



education

MPUMALANGA PROVINCE
REPUBLIC OF SOUTH AFRICA

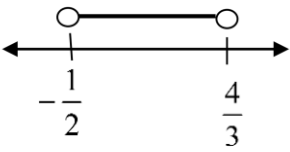
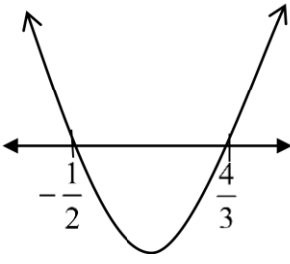
**NATIONAL
SENIOR CERTIFICATE**

GRADE 12

**MATHEMATICS PAPER 1
SEPTEMBER 2023
MARKING GUIDELINES**

MARKS: 150

This marking guideline consists of 13 pages.

| QUESTION 1 / VRAAG 1 | | |
|----------------------|--|--|
| 1.1.1 | $(3-x)(2-x) = 0$ $x = 3$ or $x = 2$ | ✓ ✓ answers (2) |
| 1.1.2 | $2x^2 + 7x - 2 = 0$ $2x^2 + 7x - 2 = 0$ $x = \frac{-(7) \pm \sqrt{7^2 - 4(2)(-2)}}{2(2)}$ $x = \frac{-7 \pm \sqrt{65}}{4}$ $x = 0,27$ or $x = -3,77$ <div style="border: 1px solid black; padding: 2px; display: inline-block;">Penalise 1 mark for incorrect rounding</div> | ✓ standard form ✓ substitution into correct formula ✓ ✓ answers (4) |
| 1.1.3 | $4 + 5x > 6x^2$ $6x^2 - 5x - 4 < 0$ $(3x - 4)(2x + 1) < 0$  $-\frac{1}{2} < x < \frac{4}{3}$ OR $x \in \left(-\frac{1}{2}; \frac{4}{3}\right)$  | ✓ factors ✓ critical values ✓ ✓ answer (4) |
| 1.1.4 | $9^x + 9 = 10 \cdot 3^x$ $3^{2x} - 10 \cdot 3^x + 9 = 0$ $(3^x - 9)(3^x - 1) = 0$ $3^x = 3^2$ or $3^x = 3^0$ $x = 2$ or $x = 0$ | ✓ 3^{2x} ✓ factors ✓ exponential law ✓ answer (4) |

| | | |
|------------|---|---|
| <p>1.2</p> | $y - 2x = -1$ $y = 2x - 1 \dots (1)$ $y^2 - 3xy = -2 \dots (2)$ $(2x - 1)^2 - 3x(2x - 1) = -2$ $4x^2 - 4x + 1 - 6x^2 + 3x = -2$ $-2x^2 - x + 3 = 0$ $2x^2 + x - 3 = 0$ $(2x + 3)(x - 1) = 0$ $x = -\frac{3}{2} \text{ or } x = 1$ $y = 2\left(-\frac{3}{2}\right) - 1 \text{ or } y = 2(1) - 1$ $y = -4 \text{ or } y = 1$ <p>OR</p> $y - 2x = -1$ $x = \frac{y + 1}{2} \dots (1)$ $y^2 - 3xy = -2 \dots (2)$ $y^2 - 3\left(\frac{y + 1}{2}\right)y = -2$ $2y^2 - 3y^2 - 3y + 4 = 0$ $-y^2 - 3y + 4 = 0$ $y^2 + 3y - 4 = 0$ $(y + 4)(y - 1) = 0$ $y = -4 \text{ or } y = 1$ $x = -\frac{3}{2} \text{ or } x = 1$ | <p>✓ equation 1</p> <p>✓ substitution</p> <p>✓ standard form</p> <p>✓ factors</p> <p>✓ x-values</p> <p>✓ y-values (6)</p> <p>✓ equation 1</p> <p>✓ substitution</p> <p>✓ standard form</p> <p>✓ factors</p> <p>✓ y-values</p> <p>✓ x-values (6)</p> |
| <p>1.3</p> | <p>Multiply every term by LCD $4xy$</p> $4x^2 + 4y^2 = 17xy$ $4x^2 - 17xy + 4y^2 = 0$ $(4x - y)(x - 4y) = 0$ $4x - y = 0$ $\frac{x}{y} = \frac{1}{4}$ $x - 4y = 0$ $\frac{x}{y} = 4$ | <p>✓ multiply by LCD</p> <p>✓ standard form</p> <p>✓ factors</p> <p>✓ answer</p> <p>✓ answer (5)</p> |

| | | |
|--|---|--|
| | <p>OR</p> $\frac{x}{y} + \frac{y}{x} = \frac{17}{4}$ $\frac{x}{y} + \frac{1}{\frac{x}{y}} = \frac{17}{4}$ $\frac{x^2}{y^2} - \frac{17x}{4y} + 1 = 0$ $\frac{x}{y} = \frac{\frac{17}{4} \pm \sqrt{\left(\frac{17}{4}\right)^2 - 4(1)(1)}}{2(1)}$ $= \frac{\frac{17}{4} \pm \sqrt{\frac{225}{16}}}{2}$ $\frac{x}{y} = 4 \text{ or } \frac{x}{y} = \frac{1}{4}$ | <p>✓ fraction</p> <p>✓ standard form</p> <p>✓ substitution into correct formula</p> <p>✓ simplification</p> <p>✓ answers (5)</p> |
| | | [25] |


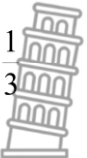
QUESTION 2 / VRAAG 2

| | | |
|-------|--|--|
| 2.1 | 319 ; 280 ; 243 ; 208 ; 175 ; 144 ; | |
| 2.1.1 | $ \begin{array}{cccc} 319 & 280 & 243 & 208 \\ & \swarrow & \searrow & \swarrow & \searrow \\ & -39 & -37 & -35 \\ & & \swarrow & \searrow & \swarrow & \searrow \\ & & 2 & 2 \end{array} $ $ \begin{array}{lll} 2a = 2 & 3a + b = -39 & a + b + c = 319 \\ a = 1 & 3(1) + b = -39 & 1 - 42 + c = 319 \\ & b = -42 & c = 360 \end{array} $ $T_n = n^2 - 42n + 360$ | <p>✓ First differences</p> <p>✓ $2a = 2$</p> <p>✓ $3(1) + b = -39$</p> |

| | | |
|-------|--|--|
| | | $\checkmark 1 - 42 + c = 319$ (4) |
| 2.1.2 | $n^2 - 42n + 360 = 0$ $(n-12)(n-30) = 0$ $n = 12 \text{ or } n = 30$ | $\checkmark T_n = 0$ \checkmark factors \checkmark answer (3) |
| 2.1.3 | $2n - 42 = 0$ OR $n = \frac{-(-42)}{2(1)}$ $n = 21$ $n = 21$ T_{21} is lowest | \checkmark derivative = 0 \checkmark answer (2) OR \checkmark substitution into $x = \frac{-b}{2a}$ \checkmark answer (2) |
| 2.2 | $3t ; 4t - 1 ; 23$ | |
| 2.2.1 | $3t ; 4t - 1 ; 23$ $4t - 1 - 3t = 23 - (4t - 1)$ $4t - 1 - 3t = 23 - 4t + 1$ $5t = 25$ $t = 5$ | \checkmark subtraction \checkmark simplification (2) |
| 2.2.2 | $15 ; 19 ; 23$ $a = 15 \quad d = 4$ $S_{50} = \frac{50}{2} [2(15) + (49)4]$ $= 5650$ | $\checkmark a = 15$ \checkmark value of d \checkmark substitution into correct formula \checkmark answer (4) |
| | | [15] |






QUESTION 3

| | | |
|-------|---|---|
| 3.1 | $x = 4 + 12 + 36 + \dots$ to 15 terms | |
| 3.1.1 | $T_n = a \cdot r^{n-1}$ $T_n = 4 \cdot 3^{n-1}$ $\sum_{n=1}^{15} (4 \cdot 3^{n-1})$  | ✓ $T_n = 4 \cdot 3^{n-1}$ ✓ answer (2) |
| 3.1.2 | $S_n = \frac{a(r^n - 1)}{r - 1}$ $S_n = \frac{4(3^{15} - 1)}{3 - 1}$ $= 28\,697\,812$ | ✓ substitution into the correct formula ✓ answer (2) |
| 3.1.3 | $r = 3$ and not between -1 and 1 \therefore not converging | ✓ $r = 3$ not in $-1 < r < 1$ ✓ answer (2) |
| 3.2.1 | $S_\infty = \frac{a}{1 - r} = 13,5$ $a = 13,5(1 - r)$ $S_\infty = \frac{ar^2}{1 - r} = 1,5$ $ar^2 = 1,5(1 - r)$ $a = \frac{1,5(1 - r)}{r^2}$ $13,5(1 - r) = \frac{1,5(1 - r)}{r^2}$ $r^2 = \frac{1,5}{13,5}$ $= \frac{1}{9}$ $r = \pm \frac{1}{3}$ But $r > 0$ $\therefore r = \frac{1}{3}$ | ✓ substitution into correct formula ✓ $a = 13,5(1 - r)$ ✓ equating ✓ $r = \frac{1}{3}$  (4) |

| | | |
|-------------------|---|---|
| 3.2.2 | $\frac{a}{1 - \frac{1}{3}} = 13,5$ $a = 9$ $9 + 3 + 1 + \dots$ | ✓ value of a ✓ answer (2) |
| | | [12] |
| QUESTION 4 | | |
| 4.1 | $0 = \frac{a}{-2+1} + 2$ $a = 2$ | ✓ substitution ✓ answer (2) |
| 4.2 | Average gradient = $\frac{h(-2) - h(-4)}{-2 - (-4)}$ $= \frac{\frac{2}{-2+1} + 2 - \left(\frac{2}{-4+1} + 2\right)}{2}$ $= -\frac{2}{3}$ | ✓ $h(-4)$ and $h(-2)$ ✓ substitution ✓ answer (3) |
| 4.3 | $-1 < x < 0$ | ✓ critical values ✓ answers (2) |
| 4.4 | $y = x + 1 + 2$ $y = x + 3$ OR $y = x + c$ $2 = -1 + c$ $c = 3$ $y = x + 3$ | ✓ gradient ✓ answer (2) OR ✓ value of c ✓ answer (2) |
| 4.5 | $g(x) = \frac{-2}{x+1} - 2$ $x = -1$ $y = -2$ | ✓ horizontal asymptote ✓ vertical asymptotes (2) |
| | | [11] |

QUESTION 5

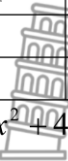

| | | |
|-------|---|--|
| | $h(x) = q^x \quad f(x) = -\frac{1}{2}x^2 + \frac{1}{2}x + k \quad g(x) = mx + c$ | |
| 5.1 | $h(x) = q^0$ $= 1$  $D(0; 1)$ | ✓ answer (1) |
| 5.2 | $k = 1$ | ✓ answer (1) |
| 5.3 | $f(x) = -\frac{1}{2}x^2 + \frac{1}{2}x + 1$ $x^2 - x - 2 = 0$ $(x - 2)(x + 1) = 0$ $x = 2 \text{ or } x = -1$ BC = 3 units | ✓ $f(x) = 0$ ✓ factors ✓ both x values ✓ answer (4) |
| 5.4 | $f(-2) = -\frac{1}{2}(-2)^2 + \frac{1}{2}(-2) + 1$ $= -2$ PR = 2 units | ✓ substitution of -2 ✓ answer (2) |
| 5.5.1 | $h(x) = q^x$ $\frac{1}{4} = q^2$ $q = \frac{1}{2}$ | ✓ $A\left(2; \frac{1}{4}\right)$ ✓ substitution of $\left(2; \frac{1}{4}\right)$ ✓ value of q (3) |
| 5.5.2 | $h(x) = \left(\frac{1}{2}\right)^x$ $h^{-1}(x) = \log_{\frac{1}{2}} x$ | ✓ equation of h ✓ equation of h^{-1} (2) |
| 5.6 | $y > 0$ | ✓ answer (1) |
| 5.7 | $f'(x) = -x + \frac{1}{2}$ $f'(-1) = -(-1) + \frac{1}{2} = \frac{3}{2}$ $y - y_1 = m(x - x_1)$ B(-1; 0) $y - 0 = \frac{3}{2}(x - (-1))$ $y = \frac{3}{2}x + \frac{3}{2}$ | ✓ derivative ✓ derivative $x = -1$ ✓ $f'(-1) = \frac{3}{2}$ ✓ substitution of B ✓ tangent  (5) |
| 5.8 | $x < -1 \text{ or } x > 2$ OR $x \in (-\infty; 1) \cup (2; \infty)$ | ✓✓ answer  (2) |
| | | [21] |

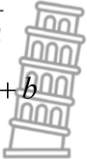
QUESTION 6

| | | |
|-----|--|--|
| 6.1 | $A = P(1+i)^n$ $2x = x(1+0,054)^n$ $2 = (1,054)^n$ $\log(2) = n \log(1,054)$ $n = \frac{\log 2}{\log(1,054)}$ $n = 13,18$ | <p>✓ substitution into correct formula</p> <p>✓ correct use of logs</p> <p>✓ answer (3)</p> |
| 6.2 | $P = \frac{x [1 - (1+i)^{-n}]}{i}$ $P = \frac{3300 \left[1 - \left(1 + \frac{0,12}{12} \right)^{-60} \right]}{\frac{0,12}{12}}$ $= R148351,63$ | <p>✓ value of i</p> <p>✓ value of n</p> <p>✓ substitution into correct formula</p> <p>✓ answer (4)</p> |
| 6.3 | $i_{eff} = \left(1 + \frac{0,05}{4} \right)^4 - 1$ $= 0,05094533691$ <p>Rate = 5,09%</p> | <p>✓ value of n</p> <p>✓ substitution into correct formula</p> <p>✓ rate (3)</p> |
| 6.4 | $10\,000\,000 = \frac{x \left[\left(1 + \frac{0,15}{12} \right)^{384} - 1 \right]}{\frac{0,15}{12}}$ $125000 = x(116,9484518)$ $x = R1068,85$ | <p>✓ value of n</p> <p>✓ substitution into correct formula</p> <p>✓ answer (3)</p> |
| | | [13] |

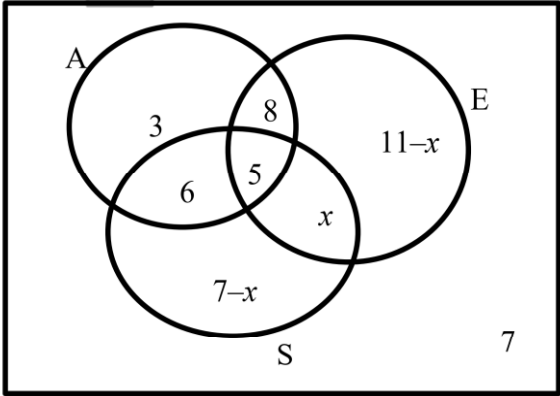


| QUESTION 7 | | Penalty for notation error in 7.1 only. |
|------------|---|--|
| 7.1 | $f(x) = -2x^2 + 1$ $f(x+h) = -2(x+h)^2 + 1$ $f(x+h) = -2x^2 - 4xh - 2h^2 + 1$ $f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h)^2 - f(x)}{h}$ $f'(x) = \lim_{h \rightarrow 0} \frac{-2x^2 - 4xh - 2h^2 + 1 - (-2x^2 + 1)}{h}$ $f'(x) = \lim_{h \rightarrow 0} \frac{-2x^2 - 4xh - 2h^2 + 1 + 2x^2 - 1}{h}$ $f'(x) = \lim_{h \rightarrow 0} \frac{-4xh - 2h^2}{h}$ $f'(x) = \lim_{h \rightarrow 0} \frac{h(-4x - 2h)}{h}$ $= -4x$ | <p>✓ $-2x^2 - 4xh - 2h^2 + 1$</p> <p>✓ correct substitution into formula</p> <p>✓ simplification</p> <p>✓ common factor</p> <p>✓ answer (5)</p> |
| 7.2.1 | $f(x) = \frac{1}{2}x^2 - 5x^{-1}$ $f'(x) = x + 5x^{-2}$ | <p>✓ $-5x^{-1}$</p> <p>✓ x</p> <p>✓ $5x^{-2}$ (3)</p> |
| 7.2.2 | $D_x \left[\frac{-2x^2 + \sqrt[4]{x}}{x^2} \right]$ $= D_x \left[-\frac{2x^2}{x^2} + \frac{x^{\frac{1}{4}}}{x^2} \right]$ $= D_x \left[-2 + x^{-\frac{7}{4}} \right]$ $= -\frac{7}{4}x^{-\frac{11}{4}}$ | <p>✓ $\sqrt[4]{x} = x^{\frac{1}{4}}$</p> <p>✓ $-2 + x^{-\frac{7}{4}}$</p> <p>✓ derivative (3)</p> |
| | | [11] |

| QUESTION 8 | | | | | | | | | | | | | | | | | |
|------------|--|--|----|-----|---|-----|--|---|----|---|----|--|----|---|----|---|---|
| 8.1 | <div></div> <table border="1"><tr><td>1</td><td>-1</td><td>5</td><td>8</td><td>-12</td></tr><tr><td></td><td>0</td><td>-1</td><td>4</td><td>12</td></tr><tr><td></td><td>-1</td><td>4</td><td>12</td><td>0</td></tr></table> $(x-1)(-x^2+4x+12)=0$ $(x-1)(x^2-4x-12)=0$ $(x-1)(x-6)(x+2)=0$ $x=1 \text{ or } x=6 \text{ or } x=-2$ $\therefore C(6; 0)$ | 1 | -1 | 5 | 8 | -12 | | 0 | -1 | 4 | 12 | | -1 | 4 | 12 | 0 | <div>✓ quadratic factor</div> <div>✓ factors</div> <div>✓ x-values</div> <div>✓ answer</div> <div>(4)</div> |
| 1 | -1 | 5 | 8 | -12 | | | | | | | | | | | | | |
| | 0 | -1 | 4 | 12 | | | | | | | | | | | | | |
| | -1 | 4 | 12 | 0 | | | | | | | | | | | | | |
| 8.2 | $-3x^2+10x+8=0$ $3x^2-10x-8=0$ $(3x+2)(x-4)=0$ $x=-\frac{2}{3} \text{ or } x=4$ $f(4)=-(4)^3+5(4)^2+8(4)-12$ $=36$ | <div>✓ derivative</div> <div>✓ $f'(x)=0$</div> <div>✓ factors</div> <div>✓ x-values</div> <div>✓ answer</div> <div>(5)</div> | | | | | | | | | | | | | | | |
| 8.3 | $-6x+10>0$ $x<\frac{5}{3}$ | <div>✓ inequality</div> <div>✓ answer</div> <div>(2)</div> | | | | | | | | | | | | | | | |
| 8.4 | $f'(x)=-3x^2+10x+8$ $f'(0)=8$ $-3x^2+10x+8=8$ $-3x^2+10x=0$ $x(-3x+10)=0$ $x=0 \text{ or } x=\frac{10}{3}$ $\therefore \text{OG}=\frac{10}{3}$ | <div>✓ $f'(0)=8$</div> <div>✓ equating</div> <div>✓ factors</div> <div>✓ x-values</div> <div>✓ value of OG</div> <div>(5)</div> | | | | | | | | | | | | | | | |
| 8.5 | $f'(x)=-3x^2+10x+8$ $f'(5)=-3(5)^2+10(5)+8$ $=-17$ $m=\frac{1}{17}$ | <div>✓ substitution</div> <div>✓ $f'(5)=-17$</div> <div>✓ answer</div> <div>(3)</div> | | | | | | | | | | | | | | | |
| | | <div></div> <div>[19]</div> | | | | | | | | | | | | | | | |

| QUESTION 9 | | |
|------------|--|---|
| 9.1 | $m_{mn} = -\frac{b}{a}$ $y = -\frac{b}{a}x + b$  | <p>✓ m_{mn}</p> <p>✓ answer (2)</p> |
| 9.2 | <p>Area of PTOR = TP x PR</p> $= x \left(-\frac{b}{a}x + b \right)$ $= -\frac{b}{a}x^2 + bx$ <p>Max: $-\frac{2b}{a}x + b = 0$</p> $x = b \left(\frac{a}{2b} \right)$ $x = \frac{a}{2}$ $y = -\frac{b}{a} \left(\frac{a}{2} \right) + b = \frac{b}{2}$ <p>Midpoint of MN: $\left(\frac{0+a}{2}; \frac{0+b}{2} \right) = \left(\frac{a}{2}; \frac{b}{2} \right)$</p> <p>∴ Max at midpoint</p> | <p>✓ substitution into area formula</p> <p>✓ simplification</p> <p>✓ derivative = 0</p> <p>✓ value of x</p> <p>✓ value of y</p> <p>✓ midpoint (6)</p> |
| | | [8] |



| QUESTION 10 | | |
|-------------|--|---|
| 10.1.1 | $45 - 38 = 7$ | ✓ answer (1) |
| 10.1.2 | $S = 45$  | ✓ 8 ✓ 6 ✓ $11 - x$ ✓ $7 - x$ (4) |
| 10.1.3 | $3 + 8 + 6 + 5 + 11 - x + x + 7 - x + 7 = 45$ $x = 2$ | ✓ calculation ✓ answer (2) |
| 10.1.4 | $P(2 \text{ languages}) = \frac{6 + 8 + 2}{45}$ $= \frac{16}{45}$ | ✓ 16 ✓ $\frac{16}{45}$ (2) |
| 10.2.1 | $8! = 40320$ | ✓ 8! (1) |
| 10.2.2 | $3! \times 5! \times 6 = 4320$ OR $3! \times 6! = 4320$ | ✓ 3! ✓ 5! ✓ $3! \times 5! \times 6$ (3) |
| 10.2.3 | $\frac{5!}{8!} = \frac{1}{336}$ | ✓ 5! ✓ $\frac{5!}{8!}$ OR $\frac{1}{336}$ (2) |
| | | [15] |



TOTAL: 150