LIMPOPO
PROVINCIAL GOVERNMENT
REPUBLIC OF SOUTH AFRICA

DEPARTMENT OF
EDUCATION

NATIONAL SENIOR CERTIFICATE

GRADE 12

MATHEMATICS PAPER 2

SEPTEMBER 2023

MARKS: 150

TIME: 3 HOURS

EMATHP2

This question paper consists of 14 pages and an information sheet.

INSTRUCTIONS AND INFORMATION

Read the following instructions carefully before answering the questions.

1. This question paper consists of 10 questions.

2. Answer ALL the questions in the SPECIAL ANSWER BOOK.

3. Clearly show ALL calculations, diagrams, graphs, et cetera that you have used in determining your answers.

4. ANSWERS ONLY will not necessarily be awarded full marks.

5. You may use an approved scientific calculator (non-programmable and non-graphical), unless stated otherwise.

6. If necessary, round answers off to TWO decimal places, unless stated otherwise.

7. Diagrams are NOT necessarily drawn to scale.

8. Write legibly and present your work neatly.

QUESTION 1

A mathematics teacher wants to make an unbiased prediction of her Grade 12 learners' final marks. She uses their SBA mark and notes the final mark. The results are as follows:

SBA MARK (%)	FINAL MARK (%)	SBA MARK (%)	FINAL MARK (%)
42	51	48	59
35	43	72	85
69	76	57	63
62	73	25	35
83	85	65	59
75	72	68	75

1.1 Draw the scatter plot for the data on the grid provided in the ANSWER BOOK. (4)

1.2 Calculate the correlation coefficient for the data. (2)

1.3 Is the SBA mark a reliable predictor of the final mark? Provide a reason for your answer. (2)

1.4 Determine the equation of the least squares regression line. (3)

1.5 Predict Toby's final mark if his SBA mark was 66%. (2)

[13]

QUESTION 2

The following set of data: $3 ; 4 ; 4 ; 4 ; 6 ; 10 ; 12 ; 12 ; y$ has a mean of 7.

2.1 Determine:

2.1.1 the value of y (2)

2.1.2 the median of this set of data points (1)

2.2 Two additional numbers, $7-n$ and $7+n$, are added to the data set.

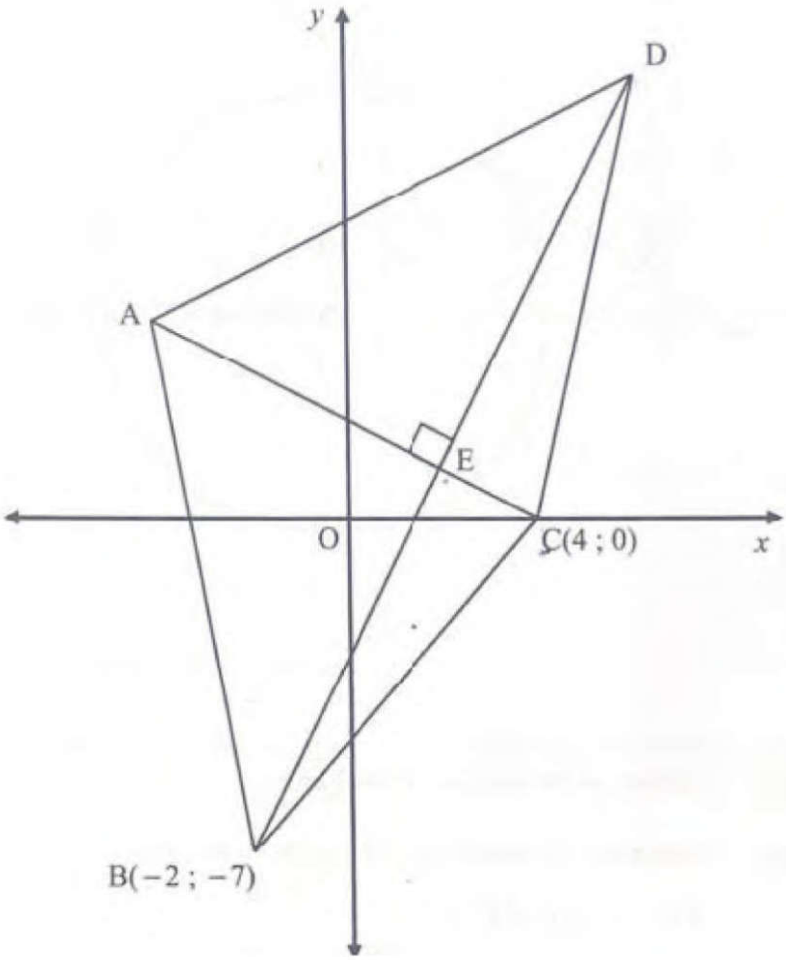
2.2.1 Calculate the mean of these eleven numbers. (2)

2.2.2 Determine the standard deviation if the data points, that are within ONE standard deviation of the mean, lie in the interval $3 \leq x \leq 11$. (2)

[7]

QUESTION 3

In the diagram below A, B $(-2\ ;\ -7)$, C $(4\ ;\ 0)$ and D are the vertices of a kite. E is the midpoint of the diagonal BD and AC $\perp$ BD at E. The equation of AC is $y=-\frac{1}{2}x+2$.



Determine:

3.1 The equation of BD. (4)

3.2 The coordinates of E. (3)

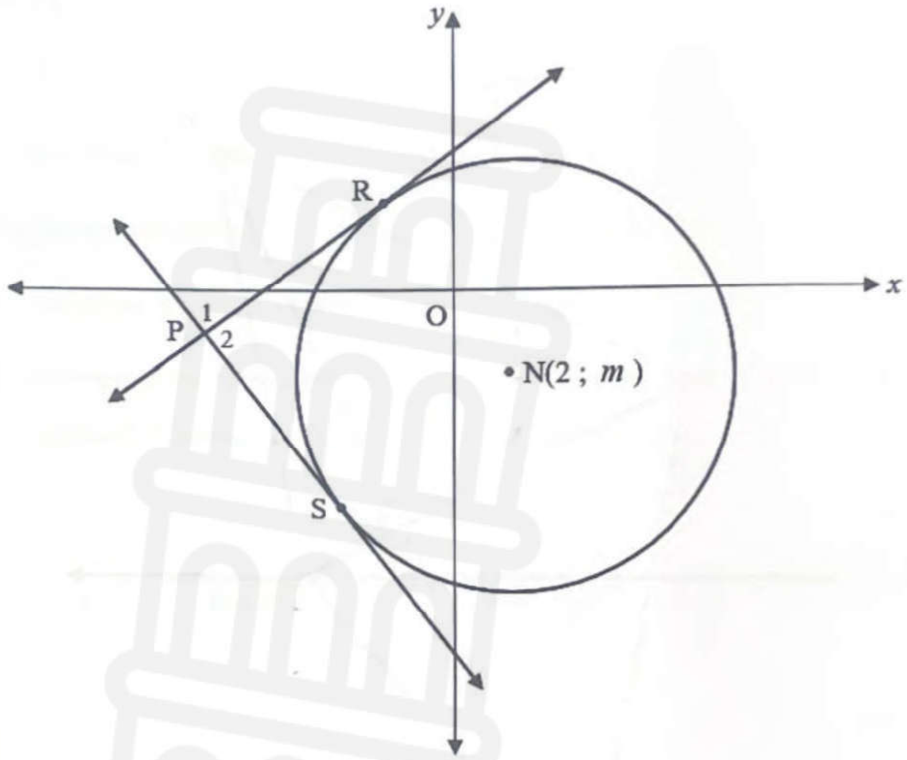
3.3 If the ratio CE : EA = 1 : 3, determine the coordinates of A. (2)

3.4 Kite PQRS is obtained after the measurements of kite ABCD is enlarged by a scale factor 2. Calculate the area of kite PQRS. (5)

[14]

QUESTION 4

4.1 In the diagram, the centre of the circle is N $(2\,;\,m)$ where $m < 0$. The radius of the circle is 17 units. R $(-13\,;\,5)$ and S $(-13\,;\,-11)$ are two points on the circle.



4.1.1 (a) Determine the numerical value of m. (4)

(b) Determine the equation of the circle in the form

$$(x-a)^2+(y-b)^2=r^2$$ (1)

4.1.2 Determine the gradients of:

(a) NR (2)

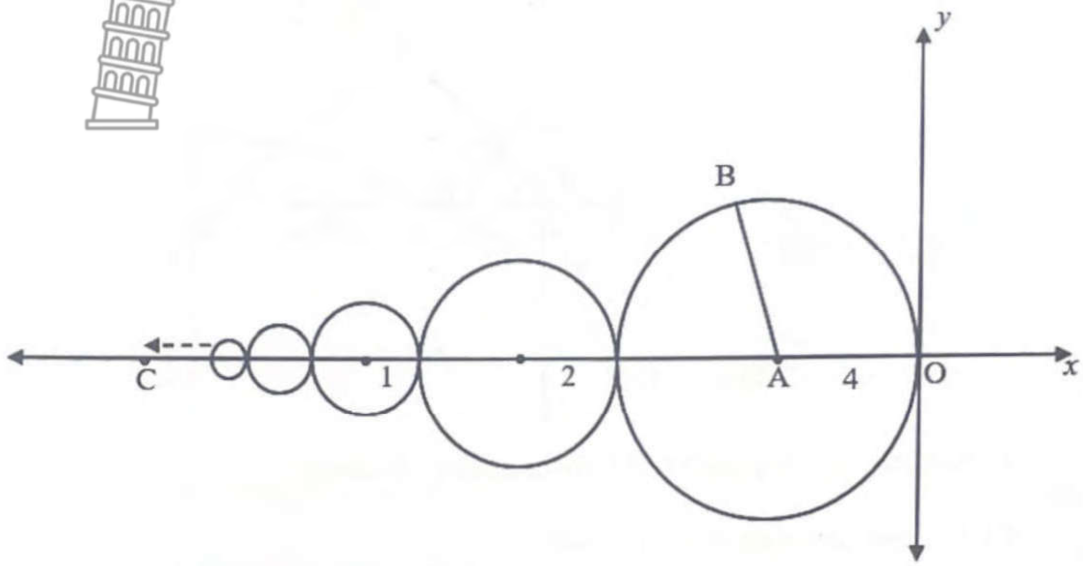
(b) NS (1)

4.1.3 The tangents at S and R intersect at P. Calculate the size of $\hat{P}_2$. (6)

4.1.4 Circle N is reflected about the $x-$axis and then translated 2 units upwards to obtain circle M. Determine the equation of circle M in the form

$$(x-c)^2+(y-d)^2=r^2$$ (2)

4.2 An infinite number of circles, each touching the next, are drawn between C and O. The centres of all the circles lie on the negative x-axis. The radius of the largest circle, centred at A, is 4 units and the radius of each circle thereafter is halved. B is a point on the largest circle.



4.2.1 Show that OC = 16 units. (2)

4.2.2 If BC is a tangent to circle A at B, write down the size of $A\hat{B}C$, providing a reason for your answer. (2)

4.2.3 Hence, determine $\tan \hat{C}$. (4)

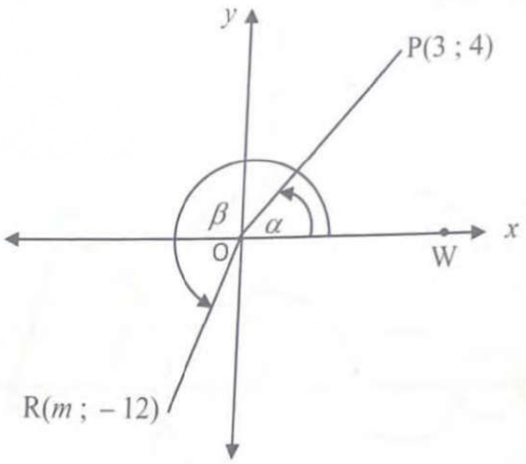
4.2.4 Determine the equation of BC. (3)

[27]

QUESTION 5

5.1 In the diagram below P (3 ; 4) and R (m ; -12) are two points as indicated.

$\hat{POW} = \alpha$ and $\hat{ROW} = \beta$.



Answer the following questions without using a calculator.

5.1.1 Write down the value of $\tan\alpha$. (1)

5.1.2 Determine the value of $\sin(90° + \alpha)$. (3)

5.1.3 Determine the value of m if it is given that $12 + 13\sin\beta = 0$. (4)

5.1.4 Determine the value of $\cos(\alpha + \beta)$. (3)

5.2 Simplify the following:

5.2.1 $\sqrt{4^{\sin 150°}.2^{3\tan 225°}}$ without using a calculator. (5)

5.2.2 $\dfrac{\tan(180° + x)\cos x}{\sin(180° + x)\cos x - \cos(540° + x)\cos(90° + x)}$ to a single trigonometric expression. (6)

5.3 Prove that: $\dfrac{1 - \cos 2x - \sin x}{\sin 2x - \cos x} = \tan x$ (4)

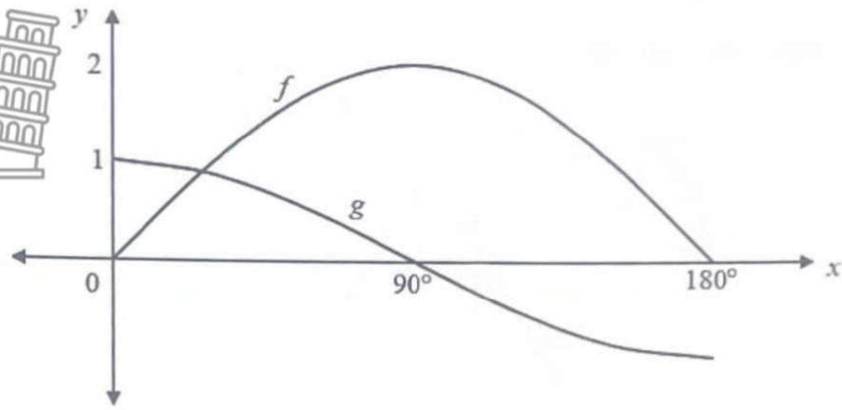
5.4 It is given that P and Q are both acute angles, solve for P and Q if:

$\sin P\sin Q - \cos P\cos Q = \dfrac{1}{2}$ and $\sin(P - Q) = \dfrac{1}{2}$ (7)

[33]

QUESTION 6

In the diagram, the graphs of $f(x) = a\sin x$ and $g(x) = \cos bx$ are drawn for $x \in [0° ; 180°]$.



6.1 Determine the values of a and b. (2)

6.2 Consider the interval $x \in [0° ; 180°]$:

6.2.1 Calculate the value(s) of x where $a\sin x - \cos bx = 0$. (2)

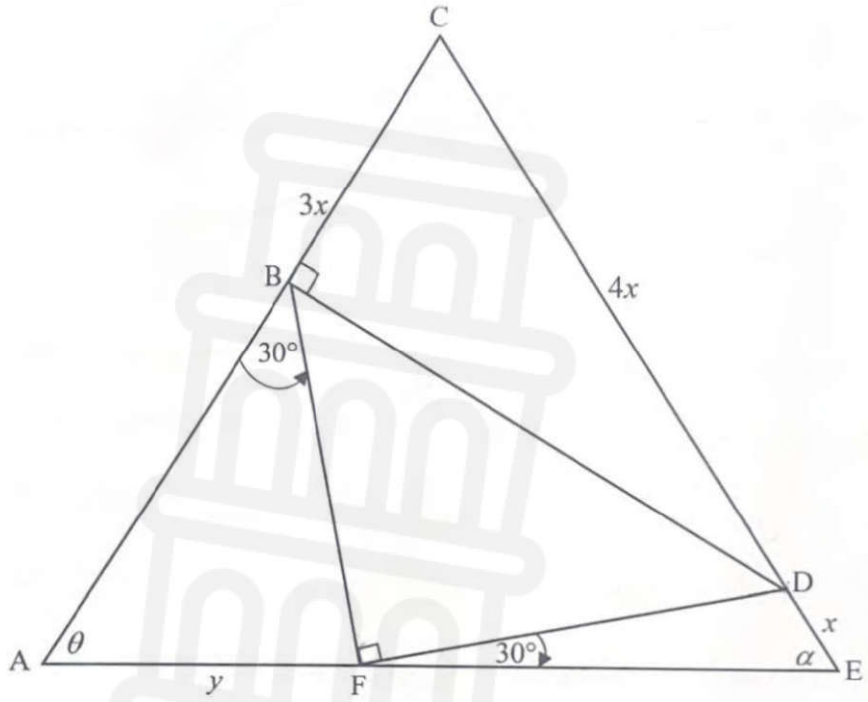
6.2.2 For which value(s) of x will $g(x).f'(x) \geq 0$. (2)

6.2.3 Determine the values(s) of y for which $y = 2^{2f(x)-1}$. (2)

[8]

QUESTION 7

The diagram shows ΔACE with $\hat{A} = \theta$ and $\hat{E} = \alpha$. Points B, D and F lie on AC, CE and AE respectively so that $BC = 3x$, $CD = 4x$, $DE = x$ and $AF = y$. $BD \perp AC$ and $B\hat{F}D = 90°$. $A\hat{B}F = 30°$ and $D\hat{F}E = 30°$.



7.1 Write BF in terms of θ and y. (3)

7.2 Write DF in terms of α and x. (2)

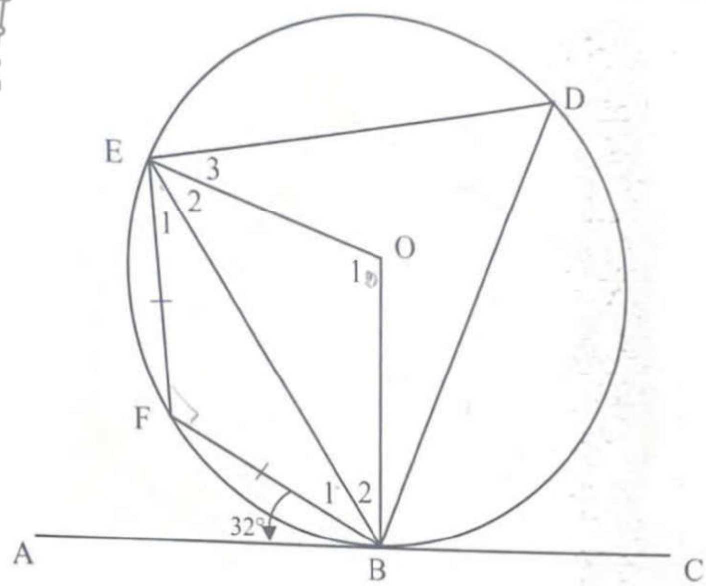
7.3 Hence, prove that $BD^2 = 4x^2.\sin^2\alpha + 4y^2.\sin^2\theta$ (1)

7.4 Hence, prove that $x = \sqrt{\dfrac{4y^2.\sin^2\theta}{7 - 4\sin^2\alpha}}$ (3)

[9]

QUESTION 8

In the diagram, ABC is the tangent to the circle centre O at B. F, E and D are points on the circle. EF = BF. $A\hat{B}F = 32°$.



Determine, with reasons, the sizes of the following:

8.1 $\hat{E}_1$ (2)

8.2 $\hat{F}$ (2)

8.3 $\hat{D}$ (2)

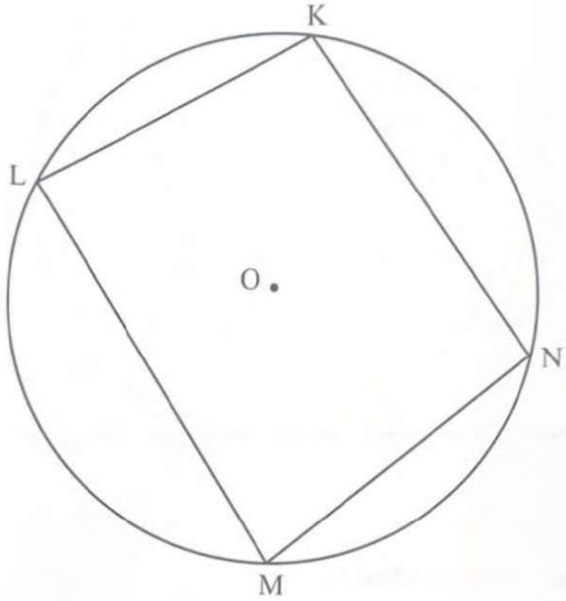
8.4 $\hat{O}_1$ (2)

8.5 $\hat{E}_2$ (2)

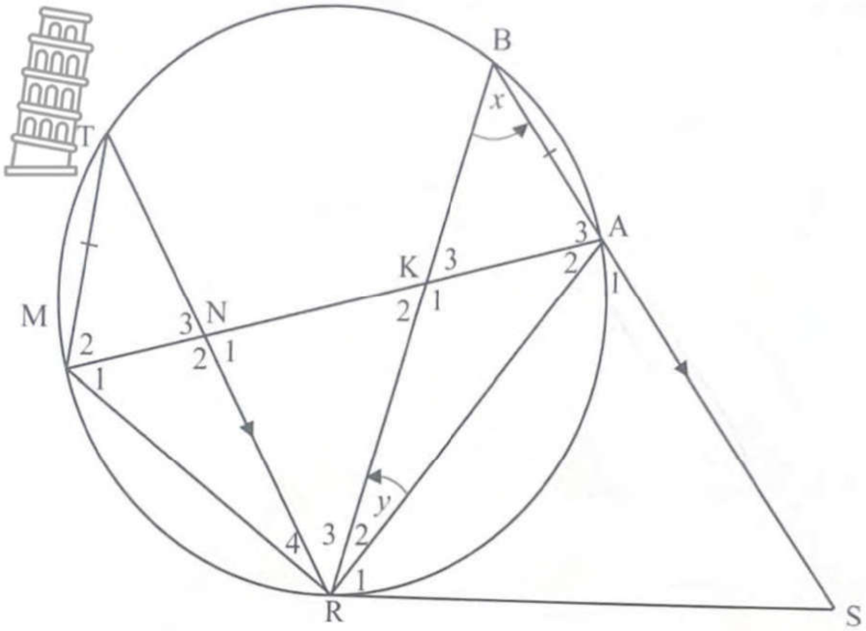
[10]

QUESTION 9

9.1 O is the centre of the circle. Points K, L, M and N are on the circle. Use the diagram to prove the theorem that states that the opposite angles of a cyclic quadrilateral are supplementary, i.e. $\hat{K}+\hat{M}=180°$ (5)



9.2 In the diagram, RS is a tangent to the circle at R. SAB is a line that passes through the circle and RT ∥ SAB. MT = AB.



9.2.1 If $A\hat{B}R = x$, write down THREE other angles in the diagram which are also equal to x. Provide reasons. (6)

9.2.2 If $A\hat{R}B = y$, provide a reason why $M\hat{R}T = y$? (1)

9.2.3 (a) Write $\hat{A}_1$ in terms of x and y. (1)

(b) Write $\hat{N}_1$ in terms of x and y. (1)

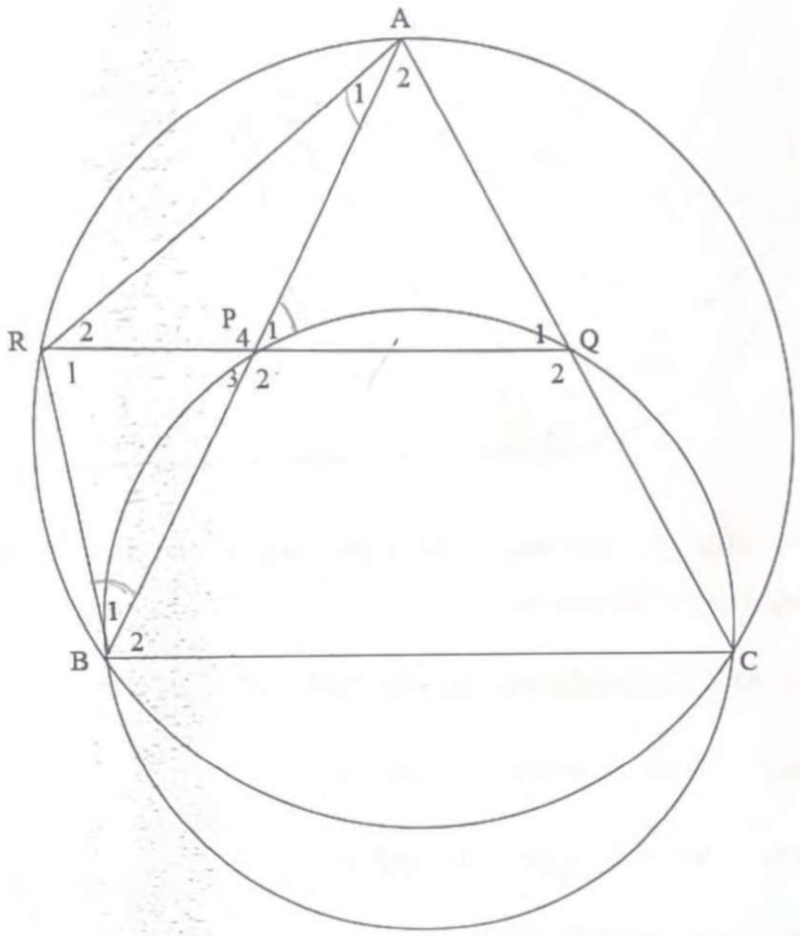
9.2.4 Prove that $\Delta SAR \,|||\, \Delta KNR$. (3)

9.2.5 Prove that SAKR is a cyclic quadrilateral. (2)

[19]

QUESTION 10

In the diagram, P is a point on side AB of ΔABC. The circle through P, B and C cuts AC at Q. QP produced cuts the circle passing through A, B and C at R.



Prove that:

10.1 $\hat{P}_1 = \hat{A}_1 + \hat{B}_1$ (5)

10.2 $AR^2 = AP.AB$ (5)

[10]

GRAND TOTAL: 150

INFORMATION SHEET: MATHEMATICS

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$A = P(1+ni)$ $\quad A = P(1-ni)$ $\quad A = P(1-i)^n$ $\quad A = P(1+i)^n$

$T_n = a + (n-1)d$ $\quad S_n = \frac{n}{2}(2a + (n-1)d)$

$T_n = ar^{n-1}$ $\quad S_n = \frac{a(r^n - 1)}{r-1}\ ;\ r \neq 1$ $\quad S_\infty = \frac{a}{1-r}\ ;\ -1 < r < 1$

$F = \frac{x[(1+i)^n - 1]}{i}$ $\quad P = \frac{x[1-(1+i)^{-n}]}{i}$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $\quad M\left(\frac{x_1 + x_2}{2};\frac{y_1 + y_2}{2}\right)$

$y = mx + c$ $\quad y - y_1 = m(x - x_1)$ $\quad m = \frac{y_2 - y_1}{x_2 - x_1}$ $\quad m = \tan\theta$

$(x-a)^2 + (y-b)^2 = r^2$

In ΔABC: $\quad \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

$a^2 = b^2 + c^2 - 2bc.\cos A$

$area\,\Delta ABC = \frac{1}{2}ab.\sin C$

$\sin(\alpha + \beta) = \sin\alpha.\cos\beta + \cos\alpha.\sin\beta$ $\quad \sin(\alpha - \beta) = \sin\alpha.\cos\beta - \cos\alpha.\sin\beta$

$\cos(\alpha + \beta) = \cos\alpha.\cos\beta - \sin\alpha.\sin\beta$ $\quad \cos(\alpha - \beta) = \cos\alpha.\cos\beta + \sin\alpha.\sin\beta$

$$\cos 2\alpha = \begin{cases} \cos^2\alpha - \sin^2\alpha \\ 1 - 2\sin^2\alpha \\ 2\cos^2\alpha - 1 \end{cases}$$

$\sin 2\alpha = 2\sin\alpha.\cos\alpha$

$\bar{x} = \frac{\sum fx}{n}$ $\quad \sigma^2 = \frac{\sum_{i=1}^{n}(x - \bar{x})^2}{n}$

$P(A) = \frac{n(A)}{n(S)}$ $\quad P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$

$\hat{y} = a + bx$ $\quad b = \frac{\sum(x - \bar{x})(y - \bar{y})}{\sum(x - \bar{x})^2}$