



**GAUTENG PROVINCE**  
EDUCATION  
REPUBLIC OF SOUTH AFRICA

**PROVINSIALE EKSAMEN**  
**NOVEMBER 2021**  
**GRAAD 11**  
**NASIENRIGLYNE**

**WISKUNDE (VRAESTEL 2)**

**17 bladsye**

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**INSTRUKSIES EN INLIGTING:**

A – Akkuraatheid

C.A. – Kontinue Akkuraatheid (volg op)

S – Bewering

R – Rede

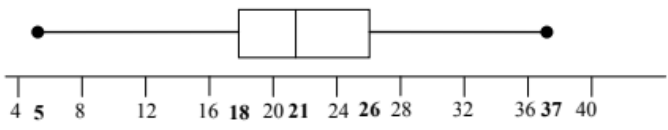
S en R – Bewering en Rede

**NOTA:**

- Indien 'n kandidaat 'n vraag TWEE keer beantwoord het, merk slegs die EERSTE poging.
- Indien 'n kandidaat 'n antwoord doodgetrek het, en dit nie weer gedoen het nie, merk die doodgetrekte antwoord.
- Deurlopende akkuraatheid moet op ALLE aspekte van die nasienriglyne toegepas word.
- Dit is onaanvaarbaar om waardes/antwoorde aan te neem om probleme op te los.

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## VRAAG 1

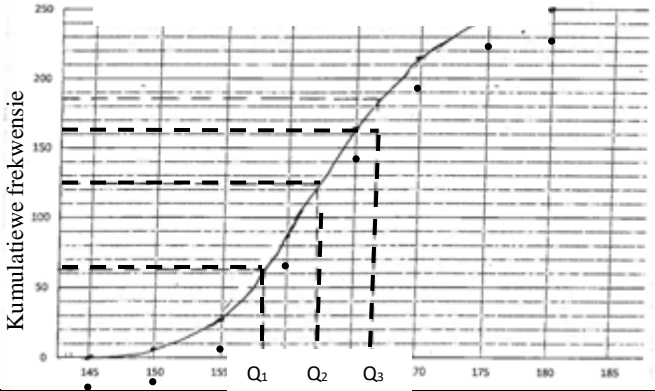
1.1	Gemiddeld $= \frac{\sum_{i=1}^n x_i}{n} = \frac{408}{19} = 21,47$	✓ antwoord	(1)
1.2	Standaardafwyking = 7,81	✓✓ antwoord	(2)
1.3	Die limiete vir die een standaardafwyking is $(\bar{x} - 1\sigma ; \bar{x} + 1\sigma)$ $= (21,47 - 7,81; 21,47 + 7,81) = (13,66 ; 29,28)$ $\therefore$ 13 mense lê binne 1 standaardafwyking van die gemiddeld af.	✓ interval ✓ 13 mense	(2)
1.4	5 12 13 15 18 18 18 19 20 21 21 22 23 23 26 29 33 35 37 IKVW = 26 – 18 = 8	✓ $Q_1 = 18$ ✓ $Q_3 = 26$ ✓ IKVW = 8	(3)
1.5		✓✓ houer / mond ✓ punt / snor	(3)
1.6	Daar is 'n merkbare verskil tussen die laagste waarde (5) en die volgende laagste waarde (12) terwyl die verskille tussen al die ander data waardes hoogstens binne 3 waardes is $\therefore$ 5 is 'n uitskieter	✓ rede ✓ 5 is 'n uitskieter	(2)
			[13]

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## VRAAG 2

2.1	<table><tr><th>Lengte (cm)</th><th>Aantal leerders (f)</th><th>Kumulatiewe frekwensie</th></tr><tr><td><math>145 \leq x &lt; 150</math></td><td>6</td><td>6</td></tr><tr><td><math>150 \leq x &lt; 155</math></td><td>23</td><td>29</td></tr><tr><td><math>155 \leq x &lt; 160</math></td><td>60</td><td><b>89</b></td></tr><tr><td><math>160 \leq x &lt; 165</math></td><td>74</td><td>163</td></tr><tr><td><math>165 \leq x &lt; 170</math></td><td>52</td><td><b>215</b></td></tr><tr><td><math>170 \leq x &lt; 175</math></td><td>32</td><td><b>247</b></td></tr><tr><td><math>175 \leq x &lt; 180</math></td><td>3</td><td>250</td></tr><tr><td><b>Totaal</b></td><td><b>250</b></td><td></td></tr></table>	Lengte (cm)	Aantal leerders (f)	Kumulatiewe frekwensie	$145 \leq x < 150$	6	6	$150 \leq x < 155$	23	29	$155 \leq x < 160$	60	<b>89</b>	$160 \leq x < 165$	74	163	$165 \leq x < 170$	52	<b>215</b>	$170 \leq x < 175$	32	<b>247</b>	$175 \leq x < 180$	3	250	<b>Totaal</b>	<b>250</b>		<div>✓ 89</div> <div>✓ 215</div> <div>✓ 247</div>	(3)
Lengte (cm)	Aantal leerders (f)	Kumulatiewe frekwensie																												
$145 \leq x < 150$	6	6																												
$150 \leq x < 155$	23	29																												
$155 \leq x < 160$	60	<b>89</b>																												
$160 \leq x < 165$	74	163																												
$165 \leq x < 170$	52	<b>215</b>																												
$170 \leq x < 175$	32	<b>247</b>																												
$175 \leq x < 180$	3	250																												
<b>Totaal</b>	<b>250</b>																													
2.2	<div>Ogief wat die verspreiding van punte toon</div> 	<div>✓ Vorm</div> <div>✓ grond op <math>x</math>- as</div> <div>✓ plot die punte</div>	(3)																											
2.3	<div><math>Q_2 \approx 162</math></div> <div><math>Q_1 \approx 158</math></div> <div><math>Q_3 \approx 167</math></div> <div><math>\therefore \text{IKVW} \approx 162</math></div>	<div>✓</div> <div>✓</div> <div>✓</div> <div>✓</div>	(4)																											
2.4	$165 \leq x < 170$	✓ antwoord	(1)																											
2.5	$\frac{35}{250} = 14\%$	✓ antwoord	(1)																											
2.6	$\sigma = 6,43$	✓✓ antwoord	(2)																											
			[14]																											

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## VRAAG 3

3.1	$E\left(\frac{3+12}{2}; \frac{0+3}{2}\right)$ $=\left(7\frac{1}{2}; 1\frac{1}{2}\right)$	✓ substitusie in middelpuntformule ✓ Antwoord	(2)
3.2	$M_{BC} = \frac{3-0}{12-3}$ $= \frac{1}{3}$	✓ substitusie in gradiëntformule ✓ Antwoord	(2)
3.3	$\tan \theta = m_{BC} = \frac{1}{3}$ $\theta = \tan^{-1}\left(\frac{1}{3}\right) = 18,43^\circ$	✓ $\tan \theta = m_{BC}$ ✓ antwoord	(2)
3.4	$M_{AD} = M_{BC} = \frac{1}{3}$ $AD \parallel BC$ , gelyke gradiënte $M_{AB} = \frac{6-0}{1-3} = -3$ $\therefore M_{AD} \times M_{AB} = \frac{1}{3} \times -3 = -1$ $\therefore AD \perp AB$	✓ $M_{AD} = \frac{1}{3}$ ✓ $M_{AB} = -3$ ✓ $M_{AD} \times M_{AB} = -1$	(3)
3.5	Inklynasie van nuwe lyn $= 45^\circ + 18,43^\circ = 63,43^\circ$ $\therefore \tan 63,43^\circ = 2 = m_{\text{lyn}}$ $\therefore y - 6 = 2(x - 1)$ $y = 2x + 4$	✓ $18,43^\circ$ ✓ $63,43^\circ$ ✓ $m = 2$ ✓ vergelyking	(4)
			[13]

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## VRAAG 4

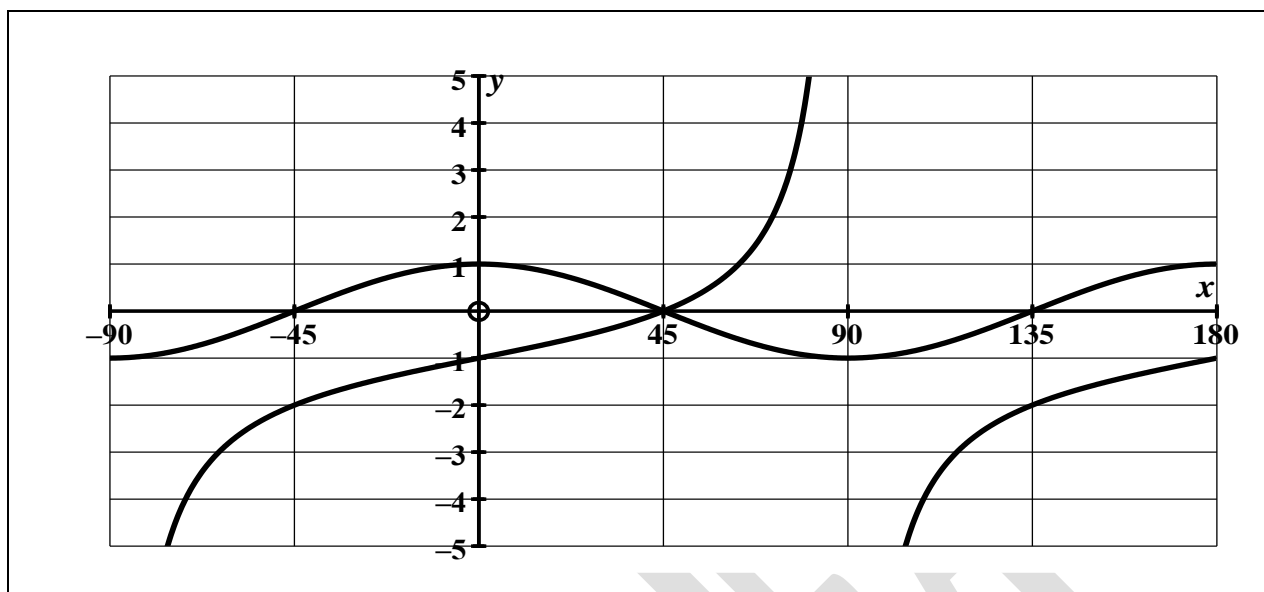
4.1	$M_{QP} = M_{OS} = 6$ $QP \parallel OS$ , gelyke gradiënte $y - 17 = 6(x + 3)$ $y = 6x + 35$	$\checkmark M_{QP} = 6$ $\checkmark$ vergelyking	(2)
4.2	$6x + 35 = -x$ $7x = -35$ $x = -5$ $y = -(-5) - 5$ $\therefore Q(-5;5)$	$\checkmark$ opstel van vergelyking $\checkmark x = -5$ $\checkmark$ koördinate van Q	(3)
4.3	$OQ^2 = (-5 - 0)^2 + (5 - 0)^2$ $= 50$ $OQ = \sqrt{50} = 5\sqrt{2}$ eenhede	$\checkmark$ substitusie in afstandsformule $\checkmark 5\sqrt{2}$	(2)
4.4	$M_{OS} = 6$ $\therefore$ inklinasie van OS is $-\tan^{-1}(4) = 80,54^\circ$ $M_{OQ} = -1$ $\therefore$ inklinasie van QO is $180^\circ - \tan^{-1}(1) = 135^\circ$ $\alpha = 135^\circ - 80,54^\circ \dots$ $= 54,46^\circ$	$\checkmark 80,54^\circ$ $\checkmark 135^\circ$ $\checkmark 54,46^\circ$	(3)
4.5	$QS^2 = OS^2 + OQ^2 - 2OS \cdot OQ \cdot \cos \alpha$ $= 148 + 50 - 2(\sqrt{148})(\sqrt{50} \cdot \cos 54,46^\circ)$ $QS = 9,90$ eenhede	$\checkmark$ korrekte gebruik van cosreël $\checkmark$ substitusie in formule $\checkmark 9,90$	(3)
			[13]

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## VRAAG 5

5.1	5.1.1	$\sin 28^\circ = \cos 62^\circ$ $= k$	✓✓ Antwoord	(2)
	5.1.2	$\cos 242^\circ = -\cos 62^\circ$ $= -k$	✓✓ Antwoord	(2)
5.2		$= -\tan x \cdot \cos x = \frac{\sin x}{\cos x} \times \frac{\cos x}{\sin x}$ $-\sin x$ $= 1$	✓ $\tan x \cdot \cos x$ ✓ $-\sin x$ ✓ $\frac{\sin x}{\cos x}$ ✓ 1	(4)
5.3		$\frac{\cos 50^\circ - \cos \theta}{\sin 40^\circ + \cos \theta} = \frac{-(\cos \theta + \cos 50^\circ)}{\cos \theta + \sin 40^\circ}$ $\cos 50^\circ = \sin 40^\circ$ $= -1$	✓✓ $\frac{\cos 50^\circ - \cos \theta}{\sin 40^\circ + \cos \theta}$ ✓✓ $-(\cos \theta + \cos 50^\circ)$ ✓ antwoord	(5)
5.4		$= \sqrt{2^{2\sin 20^\circ} \cdot 2^{3 \cdot -\tan 45^\circ}}$ $= \sqrt{2^{2 \cdot \frac{1}{2}} \cdot 2^{-3}}$ $= \sqrt{\frac{1}{4}}$ $= \frac{1}{2}$	✓ $\sqrt{2^{2\sin 20^\circ} \cdot 2^{3 \cdot -\tan 45^\circ}}$ ✓ $\sqrt{2^{2 \cdot \frac{1}{2}} \cdot 2^{-3}}$ ✓ $\frac{1}{2}$	(3)
5.5	5.5.1	$\sin^2 x = \frac{3}{4}$ $\sin x = \pm \frac{\sqrt{3}}{2}$	✓ antwoord	(1)
	5.5.2	Verwysingshoek = $60^\circ$ $60^\circ; 120^\circ; 240^\circ$	✓✓✓ antwoorde	(3)
				[20]

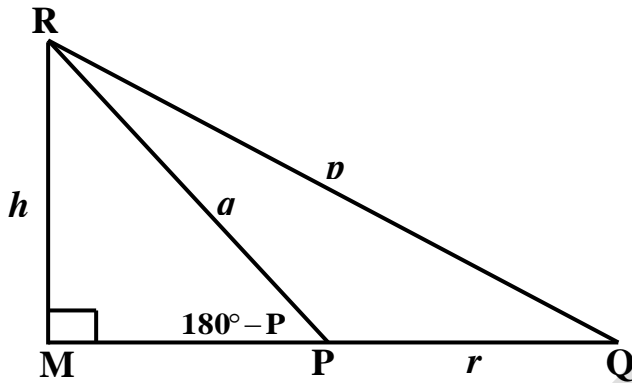
## VRAAG 6



6.1	$f(x) = \tan x - 1$  $g(x) = \cos 2x$		✓ asympote ✓ afsnitte met asse ✓ vorm ✓ periode ✓ vorm met oop punt/kol by $x = -90^\circ$	(5)
6.2	6.2.1	$\cos 2x + 1 = \tan x$ $\therefore \cos 2x = \tan x - 1$ $\therefore x = 45^\circ$	✓ $x = 45^\circ$	(1)
	6.2.2	$2\cos^2 x = 1$ $\therefore 2\cos^2 x - 1 = 0$ $\therefore \cos 2x = 0$ $\therefore x = \pm 45^\circ$ or $x = 135^\circ$	✓ $\cos 2x = 0$ ✓ oplossings	(2)
	6.2.3	$-90^\circ < x < -45^\circ$ of $90^\circ < x < 135^\circ$	✓ $-90^\circ < x < -45^\circ$ ✓ $90^\circ < x < 135^\circ$	(2)
6.3	$h(x) = \cos 2(x - 30^\circ)$		✓ $h(x) = \cos 2(x - 30^\circ)$	(1)
				[11]

## VRAAG 7

7.1



Konstruksie: Trek  $RM \perp PQ$  of  $QP$  verleng

$$RM = p \sin Q$$

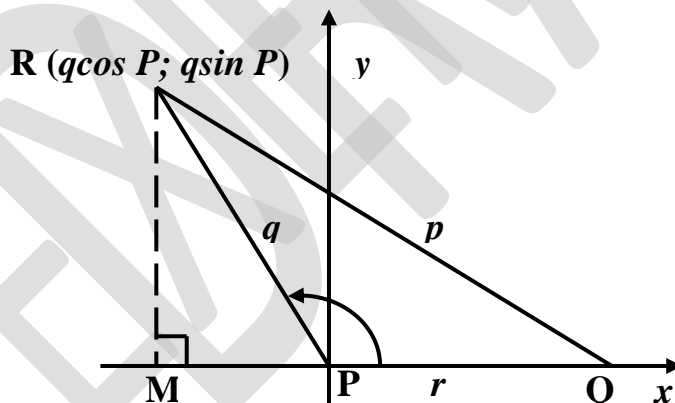
$$\begin{aligned} CM &= q \sin (180^\circ - P) \\ &= q \sin P \end{aligned}$$

$$\therefore p \sin Q = q \sin P$$

Deel deur  $\sin P \cdot \sin Q$  gee dan

$$\frac{p}{\sin P} = \frac{q}{\sin Q}$$

OF



P is in standaardposisie soos in die diagram getoon.

Konstruksie: Trek  $RM \perp PQ$  of  $QP$  verleng

$$RM = q \sin P = p \sin Q$$

Deel deur  $\sin P \cdot \sin Q$  dan kry ons

$$\frac{p}{\sin P} = \frac{q}{\sin Q}$$

✓ konstruksie

$$✓ RM = p \sin Q$$

$$✓ RM = q \sin (180^\circ - P)$$

$$✓ \therefore p \sin Q = q \sin P$$

✓ Deel deur  $\sin P \cdot \sin Q$

OF

$$✓ R(q \cos P; q \sin P)$$

✓ konstruksie

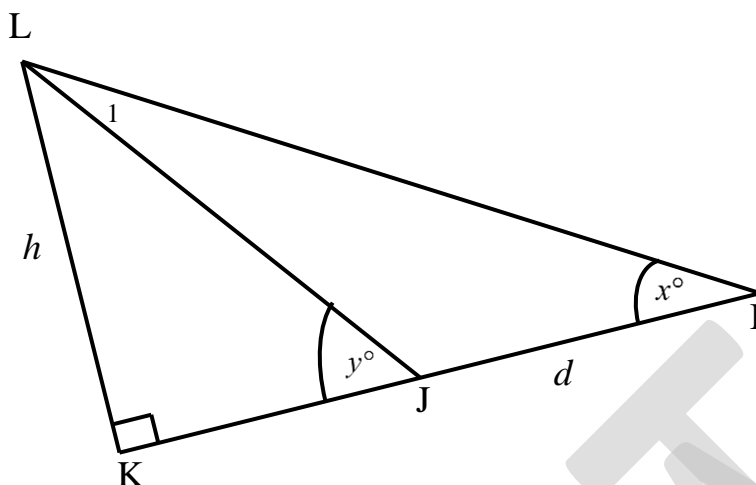
$$✓ RM = q \sin P$$

$$✓ RM = p \sin Q$$

✓ Deel deur  $\sin P \cdot \sin Q$

(5)

7.2



7.2

7.2.1

In  $\Delta LKJ$ :

$$\frac{h}{LJ} = \sin y$$

$$\therefore h = LJ \sin y \quad \text{--- (1)}$$

In  $\Delta DAB$ :

$$\frac{LJ}{\sin x} = \frac{d}{\sin L_1}$$

$$\text{maar } \hat{L}_1 = y - x$$

$$\therefore LJ = \frac{d \sin x}{\sin(y-x)} \quad \text{--- (2)}$$

sub (2) in (1):

$$h = \frac{d \sin x \sin y}{\sin(y-x)}$$

✓  $h$  substitusie in formule

✓ sinreël

✓  $\hat{L}_1 = y - x$ 

✓ LJ

✓ substitusie

(5)

7.2.2

$$h = \frac{70 \cdot \sin 12^\circ \cdot \sin 30^\circ}{\sin 18^\circ}$$

$$= 23,55 \text{ m}$$

✓ substitusie

✓ antwoord

(2)

[12]

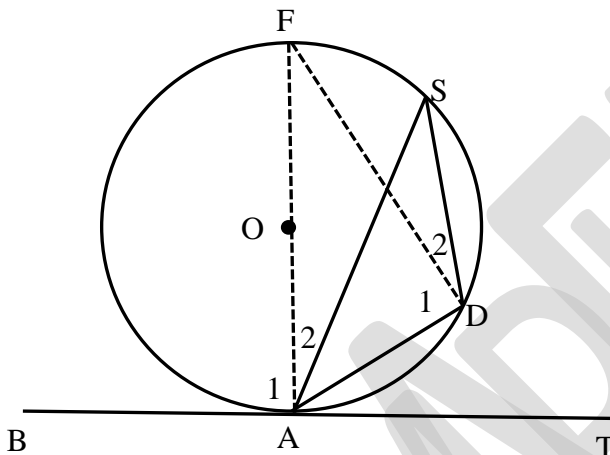
## VRAAG 8

8.1	<p>Volume van silinder:</p> $V_1 = \pi r^2 h$ $= \pi (12)^2 \cdot (19)$ $= 2736 \pi m^3$ <p>Volume van hemisfeer (halwe sfeer):</p> $V_2 = \frac{1}{2} \left( \frac{4}{3} \pi r^3 \right)$ $\text{Totale Volume} = V_1 + V_2$ $= \frac{1}{2} \left[ \left( \frac{4}{3} \pi r^3 \right) (12)^3 \right]$ $= 1152 \pi m^3$ $\text{Totale Volume} = V_1 + V_2$ $= 2736\pi + 1152\pi$ $= 3888\pi m^3$	<p>✓ Substitusie</p> <p>✓ antwoord</p> <p>✓ <math>\frac{1}{2}</math></p> <p>✓ Substitusie</p> <p>✓ antwoord</p> <p>✓ Totale Volume in <math>\pi</math></p> <p>(5 uit 6 indien numeriese waarde van <math>\pi</math> gebruik is )</p>	(6)
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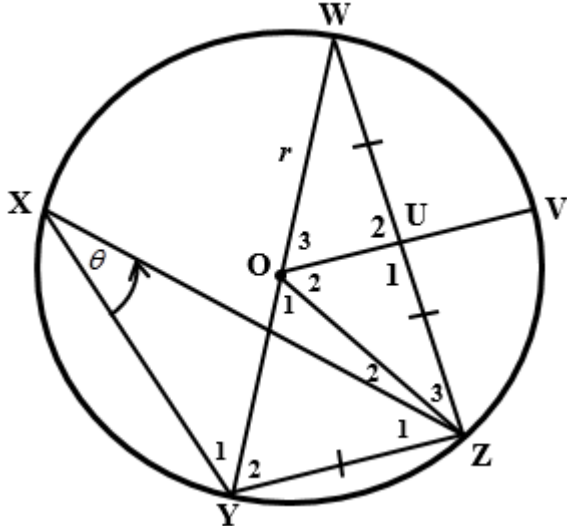
8.2	<p>Buiteoppervlak van silinder:</p> $A_1 = 2\pi r^2 + 2\pi rh$ $= 2\pi(12)^2 + 2\pi(12)(19)$ $= 2337,34 \text{ m}^2$ <p>Buiteoppervlak van hemisfeer (halwe sfeer):</p> $A_2 = \frac{1}{2}(4\pi r^2)$ $= \frac{1}{2}[(4\pi r^2)(12)^2]$ $= 452,39 \text{ m}^2$ <p><i>Totale Buiteoppervlak</i> <math>= A_1 + A_2</math></p> $= 2337,34 + 452,39$ $= 685,73 \text{ m}^2 \approx 686 \text{ m}^2$	<p>✓ Substitusie</p> <p>✓ antwoord</p> <p>✓ Substitusie</p> <p>✓ antwoord</p> <p>✓ Totale buiteoppervlak</p> <p>✓ antwoord, korrek afgerond</p>	<p>(6)</p> <p><b>[12]</b></p>
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## VRAAG 9

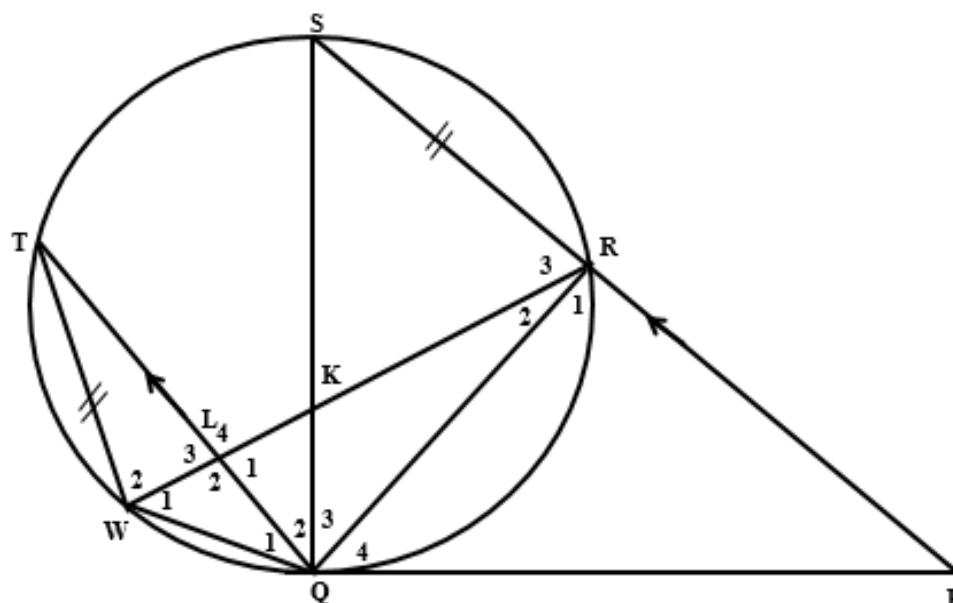
9.1	9.1.1	Die teenoorgestelde hoeke van 'n sikliese vierhoek is <i>supplementêr</i> .	✓ antwoord	(1)	
	9.1.2	Die hoek tussen die raaklyn en koord is <i>gelyk aan die hoek in die teengestelde segment</i> .	✓ antwoord	(1)	
9.2	<div>  <p>Konstrueer AF(middellyn) en FD (koord)</p> <math display="block">\hat{D}_1 = 90^\circ \quad (\angle \text{ in 'n halwe sirkel } )</math> <math display="block">\hat{A}_1 = 90^\circ \quad (rad \perp raaklyn )</math> <math display="block">\hat{A}_2 = \hat{D}_2 \quad (\angle^e \text{ in dieselfde sirkelseg. } )</math> <math display="block">\hat{A}_1 + \hat{A}_2 = \hat{D}_1 + \hat{D}_2</math> <math display="block">\therefore \hat{BAS} = \hat{D}</math> </div>			<div> <div>✓ Konstruksie</div> <div>✓ Bewering ✓ Rede</div> <div>✓ Bewering en Rede</div> <div>✓ Bewering en Rede</div> <div>✓ Gevolgtrekking</div> </div>	(6)

9.3				
9.3.1	$\hat{PQE} = \hat{H}_3 = 70^\circ$ (raaklyn – koordstelling) $\therefore \hat{Q}_1 = 70^\circ$	✓ raaklyn – koordstelling ✓ $\hat{Q}_1 = 70^\circ$	(2)	
9.3.2	$\hat{E}_1 = 70^\circ$ (verw. $\angle e, EF \parallel QH$ ) $\hat{G} = 180^\circ - \hat{E}_1$ (teenoorst. $\angle e$ kvh EFGH) $= 180^\circ - 70^\circ$ $\hat{G} = 110^\circ$	✓ $\hat{E}_1 = 70^\circ$ ✓ verw. $\angle e =$ ✓ $\hat{G} = 180^\circ - \hat{E}_1$ ✓ <b>R</b>	(4)	
9.3.3	$\hat{Q}_2 = 180^\circ - (\hat{Q}_1 + \hat{Q}_3)$ ( $\angle e$ op rtlyn PQR) $= 180^\circ - (70^\circ + 30^\circ)$ $= 80^\circ$ $\hat{EFH} = 180^\circ - \hat{Q}_2$ (teenoorst. $\angle e$ kvh EFHQ) $\hat{F}_1 = 180^\circ - 80^\circ$ $= 100^\circ$	✓ $180^\circ - (\hat{Q}_1 + \hat{Q}_3)$ ✓ $\hat{Q}_2 = 80^\circ$ ✓ teenoorst $\angle e$ van kvh ✓ $\hat{F}_1 = 100^\circ$	(4)	
			[18]	

## VRAAG 10

10.1				
10.1.1	$\angle O_1 = 2\theta$ (middelpunt $\angle = 2 \times$ omtrek $\angle$ )	✓ bewering ✓ rede	(2)	
10.1.2	$\angle W = \theta$ ( $\angle$ e in dieselfde sirkel segment)	✓ bewering ✓ rede	(2)	
10.1.3	$\angle Z = 90^\circ$ ( $\angle$ in 'n halwe sirkel) $\angle Y_2 = 90 - \theta$ (binne $\angle$ e van 'n $\Delta$ )	✓ bewering & rede ✓ bewering & rede	(2)	
10.2	$WY^2 = WZ^2 + YZ^2$ (Pythagoras stelling) $YZ = WU = UZ$ (gegee) $\therefore OW = OY = r$ (gegee) $\Rightarrow YZ = \sqrt{\frac{4r^2}{5}} = \frac{4r}{\sqrt{5}}$	✓ bewering & rede ✓ bewering & rede ✓ bewering & rede $YZ = \sqrt{\frac{4r^2}{5}}$ <b>OF</b> $YZ = \frac{4r}{\sqrt{5}}$ ✓	(4)	
10.3	$OW = OV = r = OU + OV$ (radiusse) $r^2 = 3^2 + 4^2$ (Pythagoras stelling) $r = 5 \text{ cm}$ $\therefore UV = 5 - 2 = 3 \text{ cm}$	✓ bewering ✓ rede $r^2 = 3^2 + 4^2$ ✓ $r = 5 \text{ cm}$ ✓ ✓ antwoord	(2)	
				[12]

## VRAAG 11



11.1	$\hat{S} = \hat{Q}_2 = x$ (verw. hoeke $TQ\backslash PS$ ) $\hat{S} = \hat{W}_1 = x$ ( $\angle^e$ in dieselfde sirkel segment) $\hat{S} = \hat{Q}_4 = x$ ( $\angle$ tussen raaklyn en koord)	✓ bewering & rede ✓ bewering & rede ✓ bewering & rede	(3)
11.2	11.2.1 $\hat{R}_1 = \hat{S} + \hat{Q}_3$ (buite $\angle$ van $\Delta$ ) $\hat{L}_3 = \hat{W}_1 + \hat{Q}_1$ (buite $\angle$ van $\Delta$ ) $\hat{S} = \hat{W}_1 = x$ ( $\angle^e$ in dieselfde sirkel segment) $\hat{Q}_1 = \hat{Q}_3$ (= koorde onderspan = $\angle$ ) $\therefore \hat{R}_1 = \hat{L}_3$	✓ bewering & rede ✓ bewering & rede ✓ bewering ✓ gevolgtrekking	(4)

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11.2.2	$\overset{\wedge}{W}_1 = \overset{\wedge}{Q}_4 \quad (\text{raaklyn koordstelling})$ $\overset{\wedge}{Q}_1 = \overset{\wedge}{Q}_3 \quad (= \text{koorde onderspan} = \angle)$ $\Rightarrow \overset{\wedge}{Q}_4 + \overset{\wedge}{Q}_3 = \overset{\wedge}{W}_1 + \overset{\wedge}{Q}_1$ <p>netso <math>\overset{\wedge}{L}_3 = \overset{\wedge}{W}_1 + \overset{\wedge}{Q}_1 \quad (\text{buite } \angle \text{ van } \Delta \quad)</math></p> $= \overset{\wedge}{R}_3 \quad (\text{ooreenk. } \angle e, QT // RS)$ $\therefore \overset{\wedge}{R}_3 = \overset{\wedge}{Q}_4 + \overset{\wedge}{Q}_3$ <p>PRKQ is 'n koordevierkant. (buite <math>\angle =</math> teenoorst. binne <math>\angle</math>)</p> <p><b>Alternatiewelik:</b></p> $\overset{\wedge}{Q}_1 + \overset{\wedge}{Q}_3 = \overset{\wedge}{Q}_3 + \overset{\wedge}{S}$ <p><b>OF</b></p> $\overset{\wedge}{W}_1 = \overset{\wedge}{Q}_4 \quad (\text{raaklyn koordstelling})$ $\overset{\wedge}{Q}_1 = \overset{\wedge}{Q}_3 \quad (= \text{koorde onderspan} = \angle)$ $\therefore \overset{\wedge}{Q}_4 + \overset{\wedge}{Q}_3 = \overset{\wedge}{W}_1 + \overset{\wedge}{Q}_1$	<p>✓ bewering ✓ rede</p> <p>✓ bewering &amp; rede</p> <p>✓ bewering &amp; rede</p> <p>✓ bewering &amp; rede</p>	(5)
			[12]
TOTAAL:			150