education
MPUMALANGA PROVINCE
REPUBLIC OF SOUTH AFRICA

NATIONAL
SENIOR CERTIFICATE

GRADE 12

MATHEMATICS P2

SEPTEMBER 2023

MARKS: 150

TIME: 3 hours

This question paper consists of 13 pages and 1 information sheet

and an answer book is provided.

INSTRUCTIONS AND INFORMATION

Read the following instructions carefully before answering the questions.

1. The question paper consists of 10 questions.
2. Answer ALL the questions in the SPECIAL ANSWER BOOK provided.
3. Clearly show ALL calculations, diagrams, graphs, etc. which you have used in determining the answers.
4. Answers only will NOT necessarily be awarded full marks.
5. You may use an approved scientific calculator (non-programmable and non-graphical) unless stated otherwise.
6. If necessary, round off answers to TWO decimal places, unless stated otherwise.
7. Diagrams are NOT necessarily drawn to scale.
8. Write neatly and legibly.

QUESTION 1

The table below shows the masses (in kg) of 15 randomly chosen weight lifters of a certain gymnasium.

71	83	88	91	92	92	95	97	104	108	109	110	111	115	129

1.1 Calculate the mean mass of the weight lifters. (2)

1.2 Calculate the standard deviation of the masses of the weight lifters. (1)

1.3 What percentage of weight lifters fall in the feather weight division, if the criteria is that your mass must be below one standard deviation below the mean? (3)

1.4 If all the weight lifters loose 3kg, what will be the new

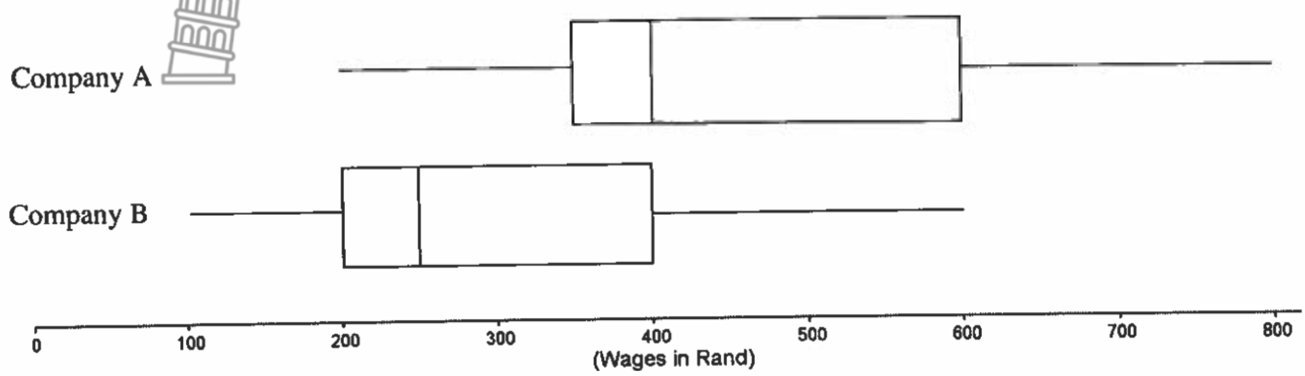
1.4.1 Mean mass of the weight lifters? (1)

1.4.2 Standard deviation of their masses? (1)

[8]

QUESTION 2

2.1 The box-and-whisker plots show the wages of workers (in Rand) at two companies for the same type of work. Both companies have 20 workers.



2.1.1 State whether the following statement is TRUE or FALSE:
All the workers at Company A earn more than 25% of the workers at Company B. (1)

2.1.2 Comment on the skewness of the data for company B. (1)

2.1.3 Which company has the biggest range? Motivate your answer with the necessary calculations. (2)

2.1.4 How many workers at Company B earn more than R200? (2)

2.2 A table of data, showing the price of crude oil at the end of each year in US Dollars ($) (to the nearest dollar) per barrel, is given.

Year	2011	2012	2013	2014	2015	2016	2017	2019	2020	2021
Price	90	92	98	54	37	54	60	61	49	75

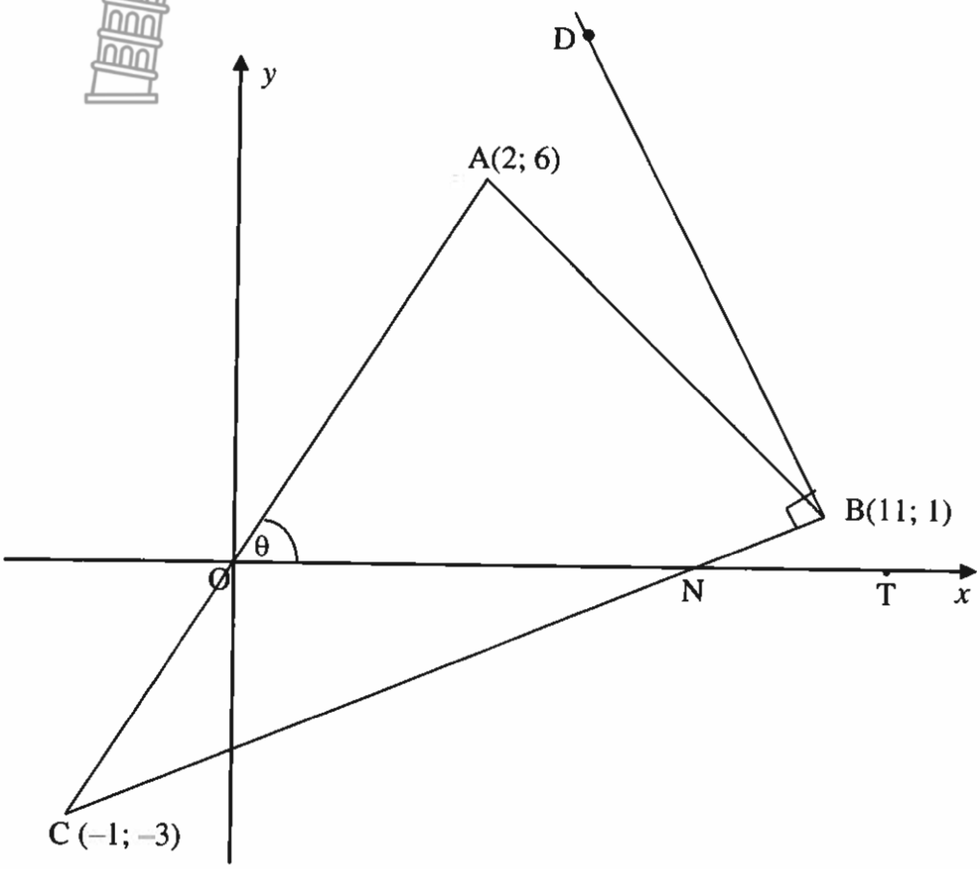
2.2.1 Determine the equation of the least squares regression line for the price of crude oil per year (3)

2.2.2 Calculate the value of the correlation coefficient. (1)

2.2.3 Use calculations to predict the price of crude oil (in US $) per barrel at the end of 2018. Discuss the validity of the answer that you obtained by using your answer in 2.2.2. (2)
[12]

QUESTION 3

In the diagram A(2 ; 6), B(11; 1) and C(–1; –3) are the vertices of ΔABC.
Point D is shown in the diagram such that $BD \perp BC$. N is the *x*-intercept of BC. $A\hat{O}N = \theta$.



3.1 If the gradient of BC is $\frac{1}{3}$ and the gradient of AC is 3, calculate

3.1.1 the *x*-coordinate of N. (2)

3.1.2 the size of $A\hat{C}B$. (5)

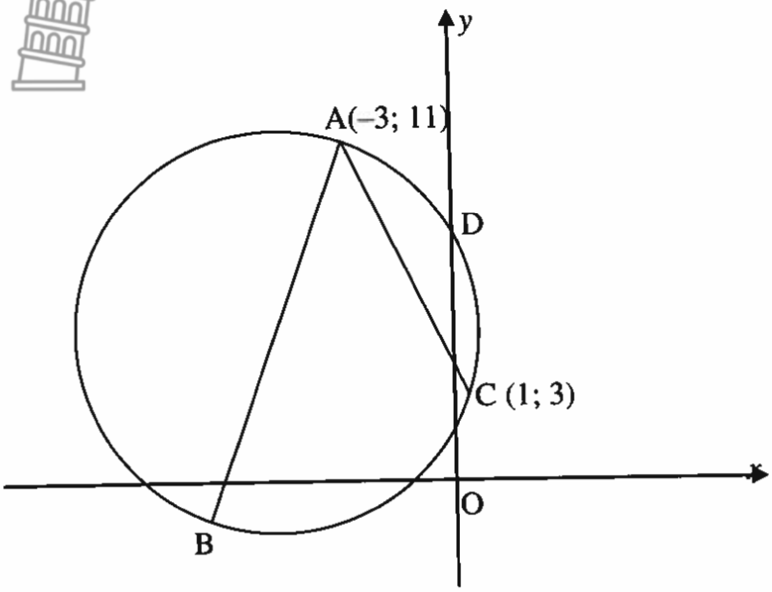
3.2 Determine the equation of AC. (2)

3.3 If it is further given that point D lies on AC produced such that $BC \perp BD$, calculate the coordinates of D. (5)

[14]

QUESTION 4

In the diagram A(–3; 11) and C(1; 3) are points on the circumference of a circle with diameter AB and centre T. The equation of AB is given by $y = 3x + 20$.



4.1 Determine the equation of the perpendicular bisector of AC. (4)

4.2 Show that the coordinates of the centre of the circle are (–5 ; 5). (3)

4.3 Calculate the length of diameter AB. (3)

4.4 Write down the equation of the circle. (2)

4.5 The tangent to the circle at A cuts the y-axis at $(0 ; p)$. Calculate the numerical value of p. (4)

4.6 If the circle through A, B and C is moved 3 units to the right and 2 units upwards, and the radius is halved, write down the equation of the new circle. (3)

4.7 A new circle with equation $(x-2)^2 + (y-3)^2 = 4$ and centre P is given. Will this circle intersect the original circle or not? Motivate your answer with the necessary calculations. (4)

[23]

QUESTION 5

5.1 If θ is a reflex angle, and $\tan\theta = -\frac{3}{4}$, determine without the use of a calculator and with the aid of a sketch, the value of:

5.1.1 $\sin\theta$ (2)

5.1.2 $\cos 2\theta$ (3)

5.1.3 $\cos(\theta + 30°)$ (3)

5.2 If $x = 4\sin\alpha$ and $y = 4\cos\alpha$, calculate the value of $x^2 + y^2$. (2)

5.3 Simplify the expression to a single trigonometric ratio:

$\sin(900° - x) \, . \cos(-x) - \sin(x - 180°) \, . \sin(90° + x)$ (6)

5.4 Given the following the identity: $\frac{\sin 7x + \sin x}{2\cos 3x} = \sin 4x$

5.4.1 Prove the identity. (4)

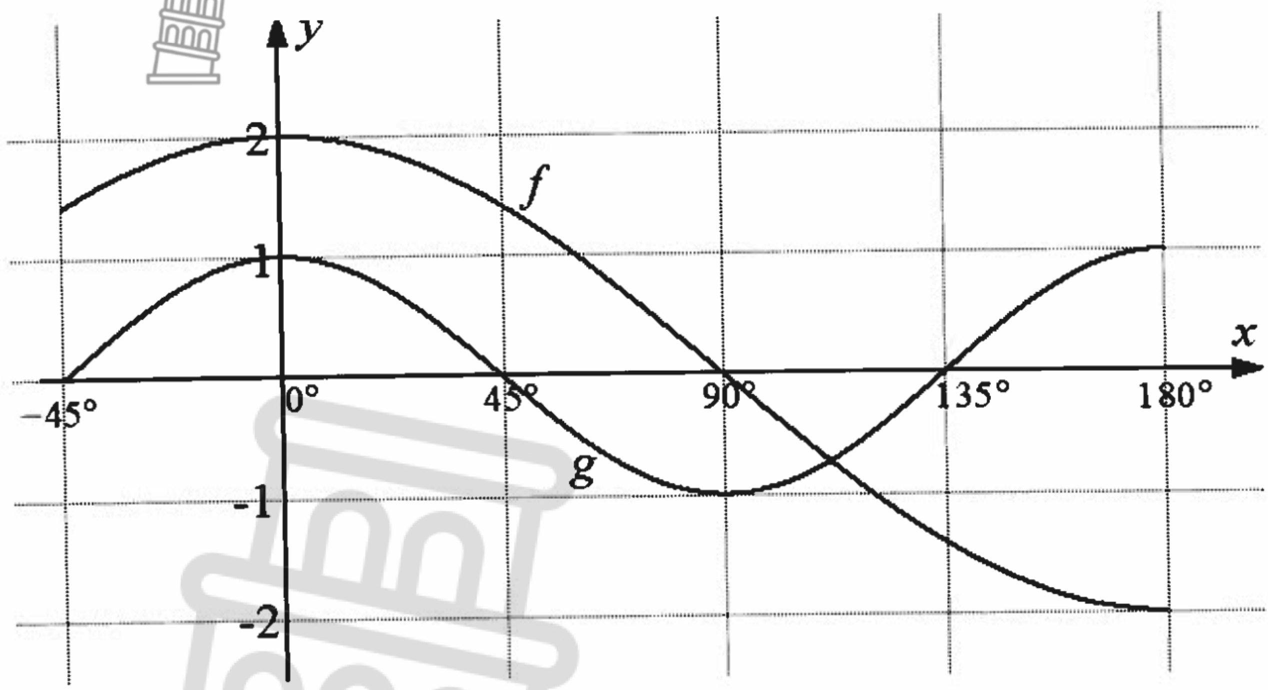
5.4.2 For which values is the identity above, undefined. Determine the general solution of x for which the identity is undefined. (3)

5.5 Calculate the general solution of x if $2\sin(3x + 20°) = 2\cos x$ (6)

[29]

QUESTION 6

In the diagram below, the graphs of $f(x) = 2\cos x$ and $g(x) = \cos 2x$ are drawn for the interval $x \in [-45°; 180°]$.



6.1 Write down the period of g. (1)

6.2 Write down the values of x for which the graph of f is increasing in the given interval. (2)

6.3 Write down the range of $y = 3g(x) - 1$. (2)

6.4 Determine the values of x for which $f(x) \geq \frac{1}{2}$ in the given interval. (4)

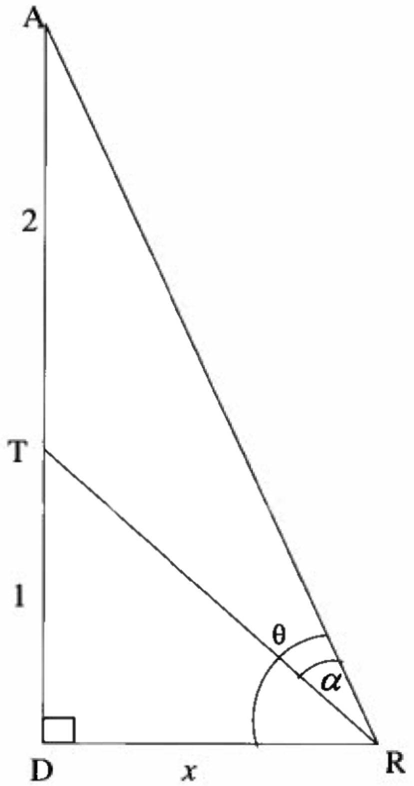
6.5 Determine the minimum value of $\frac{1}{2}\cos^2 x - \frac{1}{4}$ in the interval $x \in [0°; 180°]$ (4)

[13]

QUESTION 7

In the diagram AD is a 3m vertical pole with support cables attached at T and A to a point R which is x meters from the foot of the pole, D. D and R are in the same horizontal plane. T is 1m above the foot of the pole. The angle of elevation of A from R is θ.

$T\hat{R}A = \alpha$



7.1 Show that $TR = \dfrac{x}{\cos(\theta - \alpha)}$ (2)

7.2 Prove that $x = \dfrac{2\cos\theta\cos(\theta - \alpha)}{\sin\alpha}$ (4)

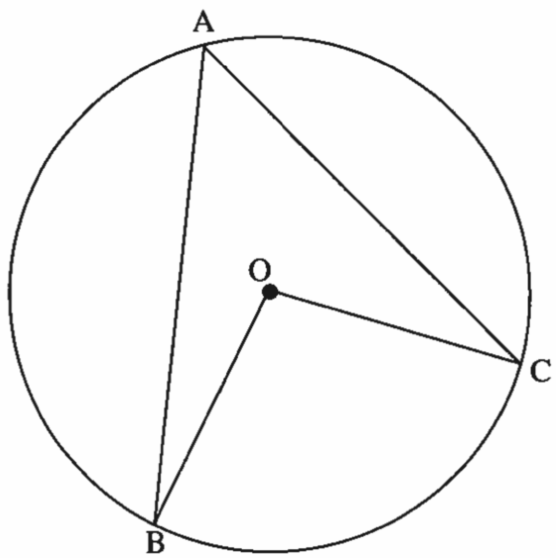
7.3 If it is further given that $\theta = 68{,}33°$ and $\alpha = 28°$ calculate the Area of ΔATR **by using the formulas in 7.1 and 7.2.** (4)

[10]

Give reasons for your statements and calculations in QUESTIONS 8, 9 and 10.

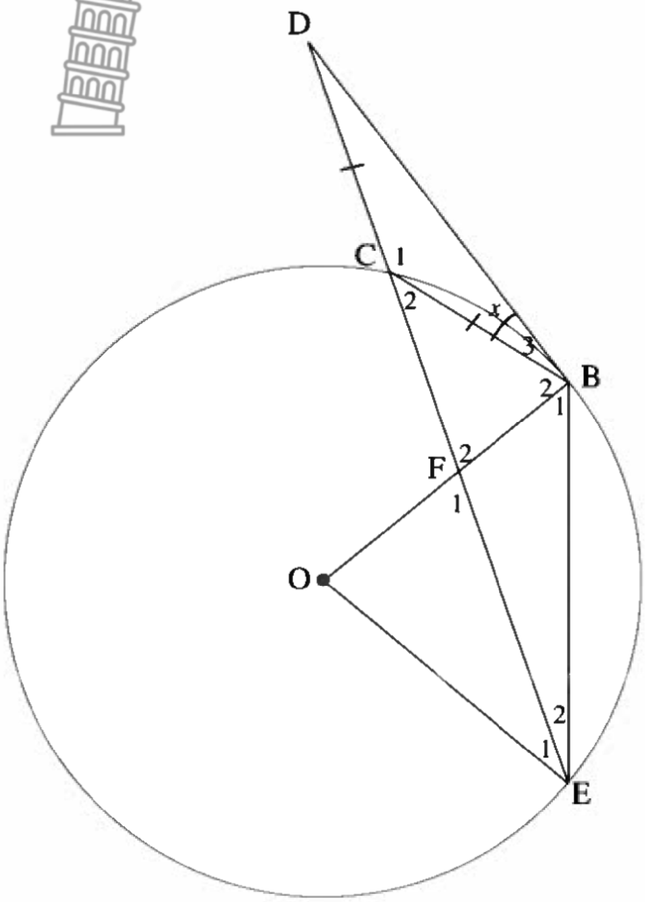
QUESTION 8

8.1 In the diagram below, O is the centre of the circle ABC. AB and AC are chords. OB and OC are joined.



Prove the theorem that states that $B\hat{O}C = 2 \times B\hat{A}C$. (5)

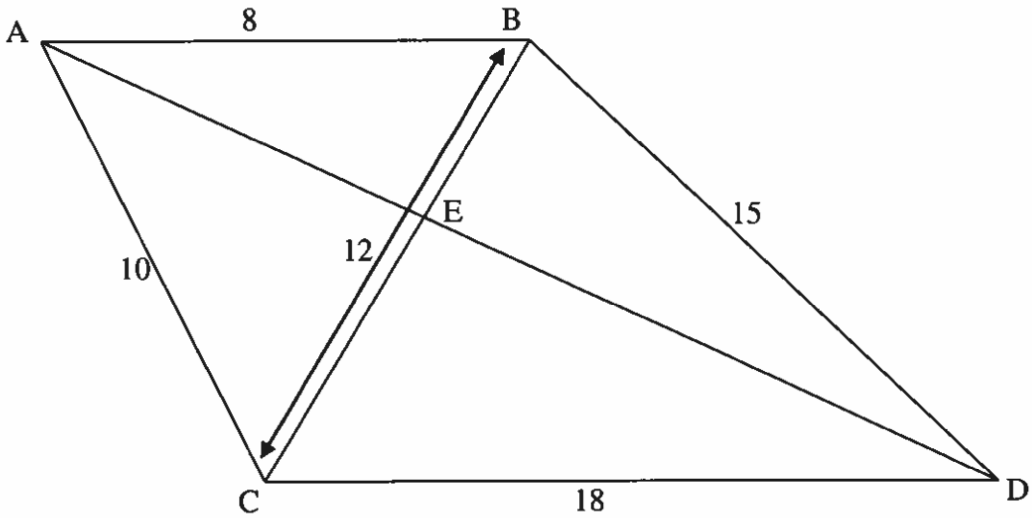
8.2 O is the centre of circle CBE. DB is a tangent to the circle at B. EC produced meets BD in D and intersects OB at F. CD = CB. OE and BE are joined. $\hat{B}_3 = x$.



8.2.1	Name, with reasons, TWO other angles each equal to x.	(4)
8.2.2	Express $E\hat{O}B$ in terms of x.	(3)
8.2.3	Determine, with reasons, the size of $\hat{B}_2$ in terms of x.	(2)
8.2.4	Show that DC = CF.	(3)
		[17]

QUESTION 9

In quadrilateral ABDC, diagonals AD and BC intersect at E.
AB = 8 cm
BD = 15 cm
DC = 18 cm
AC = 10 cm
BC = 12 cm



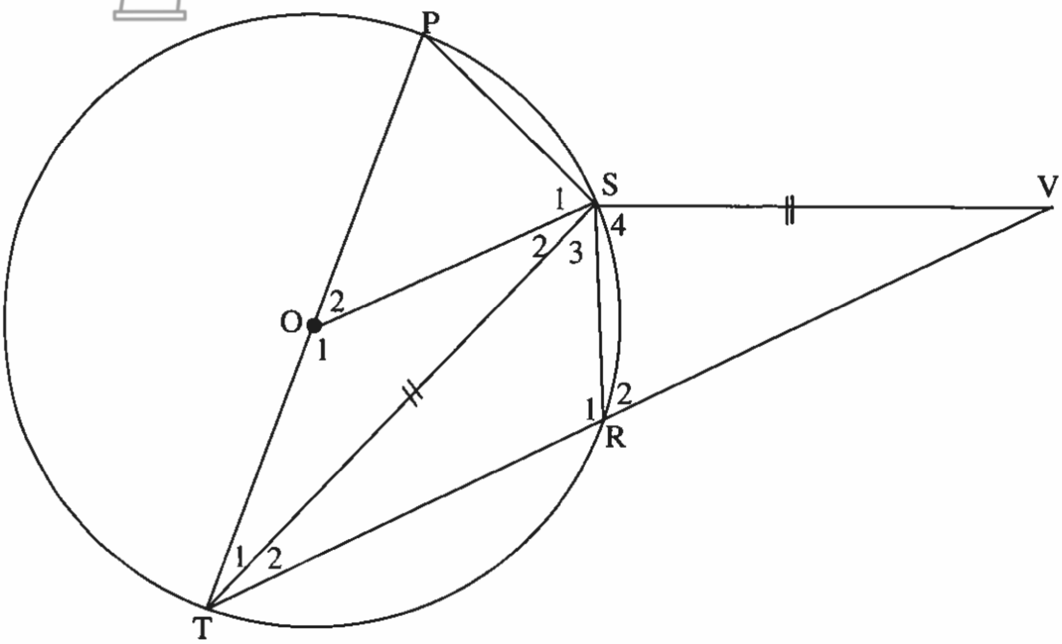
9.1 Prove that $\Delta BCA \,|||\, \Delta CDB$. (4)

9.2 Hence prove that AB $\parallel$ CD. (2)

9.3 Calculate the length of CE. (4)
[10]

QUESTION 10

In the diagram O is the centre of circle PSRT. TR produced intersects SV in V. ST bisects $P\hat{T}R$ and $TS = SV$. TOP, OS and SR are joined.



10.1 Determine with reasons the size of $P\hat{S}T$. (2)

10.2 Determine the size of $\hat{S}_4$. (5)

10.3 Prove that $\Delta TSO\ |||\ \Delta TVS$. (3)

10.4 Show that $2VS^2 = PT.TV$ (4)

[14]

TOTAL: 150

INFORMATION SHEET

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$A = P(1+ni)$ $\quad$ $A = P(1-ni)$ $\quad$ $A = P(1-i)^n$ $\quad$ $A = P(1+i)^n$

$T_n = a + (n-1)d$ $\quad$ $S_n = \frac{n}{2}\left[2a + (n-1)d\right]$

$T_n = ar^{n-1}$ $\quad$ $S_n = \frac{a\left(r^n - 1\right)}{r-1}$; $r \neq 1$ $\quad$ $S_\infty = \frac{a}{1-r}$; $-1 < r < 1$

$F = \frac{x\left[(1+i)^n - 1\right]}{i}$ $\quad$ $P = \frac{x\left[1 - (1+i)^{-n}\right]}{i}$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $\quad$ $M\left(\frac{x_1 + x_2}{2} ; \frac{y_1 + y_2}{2}\right)$

$y = mx + c$ $\quad$ $y - y_1 = m(x - x_1)$ $\quad$ $m = \frac{y_2 - y_1}{x_2 - x_1}$ $\quad$ $m = \tan\theta$

$$(x-a)^2 + (y-b)^2 = r^2$$

In ΔABC: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

$a^2 = b^2 + c^2 - 2bc.\cos A$

area $\Delta ABC = \frac{1}{2} ab.\sin C$

$\sin(\alpha + \beta) = \sin\alpha.\cos\beta + \cos\alpha.\sin\beta$ $\quad$ $\sin(\alpha - \beta) = \sin\alpha.\cos\beta - \cos\alpha.\sin\beta$

$\cos(\alpha + \beta) = \cos\alpha.\cos\beta - \sin\alpha.\sin\beta$ $\quad$ $\cos(\alpha - \beta) = \cos\alpha.\cos\beta + \sin\alpha.\sin\beta$

$$\cos 2\alpha = \begin{cases} \cos^2\alpha - \sin^2\alpha \\ 1 - 2\sin^2\alpha \\ 2\cos^2\alpha - 1 \end{cases}$$

$\sin 2\alpha = 2\sin\alpha.\cos\alpha$

$\bar{x} = \frac{\sum x}{n}$ $\quad$ $\sigma^2 = \frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n}$

$P(A) = \frac{n(A)}{n(S)}$ $\quad$ $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$

$\hat{y} = a + bx$ $\quad$ $b = \frac{\sum(x - \bar{x})(y - \bar{y})}{\sum(x - \bar{x})^2}$