FINAL

KWAZULU-NATAL PROVINCE

EDUCATION
REPUBLIC OF SOUTH AFRICA

NATIONAL
SENIOR CERTIFICATE

GRADE 12

MATHEMATICS P2

MARKING GUIDELINE

PREPARATORY EXAMINATIONS

SEPTEMBER 2023

MARKS: 150

This marking guideline consists of 14 pages.

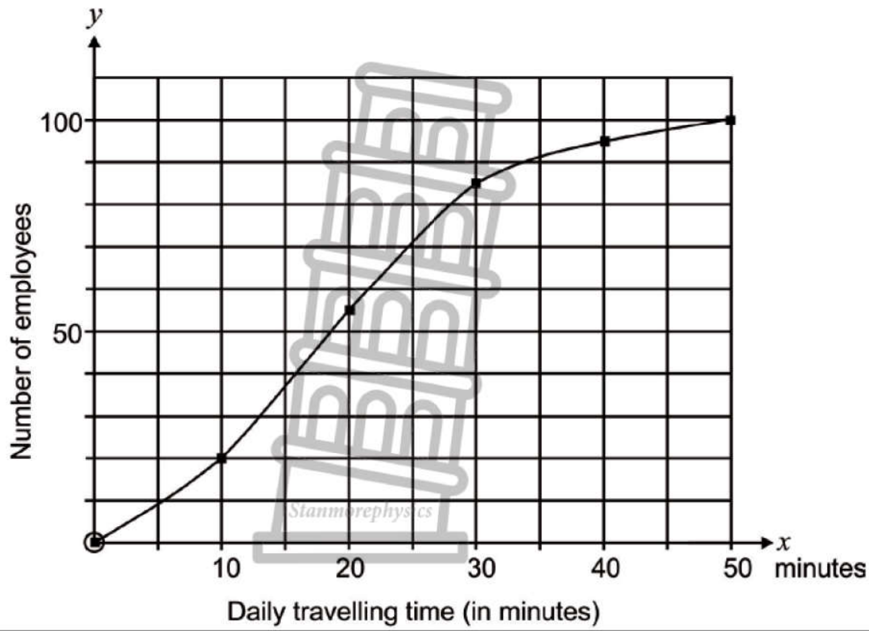
NB: CA APPLIES TO ALL SUB-QUESTIONS IN THIS MARKING GUIDELINE.

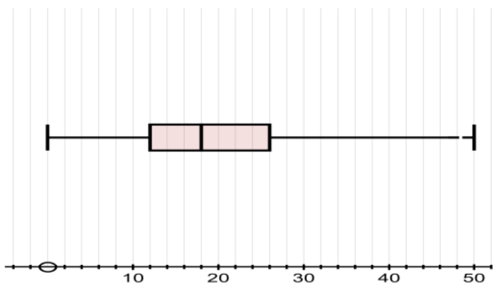
QUESTION 1

1.1	$a = 34{,}90$ $b = 0{,}08$ $y = 34{,}90 + 0{,}08x$ **Answer only: Full Marks**	✓A Value of a ✓A Value of b ✓CA Equation	(3)
1.2	$r = 0{,}88$	✓A✓A Answer	(2)
1.3	$y = 34{,}90 + 0{,}08x$ $y = 34{,}90 + 0{,}08(560)$ $y = 79.7$ minutes = 1.33 hours Total cost = R150 + R150 = R300	 ✓CA Substitution ✓CA 79,7 minutes ✓CA Answer	(3)
1.4.1	Area = 100 x 70 = 7000 square meters $y = 34{,}90 + 0{,}08(7000)$ $y = 594.9$ minutes = 9.92 hours	 ✓CA Calculation	(1)
1.4.2	No. The time taken will exceed his daily 8 hour working hours.	✓CA Justification	(1)
			[10]

QUESTION 2

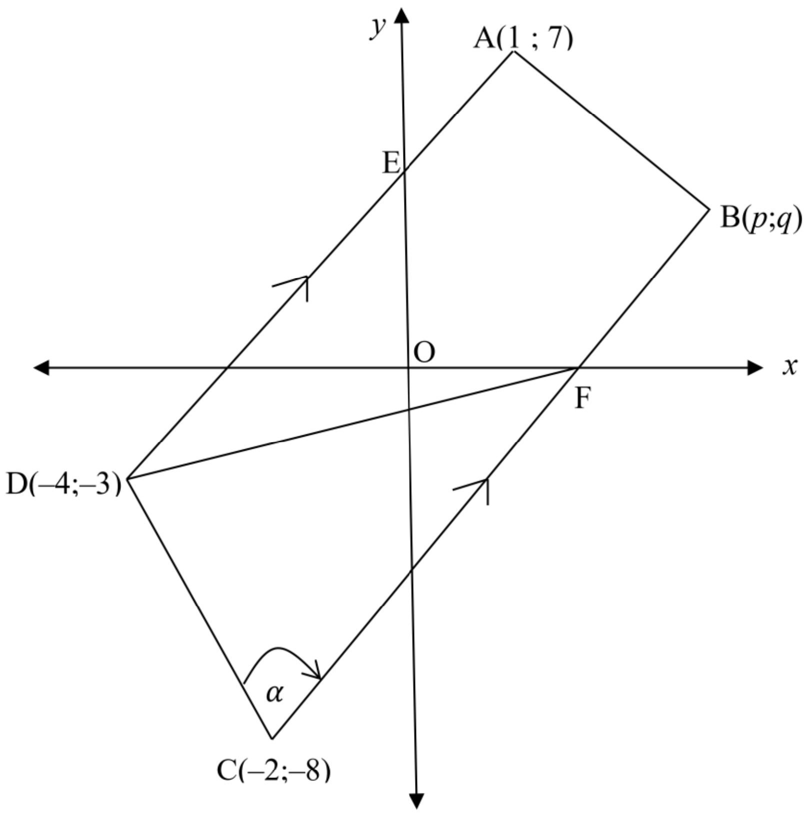
Daily travelling time x(in minutes)	Number of employees (f)	Midpoint of Interval (x)	$f.x$
$0 \leq x < 10$	20	5	100
$10 \leq x < 20$	35	15	525
$20 \leq x < 30$	30	25	750
$30 \leq x < 40$	10	35	350
$40 \leq x < 50$	5	45	225
Total	100		1950



2.1	Estimated Mean $= \frac{1950}{100} = 19{,}5$ **Answer only: Full Marks**	✓A 1950 ✓A 100 ✓CA Answer	(3)
2.2	$10 \leq x < 20$	✓A✓A Answer	(2)
2.3	See Diagram	✓A Minimum and Maximum value ✓A 1[st] and 3[rd] Quartiles ✓A 2[nd] Quartile	(3)
	 Five number summary: 0 ; 12 ; 18 ; 26 ; 50 **Accept:** ± 1 deviation on quartiles		

2.4.1	True	✓A Answer	(1)
2.4.2	False	✓A Answer	(1)
2.4.3	True	✓A Answer	(1)
			[11]

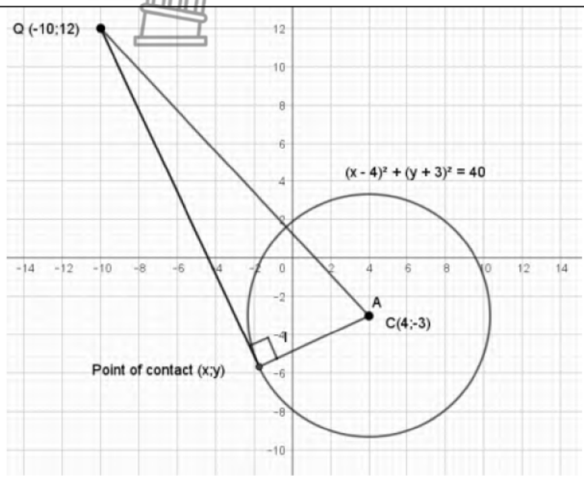
QUESTION 3



3.1	$m_{AD} = \dfrac{7+3}{1+4} = \dfrac{10}{5} = 2$	✓A Substitution of points A and D ✓CA Answer	(2)
3.2	$m_{AD} = m_{BC} = 2$ …..(AD ‖ BC) $y = mx + c$ $-8 = 2(-2) + c$ $-4 = c$ $y = 2x - 4$	✓CA Gradient of BC ✓CA Substitution of point C and gradient ✓CA Answer	(3)

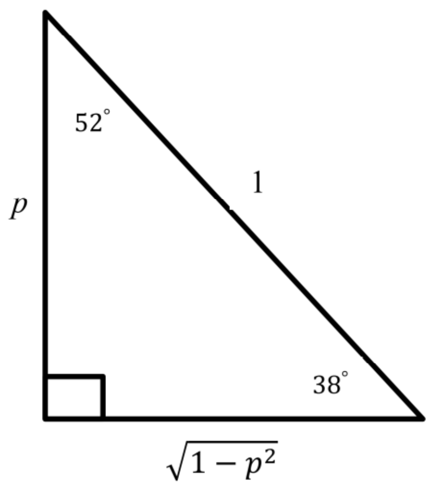
3.3	Let $y = 0$: $0 = 2x - 4$ $x = 2$ $F(2\,;0)$	✓CA $0 = 2x - 4$ ✓CA x – value	(2)
3.4	M(3 ; 2)	✓A x – value ✓A y – value	(2)
3.5	$m_{CD} = \frac{-3+8}{-4+2} = \frac{5}{-2}$ Inclination of CD: $\tan\theta_1 = 180^\circ - 68{,}2^\circ = 111{,}8^\circ$ Inclination of CF: $\tan\theta_2 = 2$ $\theta_2 = 63{,}43^\circ$ Therefore $\alpha = 111{,}8^\circ - 63{,}43^\circ$ $= 48{,}37^\circ$ **OR** DF $= \sqrt{(2\ +\ 4)2\ +\ (0\ +\ 3)2}$ $= \sqrt{45}$ DC $= \sqrt{29}$ CF $= \sqrt{80}$ $(\sqrt{45})^2 = (\sqrt{29})^2 + (\sqrt{80})^2 - 2\sqrt{29}\,\sqrt{80}\cos\alpha$ $\alpha = 48{,}37^\circ$	✓A Gradient of CD ✓A Inclination of CD ✓A Inclination of CF ✓A subtraction of the angles ✓A Distance of DF ✓A Distance of DC ✓A Distance of CF ✓A subst. into cosine rule	(4)
3.6	DC $= \sqrt{(-3+8)^2 + (-4+2)^2} = \sqrt{29}$ FC $= \sqrt{(0+8)^2 + (2+2)^2} = \sqrt{80}$ Area of ΔDCF $= \frac{1}{2}(\sqrt{29})(\sqrt{80})\sin 48{,}37^\circ$ $= 18$ square units.	✓A Length of CD ✓CA Length of FC ✓CA Substitution into area formula ✓CA Answer	(4)
			[17]

QUESTION 4

4.1.1	$x^2 + y^2 - 8x + 6y = 15$ LHS = $(2)^2 + (-9)^2 - 8(2) + 6(-9)$ $= 4 + 81 - 16 - 54$ $= 15$ = RHS	✓A Subst. of point ✓A Simplification	(2)
4.1.2	 $(x-4)^2 + (y+3)^2 = 40$ Centre: C(4 ; –3) P(2 ; –9) $m_{Radius} = \frac{-3+9}{4-2} = \frac{6}{2} = 3$ $m_{Tangent} = -\frac{1}{3}$ Equation of Tangent: $y = mx + c$ $-9 = -\frac{1}{3}(2) + c$ $-\frac{25}{3} = c$ $y = -\frac{1}{3}x - \frac{25}{3}$	✓A writing the equation as $(x-4)^2 + (y+3)^2 = 40$ ✓CA Centre of circle ✓CA Gradient of radius ✓CA Gradient of tangent ✓CA Substitution ✓CA Answer	(6)
4.1.3	r^2: $(x-4)^2 + (y+3)^2 = 40$ $r^2 = 40$ $(distance\ Q\ to\ the\ centre)^2 = (-10-4)^2 + (12+3)^2$ $= 421$ $(\text{Length of tangent})^2 = 421 - 40 = 381$ Length of tangent $= \sqrt{381}$	✓CA Calculation of r^2 ✓CA distance calculation ✓CA Tangent calculation ✓CA Answer	(4)

4.2.1	$(x-3)^2+(y+2)^2=25$ Let $x = 0$: $(0-3)^2+(y+2)^2=25$ $(y+2)^2=16$ $y+2=\pm 4$ $y=-6$ or $y=2$ B(0 ; 2)	✓A Letting $x = 0$ ✓A Simplification ✓CA y – values ✓CA Answer	(4)
4.2.2	C(6 ; 2)	✓CA x – value ✓CA y – value	(2)
4.2.3 (a)	T(3 ; –2) and M(12 ; 10) $TM^2 = (12-3)^2 + (10+2)^2 = 225$ TM = 15 units	✓A Coordinates of M ✓CA Answer	(2)
(b)	Radius, center T = 5 units and Radius, center M = 10 units Sum of radii = 15 units Circles **touch** . TM= Sum of radii	✓CA Sum of radii ✓CA Justification	(2)
			[22]

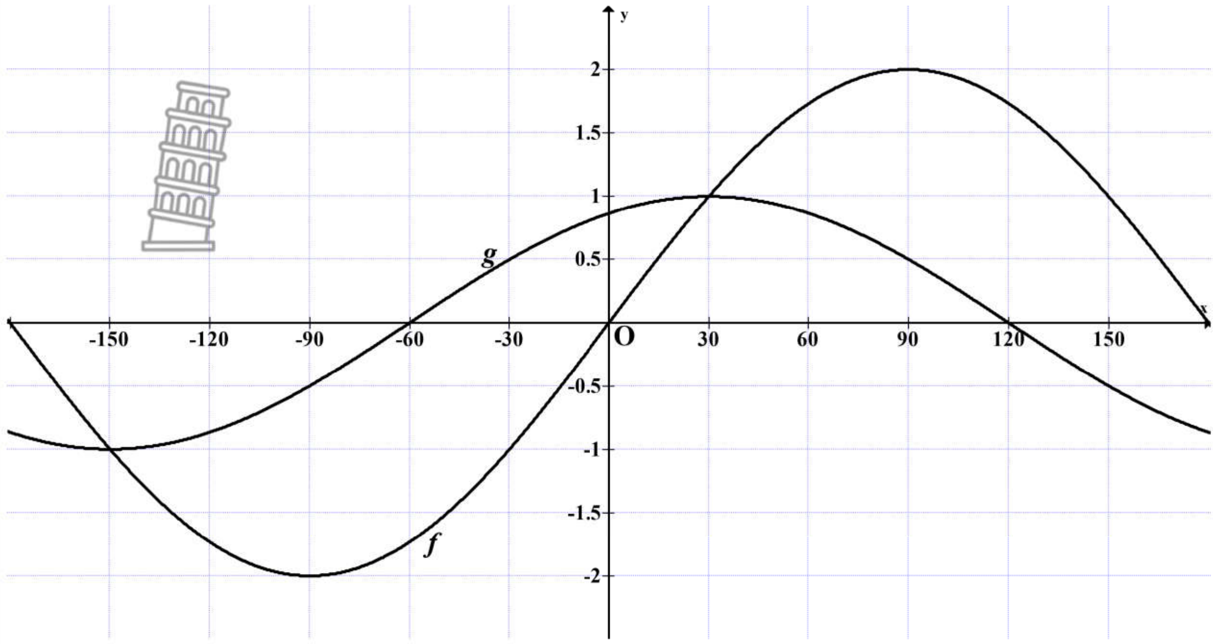
QUESTION 5

5.1			
5.1.1	$\cos 218^\circ$ $= -\cos 38^\circ$ $= -\dfrac{\sqrt{1-p^2}}{1}$	✓A Calculation of $\sqrt{1-p^2}$ ✓A Reduction ✓CA Answer	(3)

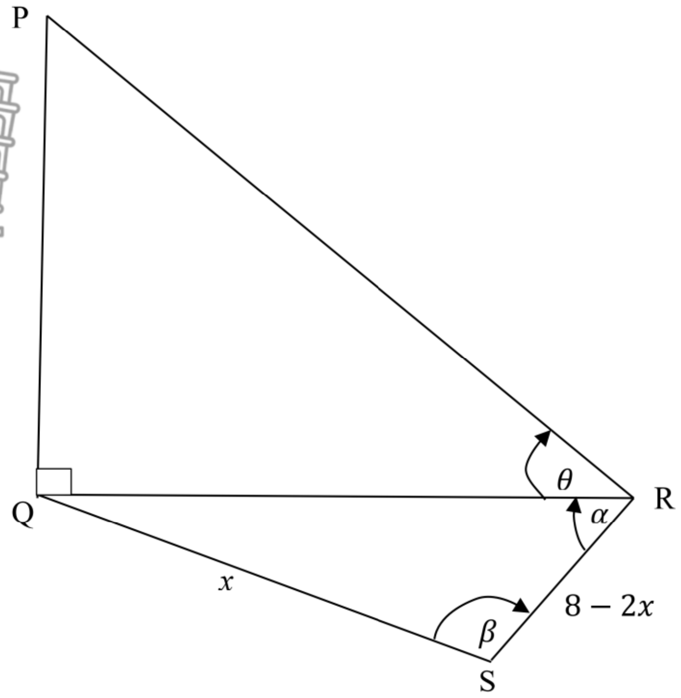
5.1.2	$\cos 14^\circ$ $= \cos(52^\circ - 38^\circ)$ $= \cos 52^\circ \cos 38^\circ + \sin 52^\circ \sin 38^\circ$ $= \left(\frac{p}{1}\right)\left(\frac{\sqrt{1-p^2}}{1}\right) + \left(\frac{\sqrt{1-p^2}}{1}\right)\left(\frac{p}{1}\right)$ $= 2p\sqrt{1-p^2}$ **OR** $\cos 24^\circ = \sin 76^\circ$ $= \sin 2(38^\circ)$ $= 2\sin 38^\circ \cos 38^\circ$ $= 2p\sqrt{1-p^2}$	✓A Writing as difference ✓A Expansion ✓CA Answer ✓ $\sin 76^\circ$ ✓double angle ✓answer	(3)
5.1.3	$\sin 26^\circ \cos 26^\circ$ $= \frac{1}{2}\sin 52^\circ$ $= \frac{1}{2}\sqrt{1-p^2}$	✓A Double angle ✓CA Answer	(2)
5.2	$\frac{2\sin 165^\circ \cos 195^\circ}{\cos 45^\circ \sin 15^\circ - \cos 15^\circ \sin 45^\circ}$ $= \frac{2\sin 15^\circ.(-\cos 15^\circ)}{\cos 45^\circ \sin 15^\circ - \cos 15^\circ \sin 45^\circ}$ $= \frac{-2\sin 30^\circ}{\sin(15^\circ - 45^\circ)}$ $= \frac{-2\sin 30^\circ}{\sin(-30^\circ)}$ $= \frac{-2\sin 30^\circ}{-\sin 30^\circ}$ $= 2$	✓A $-\cos 15^\circ$ ✓A $-2\sin 30^\circ$ ✓A $\sin(15^\circ - 45^\circ)$ ✓A $-\sin 30^\circ$ ✓CA Answer	(5)
5.3.1	$K = \sqrt{3}\cos x + \sin x$ $K = 2\left(\frac{\sqrt{3}}{2}\cos x + \frac{1}{2}\sin x\right)$ $K = 2(\sin 60^\circ \cos x + \cos 60^\circ \sin x)$ $K = 2\sin(60^\circ + x)$	✓A $2\left(\frac{\sqrt{3}}{2}\cos x + \frac{1}{2}\sin x\right)$ ✓A $\sin 60^\circ$ and $\cos 60^\circ$ ✓A $2\sin(60^\circ + x)$	(3)

5.3.2	$t = 2$ and $\theta = 60^\circ$	✓CA t – value and $\theta = 60^\circ$	(1)
5.3.3	2	✓CA Answer	(1)
5.4.1	$\text{LHS} = \dfrac{2\tan\theta - \sin 2\theta}{2\sin^2\theta}$ $= \dfrac{2\left(\dfrac{\sin\theta}{\cos\theta}\right) - 2\sin\theta\cos\theta}{2\sin^2\theta} \times \dfrac{\cos\theta}{\cos\theta}$ $= \dfrac{2\sin\theta - 2\sin\theta\cos^2\theta}{2\sin^2\theta\cos\theta}$ $= \dfrac{2\sin\theta\left(1-\cos^2\theta\right)}{2\sin^2\theta\cos\theta}$ $= \dfrac{2\sin\theta \,.\, \sin^2\theta}{2\sin^2\theta \,.\, \cos\theta}$ $= \dfrac{\sin\theta}{\cos\theta}$ $= \tan\theta$ $= \text{LHS}$	✓A $\dfrac{\sin\theta}{\cos\theta}$ ✓A $2\sin\theta\cos\theta$ ✓A simplification ✓A factorizing ✓A $1-\cos^2\theta = \sin^2\theta$ ✓A simplified to $\dfrac{\sin\theta}{\cos\theta}$	(6)
5.4.2	$2\sin^2\theta = 0$ $\sin\theta = 0$ $\therefore \theta = 180^\circ$ and 360° $\theta = 270^\circ$	✓ 180° and 360° ✓ 270°	(2)
			[26]

QUESTION 6

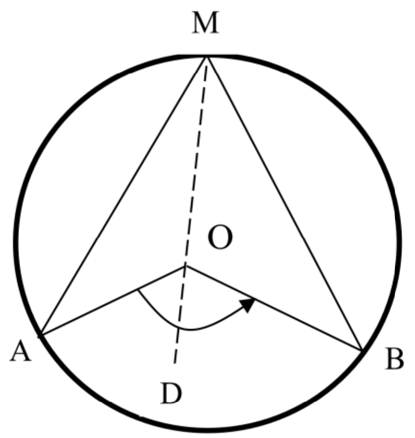
6.1			
	Graph of f: 1A mark for x – intercepts 1A marks for minimum and maximum points 1A mark for shape Graph of g: 1A mark for end points 1A mark for x – intercepts 1A mark for y – intercept		(6)
6.2.1	360°	✓A Answer	(1)
6.2.2	$x \in [-180^\circ ; -150^\circ) \cup (30^\circ ; 180^\circ]$	$[-180^\circ ; -150^\circ)$ ✓A : Notation ✓CA : values $(30^\circ ; 180^\circ]$ ✓A : Notation ✓CA : values	(4)
6.2.3	$f(x) = 1.5 + g(x)$ $f(x) - g(x) = 1.5$ $x = 90^\circ$ or $x = 150^\circ$	✓A $f(x) - g(x) = 1.5$ ✓A Answer	(2)
			[13]

QUESTION 7

			
7.1	$\frac{PQ}{QR} = \tan\theta$ $PQ = QR\tan\theta$	✓A Answer	(1)
7.2	$\frac{QR}{\sin\hat{S}} = \frac{QS}{\sin Q\hat{R}S}$ $\frac{QR}{\sin\beta} = \frac{x}{\sin\alpha}$ $QR = \frac{x\sin\beta}{\sin\alpha}$ $PQ = \frac{x\sin\beta\tan\theta}{\sin\alpha}$	✓A Sine rule formula ✓A Subs. Sine rule ✓A Making QR a subj. of the formula ✓A Subst. of QR	(4)
7.3	Area of $\Delta QSR = \frac{1}{2}(x)(8-2x)\sin 60^\circ$ $= \frac{1}{2}(x)(8-2x)\left(\frac{\sqrt{3}}{2}\right)$ $= \frac{\sqrt{3}}{4}(x)(8-2x)$ $= \sqrt{3}(x)\left(2-\frac{1}{2}x\right)$ $= 2\sqrt{3}x - \frac{\sqrt{3}}{2}x^2$	✓A Subst. into Area rule ✓A $\frac{\sqrt{3}}{2}$ ✓A Simplifying	(3)

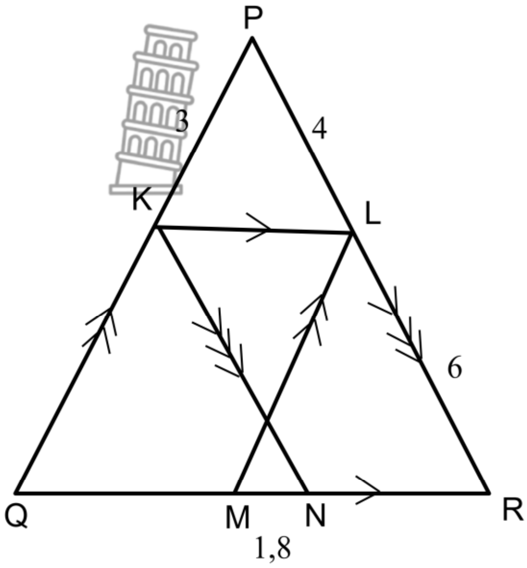
7.4	For Max Area: $x = -\dfrac{b}{2a}$ $x = -\dfrac{(2\sqrt{3})}{2\left(-\dfrac{\sqrt{3}}{2}\right)}$ $x = 2$	✓A Formula ✓ A Substitution into formula ✓ CA Answer	(3)
			[11]

QUESTION 8

8.1	 Constr: Join MO and produce to D. $\hat{AOD} = \hat{OAM} + \hat{AMO}$ … (Ext. o∠f Δ) $\hat{BOD} = \hat{OBM} + \hat{BMO}$ … (Ext. ∠ of Δ) But $\hat{OAM} = \hat{AMO}$ and $\hat{OBM} = \hat{BMO}$ …(Radii =) $\therefore \hat{AOD} + \hat{BOD} = 2\hat{AMO} + 2\hat{BMO}$ $\hat{AOB} = 2(\hat{AMO} + \hat{BMO})$ $\hat{AOB} = 2\hat{M}$ **NOTE No construction : No marks**	✓A Construction ✓A S/R ✓A S ✓A S ✓A S	(5)
8.2.1	$\hat{OVA} = \hat{ODA} = 90^\circ$ …. (Radius ⊥ Tangent) VODA is a cyclic quad. (Converse of opposite angles of quad. Supplementary)	✓A S/R ✓A R	(2)
8.2.2	$\hat{O}_1 = 40^\circ$ …(Exterior angle of cyclic quad = int. opp. Angle)	✓S ✓R	(2)

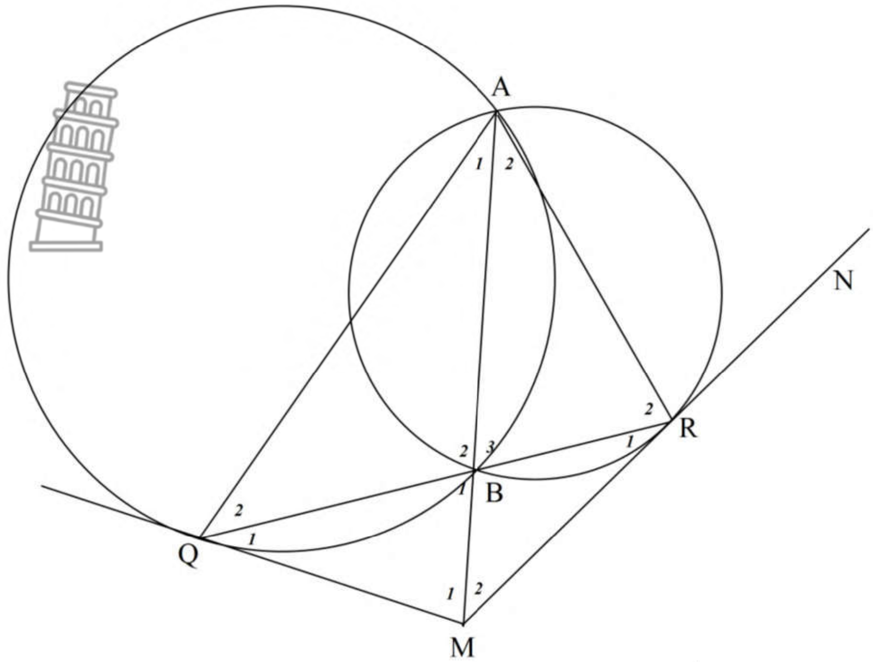
8.2.3	$\hat{V}_1 = \frac{180^\circ - 40^\circ}{2} = 70^\circ$ …..(sum of $\angle s$ of Δ; radii)	✓ S/R	
	$A\hat{D}M = \frac{180^\circ - 40^\circ}{2} = 70^\circ$ …..(sum of $\angle s$ of Δ; Tangents drawn from a common point A)	✓ S/R	
	$A\hat{D}M = \hat{O}_2 = 70^\circ$ … (Angles subtended by common chord AV)	✓S ✓R	
	BV ∥ OA ……………(Converse of Alt $\angle s$ or Alt $\angle s$ are =)	✓ R	(5)
	OR		
	In ΔOVA and ΔODA OV=OD........(radii) OA=OA.......(common) AV=AD.......(tans from the same point) $\Delta OVA \equiv \Delta ODA$.......(SSS)	✓A $\Delta OVA \equiv \Delta ODA$.......(SSS)	
	$A_1 = A_2$.......($\equiv \Delta s$) $=20^\circ$		
	$A_2 = V_2$.......($\angle$s in the same segment) $=20^\circ$	✓A S/R	
	$V_1 = 90^\circ - 20^\circ = 70^\circ$	✓A $V_1 = 70^\circ$	
	$O_2 = 180^\circ - [OVA + A_1]$ $= 180^\circ - [90^\circ + 20^\circ]$ $= 70^\circ$	✓A $O_2 = 70^\circ$	
	$\therefore V_1 = O_2$......(both $= 70^\circ$) $\therefore$ BV∥OA...... (alt$\angle$s are = or conv.alt$\angle$s)	✓A R	
			[14]

QUESTION 9

9.1	Divides the other two sides, proportionally.	✓S divides the other two sides ✓S proportionally	(2)
9.2			

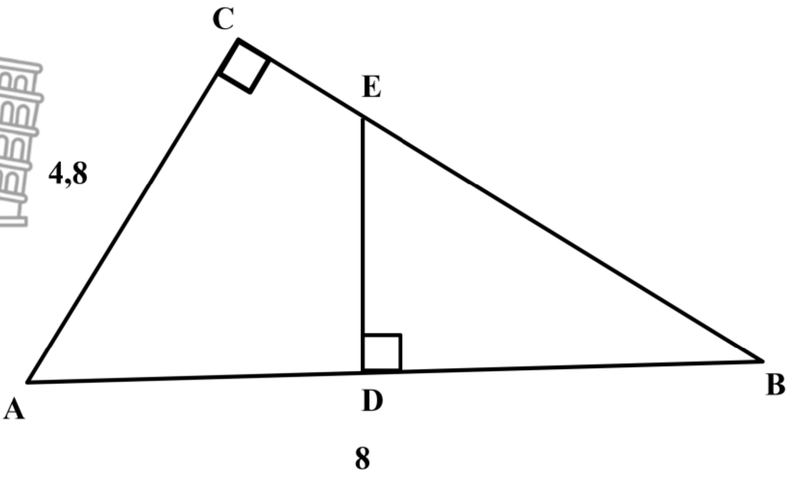
9.2.1	$\frac{KQ}{3} = \frac{6}{4}$ …………..(Prop. Thm; KL || QR) KQ = 4,5 units	✓ S/R ✓ Answer	(2)
9.2.2	KL = QM ….(Opposite sides of $\|\|^m$ QKLM are equal) KL = NR …. (Opposite sides of $\|\|^m$ NKLR are equal) Therefore QM = NR (both = KL) OR $\frac{QN}{QR} = \frac{QK}{KP}$ Prop Thm, KN||PR $\frac{4,5}{3} = \frac{3}{2}$ $\frac{MR}{QM} = \frac{RL}{PL}$ Prop Thm, ML||QP $= \frac{6}{4} = \frac{3}{2}$ $\therefore \frac{QN}{NR} = \frac{MR}{QM}$ $\frac{QM+1,8}{NR} = \frac{NR+1,8}{QM}$ $\therefore QM(QM+1,8) = MR(NR+1,8)$ $\therefore QM = NR$	✓ S/R ✓ S/R ✓ $\frac{QN}{QR} = \frac{QK}{KP}$ Prop Thm, KN||PR OR $\frac{MR}{QM} = \frac{RL}{PL}$ Prop Thm, ML||QP ✓ $\frac{QM+1,8}{NR} = \frac{NR+1,8}{QM}$	(2) **[16]**

QUESTION 10



10.1	In Δ's MQA and MBQ $\widehat{M}_1 = \widehat{M}_1$ ……….(Common) $\widehat{A}_1 = \widehat{Q}_1$ ……….(Tan-Chord Theorem) $A\widehat{Q}M = \widehat{B}_1$ ……….(Remaining ∠s of Δ's) ΔMQA ||| ΔMBQ ……(AAA)	✓ S/R ✓ S/R ✓ R	(3)
10.2	In Δ's MAR and MRB $\widehat{M}_2 = \widehat{M}_2$ ……….(Common) $\widehat{A}_2 = \widehat{R}_1$ ……….(Tan-Chord Thm) $A\widehat{R}M = R\widehat{B}M$ ……….(Remaining ∠s of Δ's) ΔMAR ||| ΔMRB ……(AAA) $\frac{MA}{MR} = \frac{MR}{MB}$ ………….(Δ's similar) $MR^2 = AM.MB$	✓ S ✓ S/R ✓ R ✓S A✓R	(5)
10.3	$\frac{MQ}{MB} = \frac{MA}{MQ}$ ………(from 10.1) $MQ^2 = MB.MA = \ldots\ldots(1)$ Also $MR^2 = AM.MB$ Now $\frac{MQ^2}{MR^2} = \frac{MB.MA}{AM.MB}$ $\therefore \frac{MQ^2}{MR^2} = 1$	✓ $\frac{MQ}{MB} = \frac{MA}{MQ}$ ✓ $MQ^2 = MB.MA$ ✓✓ $\frac{MQ^2}{MR^2} = \frac{MB.MA}{AM.MB}$	(4)
			[12]

QUESTION 11



11.1	$BC^2 = 8^2 - 4{,}8^2$ ……..(TOP) $BC = 6{,}4$ cm	✓ S ✓ S	(2)
11.2	ΔBED	✓ S	(1)
11.3	$\frac{BA}{BE} = \frac{AC}{ED} = \frac{BC}{BD}$ $\quad \Delta BAC \,\|\|\|\, \Delta BED$ $\frac{8}{BE} = \frac{4{,}8}{ED} = \frac{6{,}4}{4}$ $ED = \frac{4{,}8 \times 4}{6{,}4} = 3\ cm$ Area of ΔEDB $= \frac{1}{2}(4)(3) = 6\ cm^2$ Area of ΔABC $= \frac{1}{2}(4{,}8)(6{,}4) = 15{,}6\ cm^2$ Therefore, Area of ADEC $= 15{,}6 - 6 = 9{,}6\ cm^2$	✓A S ✓CA Value of ED ✓CA Area of ΔEDB ✓CA Area of ΔABC ✓CA Area of ADEC	(5)
			[8]

GRAND TOTAL: 150