Education

KwaZulu-Natal Department of Education
REPUBLIC OF SOUTH AFRICA

MATHEMATICS

COMMON TEST

MARCH 2018

NATIONAL SENIOR CERTIFICATE

GRADE 11

MARKS: 75

TIME: 1½ hours

This question paper consists of 6 pages.

INSTRUCTIONS AND INFORMATION

Read the following instructions carefully before answering the questions:

1. This question paper consists of 4 questions.
2. Answer ALL the questions.
3. Number the answers correctly according to the numbering system used in this question paper.
4. Clearly show ALL calculations, diagrams, graphs, et cetera, which you have used in determining the answers.
5. Answers only will NOT necessarily be awarded full marks.
6. You may use an approved scientific calculator (non-programmable and non-graphical) unless stated otherwise.
7. If necessary, round off answers to TWO decimal places, unless stated otherwise.
8. Diagrams are NOT necessarily drawn to scale.
9. Write neatly and legibly.

QUESTION 1

1.1 Solve for x in each of the following:

1.1.1 $3x^2 + 10x - 5 = 0$ (correct to TWO decimal places) (4)

1.1.2 $3^{x+1} - 3^{x+3} = -\frac{8}{27}$ (4)

1.1.3 $5 - x = \sqrt{4x + 1}$ (5)

1.2 Solve for x and y if:
$y + 7 = 2x$ and $x^2 - xy + 3y^2 = 15$ (6)

1.3 Given: $f(x) = 9x^2 - 15x + 4$

1.3.1 Solve for x if $f(x) > 0$. (3)

1.3.2 Show that $f(x) = -3$ has no real roots. (4)

[26]

QUESTION 2

Simplify fully, WITHOUT using a calculator:

2.1 $\frac{12^{n+1}.27^{n-2}}{18^{2n-1}.\sqrt{9^{-3}}} + 8^0$ (5)

2.2 If $y = \sqrt[6]{100\,000}$, WITHOUT USING A CALCULATOR, determine the value of $\sqrt[3]{16} \times \sqrt[3]{625} \times \sqrt{10}$ in terms of y. (4)

[9]

QUESTION 3

3.1 Consider the following quadratic number pattern: 64 ; 42 ; 24 ; ...

3.1.1 Write down the next TWO terms of the number pattern. (2)

3.1.2 Determine an expression for the general term, T_n, in the form $T_n = an^2 + bn + c$. (4)

3.1.3 Calculate the value of the 20^{th} term of this number pattern. (2)

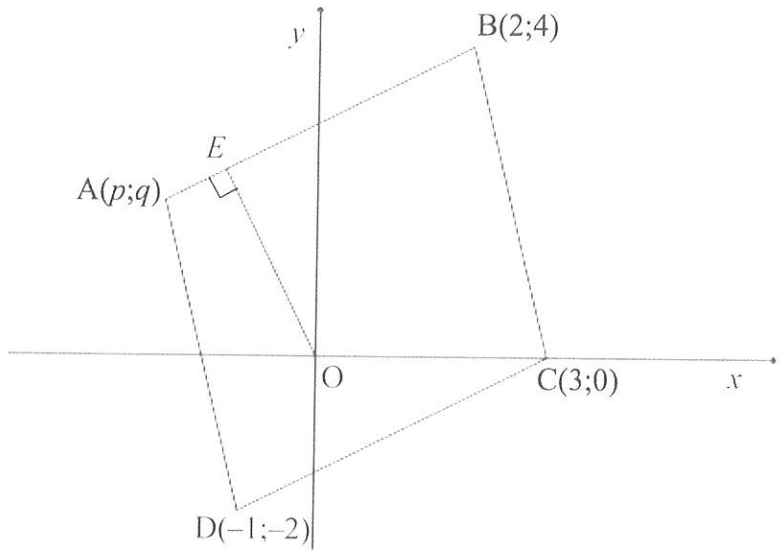
3.1.4 Determine the general term of the sequence of first differences of this number pattern. (2)

3.1.5 Between which two consecutive terms of the quadratic number pattern will the difference be equal to 174? (2)

3.2 If p ; 11 ; 21 ; $6p$ form a quadratic number pattern.
Calculate the value of p. (4)
[16]

QUESTION 4

4.1 A(p;q), B(2;4), C(3;0) and D(–1; –2) are the vertices of parallelogram ABCD. O is the origin and OE is perpendicular to AB.



4.1.1 Calculate the length of DC. (leave your answer in surd form) (2)

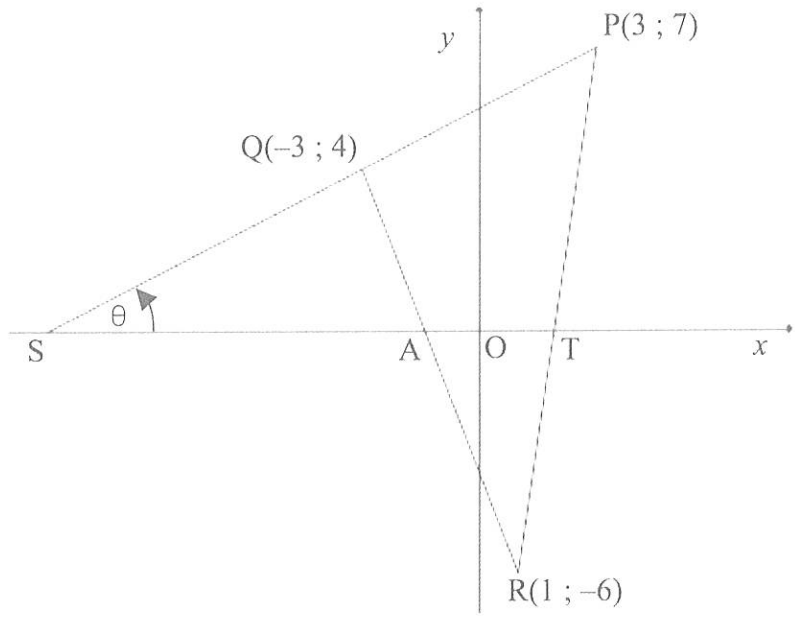
4.1.2 Hence, write down the length of AB. (1)

4.1.3 Calculate the values of p and q. (2)

4.1.4 Determine the equation of OE. (4)

4.1.5 Calculate the coordinates of E. (4)

4.2 In the diagram P (3 ; 7), Q (–3 ; 4) and R (1 ; –6) are the vertices of a triangle. PQ is produced to cut the x-axis at S. PR cuts the x-axis at T. QR cuts the x-axis at A.



4.2.1 Calculate θ the angle of inclination of the line PS. (3)

4.2.2 Calculate the size of $R\hat{Q}S$. (4)

4.2.3 N is the point (3 ; –11). Are the points N, R and Q collinear? Justify your answer by means of calculations. (4)
[24]

TOTAL MARKS: 75

Education

KwaZulu-Natal Department of Education
REPUBLIC OF SOUTH AFRICA

MATHEMATICS

COMMON TEST

MARCH 2018

MARKING GUIDELINE

NATIONAL SENIOR CERTIFICATE

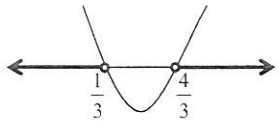
GRADE 11

MARKS: 75

This marking guideline consists of 8 pages.

QUESTION 1

1.1.1	$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ $x = \frac{-10 \pm \sqrt{(10)^2 - 4(3)(-5)}}{2(3)}$ $x = 0,44 \quad or \quad x = -3,77$	✓ quadratic formula ✓ substitution ✓ ✓ answers (4)
1.1.2	$3^{x+1} - 3^{x+3} = -\frac{8}{27}$ $3^x(3^1 - 3^3) = -\frac{8}{27}$ $3^x = -\frac{8}{27} \div -24$ $3^x = \frac{1}{81}$ $3^x = 3^{-4}$ $x = -4$	✓ factorising LHS ✓ dividing by –24 ✓ simplifying RHS ✓ answer (4)
1.1.3	$5 - x = \sqrt{4x+1}$ $(5-x)^2 = \left(\sqrt{4x+1}\right)^2$ $25 - 10x + x^2 = 4x + 1$ $x^2 - 14x + 24 = 0$ $(x-12)(x-2) = 0$ $x \neq 12$ or $x = 2$	✓ squaring both sides ✓ standard form ✓ factors ✓ both answers ✓ rejecting $x = 12$ (5)
1.2	$y + 7 = 2x$ $y = 2x - 7$ $x^2 - x(2x-7) + 3(2x-7)^2 = 15$ $x^2 - 2x^2 + 7x + 3(4x^2 - 28x + 49) = 15$ $x^2 - 2x^2 + 7x + 12x^2 - 84x + 147 = 15$ $11x^2 - 77x + 132 = 0$ $x^2 - 7x + 12 = 0$ $(x-4)(x-3) = 0$ $x = 4 \quad or \quad x = 3$ $y = 2(4) - 7$ $y = 1$ $y = 2(3) - 7$ $y = -1$	✓ rewriting y in terms of x ✓ substitution ✓ standard form ✓ factors ✓ both answers for x ✓ both answers for y (6)

1.3.1	$9x^2-15x+4>0$ $(3x-4)(3x-1)>0$ $CVs: x=\frac{4}{3}$ *or* $x=\frac{1}{3}$  $x<\frac{1}{3}$ *or* $x>\frac{4}{3}$	✓ factors ✓✓ $x<\frac{1}{3}$ or $x>\frac{4}{3}$ (3)
1.3.2	$9x^2-15x+4=3$ $9x^2-15x+7=0$ $x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$ $=\frac{15\pm\sqrt{(-15)^2-4(9)(7)}}{2(9)}$ $=\frac{15\pm\sqrt{-27}}{18}$ Because $\sqrt{-27}$ is not real, both roots are not real. **OR** $\Delta=b^2-4ac$ $=(-15)^2-4(9)(7)$ $=-27$ Because $\Delta<0$, the equation has no real roots.	✓ standard form ✓ substituting in Δ or formula for roots of quadratic equation ✓ $x=\frac{15\pm\sqrt{-27}}{18}$ or $\Delta=-27$ ✓ conclusion (4)
		[26]

QUESTION 2

2.1	$\frac{12^{n+1}.27^{n-2}}{18^{2n-1}.\sqrt{9^{-3}}}+8^0$ $=\frac{\left(3.2^2\right)^{n+1}.\left(3^3\right)^{n-2}}{\left(2.3^2\right)^{2n-1}.\left(3^2\right)^{-\frac{3}{2}}}+1$ $=\frac{3^{n+1}.2^{2n+2}.3^{3n-6}}{2^{2n-1}.3^{4n-2}.3^{-3}}+1$ $=2^{(2n+2)-(2n-1)}.3^{(n+1)+(3n-6)-(4n-2)-(-3)}+1$ $=2^3.3^0+1$ $=8+1$ $=9$	✓ $8^0=1$ ✓ writing as prime bases ✓ converting surd to exponent ✓ simplification using laws ✓ answer (5)
2.2	$\sqrt[3]{16}\times\sqrt[3]{625}\times\sqrt{10}$ $=\sqrt[3]{2^4}\times\sqrt[3]{5^4}\times\sqrt{10}$ $=(2)^{\frac{4}{3}}\times(5)^{\frac{4}{3}}\times(10)^{\frac{1}{2}}$ $=(10)^{\frac{4}{3}}\times(10)^{\frac{1}{2}}$ $=(10)^{\frac{11}{6}}$ $=10^1\times(10)^{\frac{5}{6}}$ $=10y$	✓ writing with bases of 2 and 5 ✓ surd to exponential form with base 10 ✓ simplification ✓ answer (4)
		[9]

QUESTION 3

3.1.1	10; 0	✓✓ answers (2)
3.1.2	64 42 24 10 -22 -18 -14 4 4 $2a = 4$ $a = 2$ $3(2) + b = -22$ $b = -28$ $2 + (-28) + c = 64$ $c = 90$ $T_n = 2n^2 - 28n + 90$	✓ value of a ✓ value of b ✓ value of c ✓ answer (4)
3.1.3	$T_{20} = 2(20)^2 - 28(20) + 90$ $= 330$	✓ substitution of 20 into T_n ✓ answer (2)
3.1.4	The sequence of first differences form the linear pattern: $-22 ; -18 ; -14 ; \ldots$ The general term for the sequence of first differences is : $T_n = 4n - 26$	✓ $4n$ ✓ -26 (2)
3.1.5	$4n - 26 = 174$ $4n = 200$ $n = 50$ ∴ the difference between T_{50} and T_{51} of the quadratic sequence is 174. **OR** $T_{n+1} - T_n = 174$ $2(n+1)^2 - 28(n+1) + 90 - (2n^2 - 28n + 90) = 174$ $2n^2 + 4n + 2 - 28n - 28 + 90 - 2n^2 + 28n - 90 = 174$ $4n - 26 = 174$ $4n = 200$ $n = 50$ ∴ the difference between T_{50} and T_{51} of the quadratic sequence is 174.	✓ equating T_n to 174 ✓answer (2) **OR** ✓ substituting into $T_{n+1} - T_n = 174$ ✓answer (2)

3.2	p 11 21 $6p$ $11 - p$ 10 $6p - 21$ $10 - (11-p)$ $6p - 21 - 10$ $10 - (11 - p) = 6p - 21 - 10$ $10 - 11 + p = 6p - 31$ $p - 6p = -31 + 1$ $-5p = -30$ $p = 6$	✓ calculating first differences ✓ calculating second differences ✓ equating second differences ✓answer (4)
		[16]

QUESTION 4

4.1.1	DC $= \sqrt{(3-(-1))^2+(0-(-2))^2}$ $= \sqrt{20}$	✓ substitution into distance formula ✓ answer (2)
4.1.2	AB $= \sqrt{20}$	✓ correct answer (1)
4.1.3	By inspection: $p=-2$ and $q=2$ (D is 4 units to left of C; therefore A also 4 units to left of B. D is 2 units below C; therefore A also 2 units below B.) **OR** Midpoint of BD is $M\left(\frac{2-1}{2};\frac{4-2}{2}\right)=M\left(\frac{1}{2};1\right)$ $\frac{1}{2}=\frac{p+3}{2}$ *and* $1=\frac{q+0}{2}$ $p=-2$ *and* $q=2$	✓ value of p ✓ value of q (2) **OR** ✓ value of p ✓ value of q (2)
4.1.4	$m_{AB}=\frac{4-2}{2+2}=\frac{1}{2}$ $\therefore m_{OE}=-2$ The equation of OE is $y=-2x$	✓ substitution ✓ gradient of AB ✓ gradient of OE ✓ answer (4)
4.1.5	Equation of AB: Substitute (2 ; 4) in $y=\frac{1}{2}x+c$: $4=\frac{1}{2}(2)+c$ $c=3$ $y=\frac{1}{2}x+3$ Equate equations of AB and OE: $-2x=\frac{1}{2}x+3$ $-\frac{5}{2}x=3$ $x=-\frac{6}{5}$ $y=\frac{1}{2}\left(-\frac{6}{5}\right)+3=\frac{12}{5}$ $E\left(-\frac{6}{5};\frac{12}{5}\right)$	✓ equation of AB ✓ equating equations of AB and OE ✓ value of x-coordinate ✓ value of y-coordinate (4)

4.2.1	$m_{PS}=\frac{7-4}{3+3}=\frac{1}{2}$ $\tan A\hat{S}Q=m_{PS}=\frac{1}{2}$ $A\hat{S}Q=26{,}57^\circ$	✓ gradient of PS ✓ $\tan A\hat{S}Q=m_{PS}$ ✓ answer (3)
4.2.2	$m_{RQ}=\frac{-6-4}{1+3}=-\frac{5}{2}$ $\tan T\hat{A}Q=m_{RQ}$ $T\hat{A}Q=180^\circ-68{,}20^\circ$ $=111{,}80^\circ$ $R\hat{Q}S=111{,}80^\circ-68{,}20^\circ$ $=85{,}23^\circ$	✓ gradient of RQ ✓ size of $T\hat{A}Q$ ✓ subtracting ✓ answer (4)
4.2.3	$m_{NR}=\frac{-6-(-11)}{1-3}$ $=-\frac{5}{2}$ $m_{RQ}=-\frac{5}{2}$ Because $m_{RQ}=m_{NR}$, N, R and Q are collinear. Take note: Alternatively the gradients of NQ and NR may also be shown to be equal. Or: NQ and QR.	✓ substitution ✓ value of gradient ✓ equal gradients ✓ concluding (4)
		[24]

TOTAL: 75