$  $D(0 ; 1)$	✓ answer (1)
5.2	$k = 1$	✓ answer (1)
5.3	$f(x) = -\frac{1}{2}x^2 + \frac{1}{2}x + 1$ $x^2 - x - 2 = 0$ $(x - 2)(x + 1) = 0$ $x = 2$ or $x = -1$ BC = 3 units	✓ $f(x) = 0$ ✓ factors ✓ both x values ✓ answer (4)
5.4	$f(-2) = -\frac{1}{2}(-2)^2 + \frac{1}{2}(-2) + 1$ $= -2$ PR = 2 units	✓ substitution of -2 ✓ answer (2)
5.5.1	$h(x) = q^x$ $\frac{1}{4} = q^2$ $q = \frac{1}{2}$	✓ $A\left(2; \frac{1}{4}\right)$ ✓ substitution of $\left(2; \frac{1}{4}\right)$ ✓ value of q (3)
5.5.2	$h(x) = \left(\frac{1}{2}\right)^x$ $h^{-1}(x) = \log_{\frac{1}{2}} x$	✓ equation of h ✓ equation of h^{-1} (2)
5.6	$y > 0$	✓ answer (1)
5.7	$f'(x) = -x + \frac{1}{2}$ $f'(-1) = -(-1) + \frac{1}{2} = \frac{3}{2}$ $y - y_1 = m(x - x_1)$ B(-1 ; 0) $y - 0 = \frac{3}{2}(x - (-1))$ $y = \frac{3}{2}x + \frac{3}{2}$	✓ derivative ✓ derivative $x = -1$ ✓ $f'(-1) = \frac{3}{2}$ ✓ substitution of B ✓ tangent  (5)
5.8	$x < -1$ or $x > 2$ OR $x \in (-\infty ; 1) \cup (2 ; \infty)$	✓✓ answer  (2)
		[21]

QUESTION 6

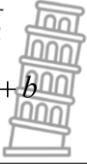
<p>6.1</p>	$A = P(1+i)^n$ $2x = x(1+0,054)^n$ $2 = (1,054)^n$ $\log(2) = n \log(1,054)$ $n = \frac{\log 2}{\log(1,054)}$ $n = 13,18$	<ul style="list-style-type: none"> ✓ substitution into correct formula ✓ correct use of logs ✓ answer (3)
<p>6.2</p>	$P = \frac{x [1 - (1+i)^{-n}]}{i}$ $P = \frac{3300 \left[1 - \left(1 + \frac{0,12}{12} \right)^{-60} \right]}{\frac{0,12}{12}}$ $= R148351,63$	<ul style="list-style-type: none"> ✓ value of i ✓ value of n ✓ substitution into correct formula ✓ answer (4)
<p>6.3</p>	$i_{eff} = \left(1 + \frac{0,05}{4} \right)^4 - 1$ $= 0,05094533691$ <p>Rate = 5,09%</p>	<ul style="list-style-type: none"> ✓ value of n ✓ substitution into correct formula ✓ rate (3)
<p>6.4</p>	$10\,000\,000 = \frac{x \left[\left(1 + \frac{0,15}{12} \right)^{384} - 1 \right]}{\frac{0,15}{12}}$ $125000 = x(116,9484518)$ $x = R1068,85$	<ul style="list-style-type: none"> ✓ value of n ✓ substitution into correct formula ✓ answer (3)
<p>[13]</p>		



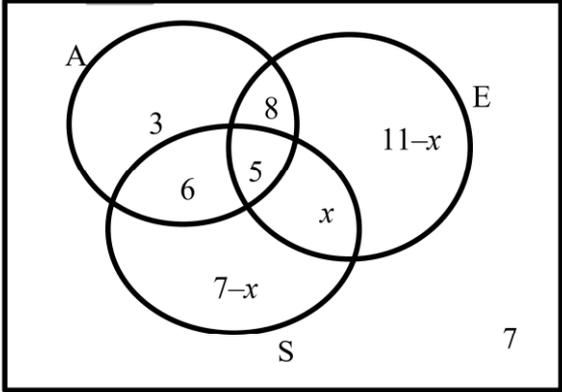
QUESTION 7		Penalty for notation error in 7.1 only.
7.1	$f(x) = -2x^2 + 1$ $f(x+h) = -2(x+h)^2 + 1$ $f(x+h) = -2x^2 - 4xh - 2h^2 + 1$ $f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$ $f'(x) = \lim_{h \rightarrow 0} \frac{-2x^2 - 4xh - 2h^2 + 1 - (-2x^2 + 1)}{h}$ $f'(x) = \lim_{h \rightarrow 0} \frac{-2x^2 - 4xh - 2h^2 + 1 + 2x^2 - 1}{h}$ $f'(x) = \lim_{h \rightarrow 0} \frac{-4xh - 2h^2}{h}$ $f'(x) = \lim_{h \rightarrow 0} \frac{h(-4x - 2h)}{h}$ $= -4x$	<p>✓ $-2x^2 - 4xh - 2h^2 + 1$</p> <p>✓ correct substitution into formula</p> <p>✓ simplification</p> <p>✓ common factor</p> <p>✓ answer (5)</p>
7.2.1	$f(x) = \frac{1}{2}x^2 - 5x^{-1}$ $f'(x) = x + 5x^{-2}$	<p>✓ $-5x^{-1}$</p> <p>✓ x</p> <p>✓ $5x^{-2}$ (3)</p>
7.2.2	$D_x \left[\frac{-2x^2 + \sqrt[4]{x}}{x^2} \right]$ $= D_x \left[-\frac{2x^2}{x^2} + \frac{x^{\frac{1}{4}}}{x^2} \right]$ $= D_x \left[-2 + x^{-\frac{7}{4}} \right]$ $= -\frac{7}{4}x^{-\frac{11}{4}}$	<p>✓ $\sqrt[4]{x} = x^{\frac{1}{4}}$</p> <p>✓ $-2 + x^{-\frac{7}{4}}$</p> <p>✓ derivative (3)</p>
		[11]

QUESTION 8

<p>8.1</p>	<table border="1" style="margin-left: auto; margin-right: auto;"> <tr> <td>1</td> <td>-1</td> <td>5</td> <td>8</td> <td>-12</td> </tr> <tr> <td>0</td> <td>-1</td> <td>4</td> <td>12</td> <td></td> </tr> <tr> <td>-1</td> <td>4</td> <td>12</td> <td>0</td> <td></td> </tr> </table> <p> $(x-1)(-x^2+4x+12)=0$ $(x-1)(x^2-4x-12)=0$ $(x-1)(x-6)(x+2)=0$ $x=1$ or $x=6$ or $x=-2$ $\therefore C(6; 0)$ </p>	1	-1	5	8	-12	0	-1	4	12		-1	4	12	0		<p> ✓ quadratic factor ✓ factors ✓ x-values ✓ answer (4) </p>
1	-1	5	8	-12													
0	-1	4	12														
-1	4	12	0														
<p>8.2</p>	<p> $-3x^2+10x+8=0$ $3x^2-10x-8=0$ $(3x+2)(x-4)=0$ $x=-\frac{2}{3}$ or $x=4$ $f(4)=-(-4)^3+5(4)^2+8(4)-12$ $=36$ </p>	<p> ✓ derivative ✓ $f'(x)=0$ ✓ factors ✓ x-values ✓ answer (5) </p>															
<p>8.3</p>	<p> $-6x+10>0$ $x<\frac{5}{3}$ </p>	<p> ✓ inequality ✓ answer (2) </p>															
<p>8.4</p>	<p> $f'(x)=-3x^2+10x+8$ $f'(0)=8$ $-3x^2+10x+8=8$ $-3x^2+10x=0$ $x(-3x+10)=0$ $x=0$ or $x=\frac{10}{3}$ $\therefore OG=\frac{10}{3}$ </p>	<p> ✓ $f'(0)=8$ ✓ equating ✓ factors ✓ x-values ✓ value of OG (5) </p>															
<p>8.5</p>	<p> $f'(x)=-3x^2+10x+8$ $f'(5)=-3(5)^2+10(5)+8$ $=-17$ $m=\frac{1}{17}$ </p>	<p> ✓ substitution ✓ $f'(5)=-17$ ✓ answer (3) </p>															
<p>[19]</p>																	

QUESTION 9		
9.1	$m_{mn} = -\frac{b}{a}$ $y = -\frac{b}{a}x + b$ 	✓ m_{mn} ✓ answer (2)
9.2	Area of PTOR = TP x PR $= x\left(-\frac{b}{a}x + b\right)$ $= -\frac{b}{a}x^2 + bx$ Max: $-\frac{2b}{a}x + b = 0$ $x = b\left(\frac{a}{2b}\right)$ $x = \frac{a}{2}$ $y = -\frac{b}{a}\left(\frac{a}{2}\right) + b = \frac{b}{2}$ Midpoint of MN: $\left(\frac{0+a}{2}; \frac{0+b}{2}\right) = \left(\frac{a}{2}; \frac{b}{2}\right)$ ∴ Max at midpoint	✓ substitution into area formula ✓ simplification ✓ derivative = 0 ✓ value of x ✓ value of y ✓ midpoint (6)
		[8]



QUESTION 10		
10.1.1	$45 - 38 = 7$	✓ answer (1)
10.1.2	$S = 45$ 	✓ 8 ✓ 6 ✓ $11-x$ ✓ $7-x$ (4)
10.1.3	$3 + 8 + 6 + 5 + 11 - x + x + 7 - x + 7 = 45$ $x = 2$	✓ calculation ✓ answer (2)
10.1.4	$P(2\text{languages}) = \frac{6 + 8 + 2}{45}$ $= \frac{16}{45}$	✓ 16 ✓ $\frac{16}{45}$ (2)
10.2.1	$8! = 40320$	✓ $8!$ (1)
10.2.2	$3! \times 5! \times 6 = 4320$ OR $3! \times 6! = 4320$	✓ $3!$ ✓ $5!$ ✓ $3! \times 5! \times 6$ (3)
10.2.3	$\frac{5!}{8!} = \frac{1}{336}$	✓ $5!$ ✓ $\frac{5!}{8!}$ OR $\frac{1}{336}$ (2)
		[15]



TOTAL: 150