KWAZULU-NATAL PROVINCE

EDUCATION
REPUBLIC OF SOUTH AFRICA

NATIONAL SENIOR CERTIFICATE

GRADE 12

MATHEMATICS P2

PREPARATORY EXAMINATIONS

SEPTEMBER 2023

MARKS: 150

TIME: 3 hours

N.B. This question paper consists of 12 pages and 1 information sheet.
This paper has an Answer Booklet.

INSTRUCTIONS AND INFORMATION

Read the following instructions carefully before answering the questions.

1. This question paper consists of **11** questions.

2. Answer **ALL** the questions.

3. Number the answers correctly according to the numbering system used in this question paper.

4. Clearly show **ALL** calculations, diagrams, graphs, etc. that you have used in determining your answers.

5. Answers only will **NOT** necessarily be awarded full marks.

6. You may use an approved scientific calculator (non-programmable and non-graphical), unless stated otherwise.

7. If necessary, round off answers to **TWO** decimal places, unless stated otherwise.

8. Diagrams are **NOT** necessarily drawn to scale.

9. An information sheet with formulae is included at the end of the question paper.

10. Write neatly and legibly.

QUESTION 1

Mr Siphokazi supplements his pension by mowing lawns for customers. He measures the areas (x) (in m^2) of 12 of his customers' lawns and the time (y) in minutes, it takes him to mow these lawns. He works 8 hours a day. He recorded the data.

Area (x) (m^2)	360	120	845	602	1 190	530	245	486	350	1 005	320	250
Time (y) (minutes)	50	28	130	75	120	95	55	70	48	110	55	60

1.1 Determine the equation of the least squares regression line. (3)

1.2 Calculate the value of r, the correlation coefficient for the data. (2)

1.3 Given that Mr Siphokazi charges a flat call out fee of R150, as well as R50 per half hour (or part thereof), estimate the charge for mowing a customer's lawn that has an area of 560 m^2.

(For example: 100 minutes would be taken as 2 hours). (3)

1.4 The local high school wants Mr Siphokazi to mow their rugby field which is rectangular, 100 meters long by 70 meters wide.

1.4.1 Use the regression equation found in 1.1 to calculate the time it would take to mow this area. (1)

1.4.2 Is it possible for him to complete this job in a day? Give a reason for your answer. (1)

[10]

QUESTION 2

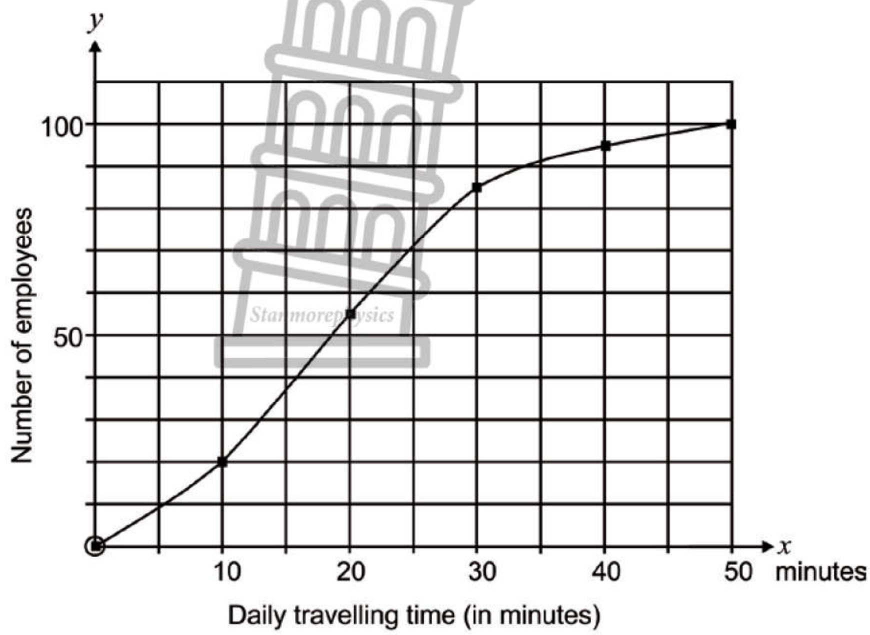
The following table gives the frequency distribution of the daily travelling time (in minutes) from home to work for the employees of a certain company.

Daily travelling time x (in minutes)	**Number of employees (f)**	**Midpoint of Interval**	
$0 \leq x < 10$	20		
$10 \leq x < 20$	35		
$20 \leq x < 30$	30		
$30 \leq x < 40$	10		
$40 \leq x < 50$	5		

2.1 Calculate the estimated mean travelling time. (3)

2.2 Write the modal class of the data. (2)

2.3 An ogive was drawn for the given data.
Construct a box-whisker plot for the data in the ANSWER BOOK. (3)



2.4 State whether the following statements are TRUE or FALSE.

2.4.1 The distribution of these travelling times is positively skewed. (1)

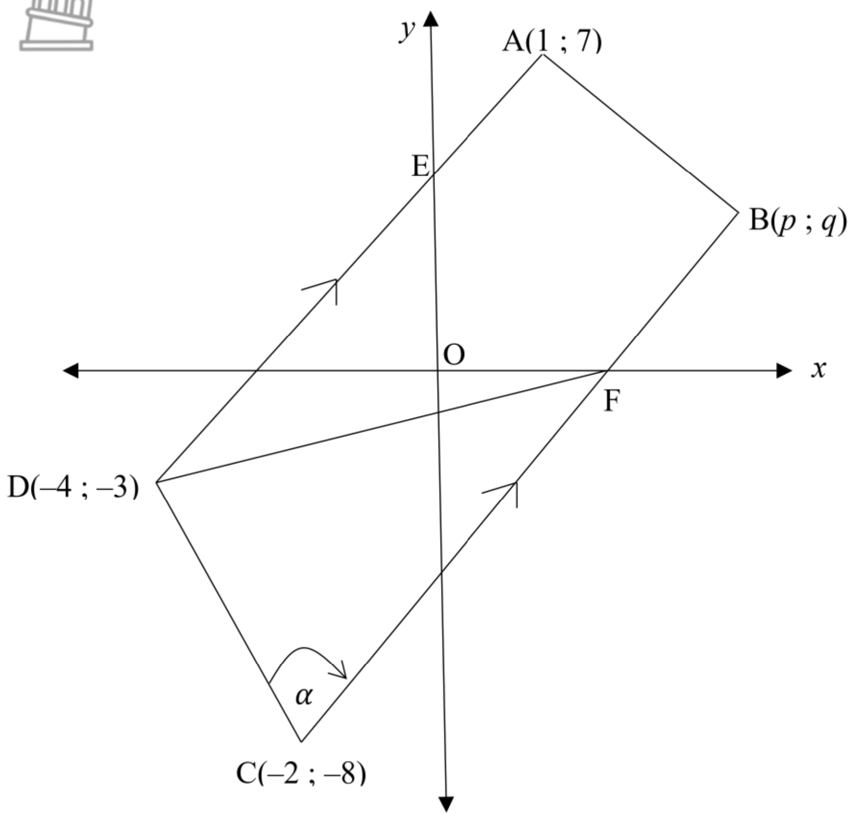
2.4.2 The inter-quartile range for the data is 25. (1)

2.4.3 35 employees take less than 20 minutes. (1)

[11]

QUESTION 3

Trapezium ABCD is drawn below with AD || BC is drawn. The coordinates of the vertices are A(1 ; 7), B(p ; q), C(–2 ; –8) and D(–4 ; –3). BC intersects the x – axis at F. $D\hat{C}B = \alpha$.
AD intersects the y – axis at E.



3.1	Calculate the gradient of AD.	(2)
3.2	Determine the equation of BC in the form $y = mx + c$.	(3)
3.3	Determine the coordinates of F.	(2)
3.4	AMCD is a parallelogram, with M on BC. Determine the coordinates of M.	(2)
3.5	Show that $\alpha = 48{,}37^{\circ}$.	(4)
3.6	Calculate the area of ΔDCF.	(4)
		[17]

QUESTION 4

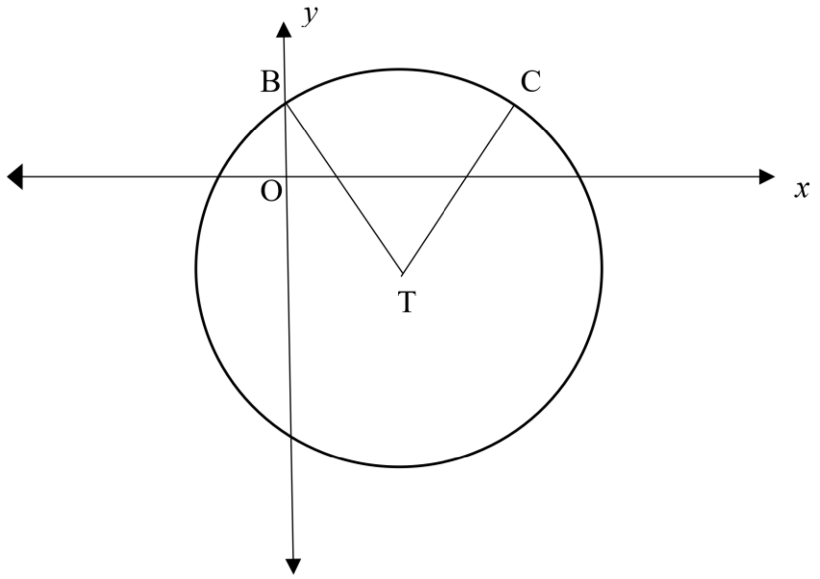
4.1 The equation of a circle is $x^2 + y^2 - 8x + 6y = 15$.

4.1.1 Show that P(2 ; –9) lies on the circle. (2)

4.1.2 Determine the equation of the tangent to the circle at point P(2 ; –9). (6)

4.1.3 A tangent is drawn from Q(–10 ; 12) to the circle. Calculate the length of the tangent. (4)

4.2 The circle, with centre T, and equation $(x - 3)^2 + (y + 2)^2 = 25$ is given below.
B is the y – intercept of the circle.



4.2.1 Determine the coordinates of B. (4)

4.2.2 Write down the coordinates of C, if C is the reflection of B in the line $x = 3$. (2)

4.2.3 Another circle with centre M and equation $(x - 12)^2 + (y - 10)^2 = 100$ is given.

(a) Calculate the distance, TM, between the centres. (2)

(b) Do these circles touch or intersect each other? Justify your answer. (2)

[22]

QUESTION 5

5.1 If $\sin 38^\circ = p$, determine the value of the following, **without using a calculator:**

5.1.1 $\cos 218^\circ$ (3)

5.1.2 $\cos 14^\circ$ (3)

5.1.3 $\sin 26^\circ \cos 26^\circ$ (2)

5.2 Evaluate the following trigonometric expression **without using a calculator:**

$$\frac{2\sin 165^\circ \cos 195^\circ}{\cos 45^\circ \sin 15^\circ - \cos 15^\circ \sin 45^\circ}$$ (5)

5.3 Given: $K = \sqrt{3}\cos x + \sin x$.

5.3.1 Write K in the form of $t\sin(x+\theta)$. (3)

5.3.2 Hence, calculate the value of t and θ. (1)

5.3.3 Write down the maximum value of K. (1)

5.4 Prove the identity:

5.4.1 $\dfrac{2\tan\theta - \sin 2\theta}{2\sin^2\theta} = \tan\theta$ (6)

5.4.2 Hence, determine the values of θ, $\theta \in [180^\circ; 360^\circ]$ which will make the above identity undefined. (2)

[26]

QUESTION 6

6.1 Sketch the graphs of $f(x) = 2\sin x$ and $g(x) = \cos(x - 30^\circ)$ for $x \in [-180^\circ; 180^\circ]$ on the grid in the ANSWER BOOK. Indicate the intercepts with the axes and also the turning points. (6)

6.2 Use your graphs to answer the following questions:

6.2.1 Write down the period of g. (1)

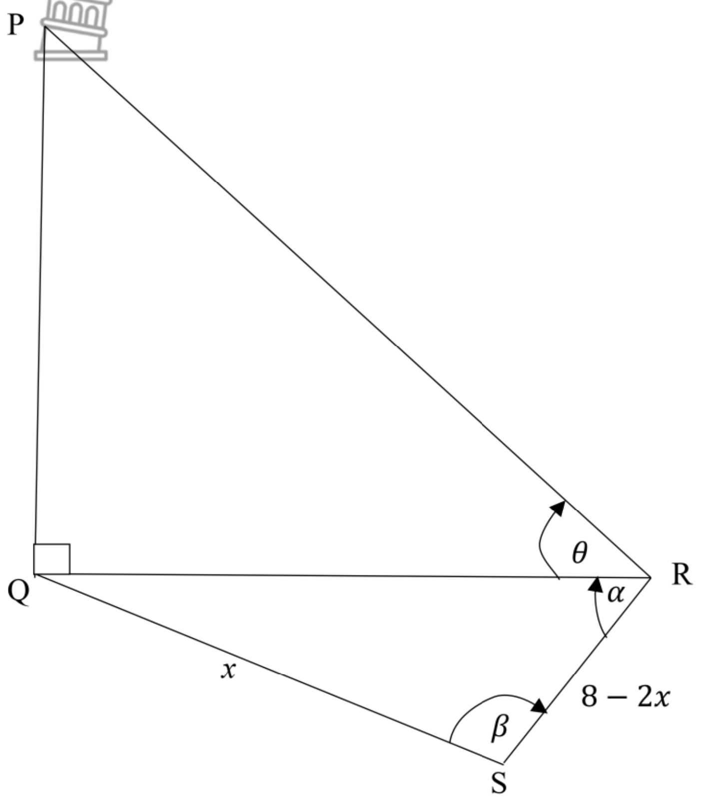
6.2.2 Determine the values of x for which $f(x) > g(x)$. (4)

6.2.3 Write down the values of x for which $f(x) = 1{,}5 + g(x)$. (2)

[13]

QUESTION 7

In the diagram below, PQ is a vertical mast. R and S are two points in the same horizontal plane as the foot of the mast, Q. $Q\hat{R}S = \alpha$, $Q\hat{S}R = \beta$, SR = $8 - 2x$ and SQ = x.
The angle of elevation of P, the top of the mast from R, is θ.



7.1 Express PQ in terms QR and a trigonometric ratio of θ. (1)

7.2 Show that: $PQ = \frac{x \sin\beta \tan\theta}{\sin\alpha}$ (4)

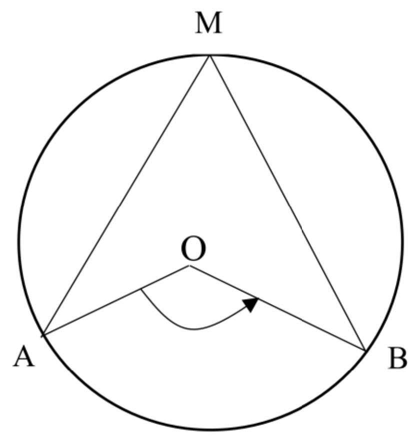
7.3 If $\beta = 60^\circ$, show that the area of $\Delta QSR = 2\sqrt{3}x - \frac{\sqrt{3}}{2}x^2$. (3)

7.4 Determine the value of x for which the area of ΔQSR will be at a maximum. (3)

[11]

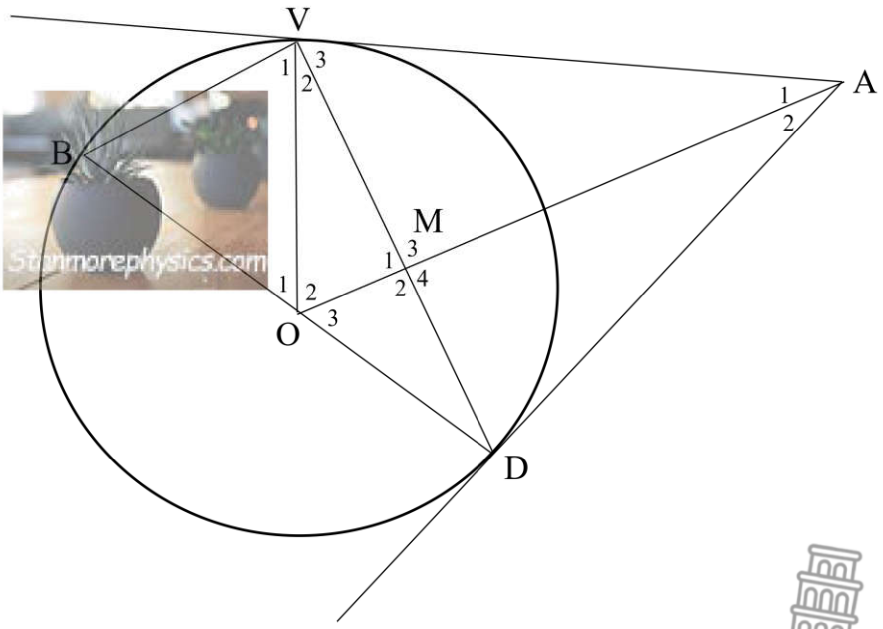
QUESTION 8

8.1 In the diagram O is the centre of the circle and M is a point on the circumference of the circle. Arc AB subtends $\hat{AOB}$ at the centre of the circle and $\hat{M}$ at the circumference of the circle.



Use the diagram to prove the theorem that states that $\hat{AOB} = 2\hat{M}$. (5)

8.2 From a point A outside the circle, center O, two tangents AD and AV are drawn. AO and VD intersect at M. BOD is a diameter of the circle. BV and VO are drawn. $\hat{VAD} = 40°$



8.2.1 Prove that quadrilateral VODA is cyclic. (2)

8.2.2 Calculate the magnitude of $\hat{O}_1$. (2)

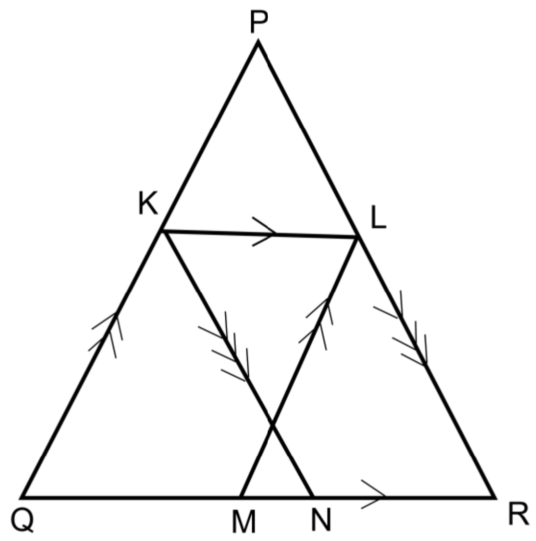
8.2.3 Prove that BV || OA. (5)

[14]

QUESTION 9

9.1 Complete the following statement: A line drawn parallel to one side of a triangle … (2)

9.2 In the figure, KL || QR. M and N are points on QR such that KN || PR and LM || PQ. PK = 3 units, PL = 4 units, LR = 6 units and MN = 1,8 units.



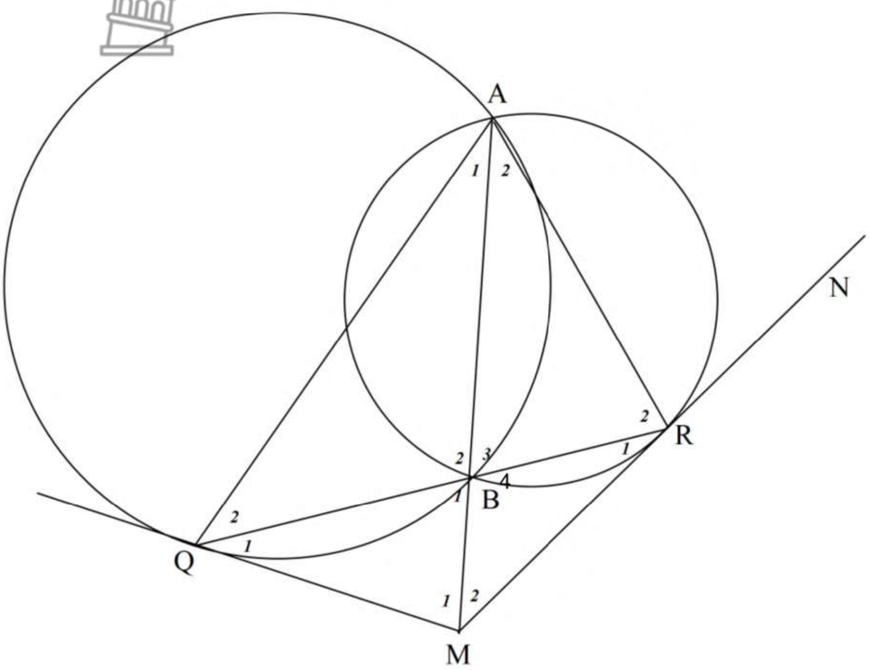
9.2.1 Calculate the length of KQ. (2)

9.2.2 Prove that QM = NR. (2)

[6]

QUESTION 10

In the figure, two circles intersect at A and B. AB produced to M bisects $Q\hat{A}R$. Tangents MQ and MR meet the circles at Q and R respectively. QBR is a straight line. AQ and AR are drawn.



Prove:

10.1 $\Delta MQA \parallel\!| \Delta MBQ$. (3)

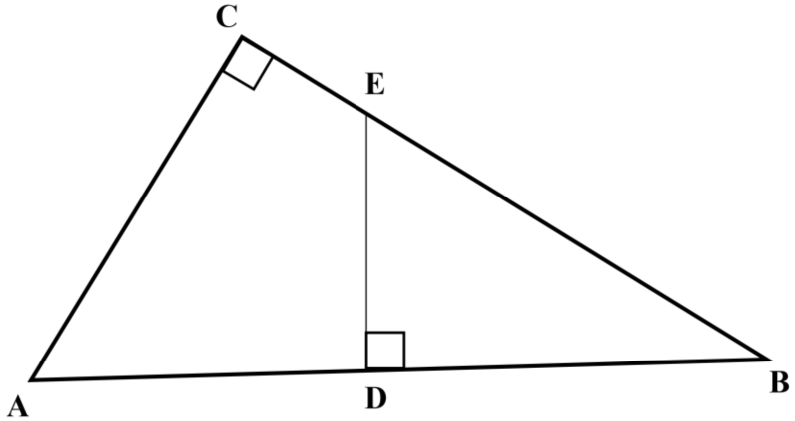
10..2 $MR^2 = AM.MB$ (5)

10.3 $\dfrac{MQ^2}{MR^2} = 1$ (4)

[12]

QUESTION 11

11 ΔABC is right angled at C. $ED \perp AB$ with E on CB and D on AB.
AC = 4,8 cm and AB = 8 cm. AD = DB.



11.1	Calculate BC, correct to 1 decimal digit.	(2)
11.2	Complete: $\Delta BAC \parallel\!\mid$ ….	(1)
11.3	Hence, or otherwise calculate the area of ADEC.	(5)

[8]

GRAND TOTAL: 150

INFORMATION SHEET: MATHEMATICS
INLIGTING BLADSY

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$A = P(1+ni)$ $\quad A = P(1-ni)$ $\quad A = P(1-i)^n$ $\quad A = P(1+i)^n$

$T_n = a + (n-1)d$ $\quad S_n = \frac{n}{2}\left(2a + (n-1)d\right)$

$T_n = ar^{n-1}$ $\quad S_n = \frac{a\left(r^n - 1\right)}{r-1}$; $\quad r \neq 1$ $\quad S_\infty = \frac{a}{1-r}$; $-1 < r < 1$

$F = \frac{x\left[(1+i)^n - 1\right]}{i}$ $\quad P = \frac{x[1-(1+i)^{-n}]}{i}$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $\quad M\left(\frac{x_1 + x_2}{2} ; \frac{y_1 + y_2}{2}\right)$

$y = mx + c$ $\quad y - y_1 = m(x - x_1)$ $\quad m = \frac{y_2 - y_1}{x_2 - x_1}$ $\quad m = \tan\theta$

$(x-a)^2 + (y-b)^2 = r^2$

In ΔABC: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$ $\quad a^2 = b^2 + c^2 - 2bc.\cos A$ $\quad area\ \Delta ABC = \frac{1}{2}ab.\sin C$

$\sin(\alpha + \beta) = \sin\alpha.\cos\beta + \cos\alpha.\sin\beta$ $\quad \sin(\alpha - \beta) = \sin\alpha.\cos\beta - \cos\alpha.\sin\beta$

$\cos(\alpha + \beta) = \cos\alpha.\cos\beta - \sin\alpha.\sin\beta$ $\quad \cos(\alpha - \beta) = \cos\alpha.\cos\beta + \sin\alpha.\sin\beta$

$$\cos 2\alpha = \begin{cases} \cos^2\alpha - \sin^2\alpha \\ 1 - 2\sin^2\alpha \\ 2\cos^2\alpha - 1 \end{cases}$$

$\sin 2\alpha = 2\sin\alpha.\cos\alpha$

$\bar{x} = \frac{\sum f.x}{n}$ $\quad \sigma^2 = \frac{\sum_{i=1}^{n}\left(x_i - \bar{x}\right)^2}{n}$

$P(A) = \frac{n(A)}{n(S)}$ $\quad P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$

$\hat{y} = a + bx$ $\quad b = \frac{\sum(x - \bar{x})(y - \bar{y})}{\sum(x - \bar{x})^2}$