



**GAUTENG PROVINCE**

EDUCATION  
REPUBLIC OF SOUTH AFRICA

**PROVINCIAL EXAMINATION**  
**NOVEMBER 2021**  
**GRADE 11**  
**MARKING GUIDELINES**

**MATHEMATICS (PAPER 2)**

**17 pages**

**INSTRUCTIONS AND INFORMATION:**

A – Accuracy

C.A. – Continued Accuracy (follow up)

S – Statement

R – Reason

S and R – Statement and Reason

**NOTE:**

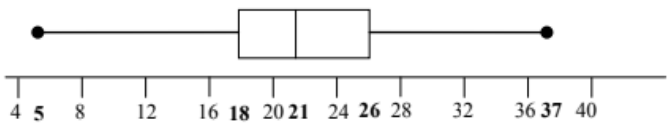
- If a candidate answered a question TWICE, mark only the FIRST attempt.
- If a candidate crossed out an answer and did not redo it, mark the crossed-out answer.
- Consistent accuracy applies to ALL aspects of the marking guidelines.
- It is unacceptable to adopt values/answers in order to solve a problem.

## MARKING GUIDELINES

MATHEMATICS  
(Paper 2)

GRADE 11

## QUESTION 1

1.1	Mean = $\frac{\sum_{i=1}^n x_i}{n} = \frac{408}{19} = 21,47$	✓ answer	(1)
1.2	Standard deviation = 7,81	✓✓ answer	(2)
1.3	The one standard deviation limits are $(\bar{x} - 1\sigma ; \bar{x} + 1\sigma)$ $= (21,47 - 7,81; 21,47 + 7,81) = (13,66 ; 29,28)$ $\therefore$ 13 people lie within 1 standard deviation of the mean	✓ interval ✓ 13 people	(2)
1.4	5 12 13 15 18 18 18 19 20 21 21 22 23 23 26 29 33 35 37 IQR = 26 – 18 = 8	✓ $Q_1 = 18$ ✓ $Q_3 = 26$ ✓ IQR = 8	(3)
1.5		✓✓ box ✓ whiskers	(3)
1.6	There is a marked difference between the lowest value (5) and the next lowest value (12) whilst the differences between all other data points are within at most 3 values > $\therefore$ 5 is an outlier	✓ reason ✓ 5 is an outlier	(2)
			<b>[13]</b>

## MARKING GUIDELINES

MATHEMATICS  
(Paper 2)

GRADE 11

## QUESTION 2

2.1	<table><tr><th>Height (cm)</th><th>Number of pupils (f)</th><th>Cumulative frequency</th></tr><tr><td><math>145 \leq x &lt; 150</math></td><td>6</td><td>6</td></tr><tr><td><math>150 \leq x &lt; 155</math></td><td>23</td><td>29</td></tr><tr><td><math>155 \leq x &lt; 160</math></td><td>60</td><td><b>89</b></td></tr><tr><td><math>160 \leq x &lt; 165</math></td><td>74</td><td>163</td></tr><tr><td><math>165 \leq x &lt; 170</math></td><td>52</td><td><b>215</b></td></tr><tr><td><math>170 \leq x &lt; 175</math></td><td>32</td><td><b>247</b></td></tr><tr><td><math>175 \leq x &lt; 180</math></td><td>3</td><td>250</td></tr><tr><td><b>Total</b></td><td><b>250</b></td><td></td></tr></table>	Height (cm)	Number of pupils (f)	Cumulative frequency	$145 \leq x < 150$	6	6	$150 \leq x < 155$	23	29	$155 \leq x < 160$	60	<b>89</b>	$160 \leq x < 165$	74	163	$165 \leq x < 170$	52	<b>215</b>	$170 \leq x < 175$	32	<b>247</b>	$175 \leq x < 180$	3	250	<b>Total</b>	<b>250</b>		<div>✓ 89</div> <div>✓ 215</div> <div>✓ 247</div>	(3)
Height (cm)	Number of pupils (f)	Cumulative frequency																												
$145 \leq x < 150$	6	6																												
$150 \leq x < 155$	23	29																												
$155 \leq x < 160$	60	<b>89</b>																												
$160 \leq x < 165$	74	163																												
$165 \leq x < 170$	52	<b>215</b>																												
$170 \leq x < 175$	32	<b>247</b>																												
$175 \leq x < 180$	3	250																												
<b>Total</b>	<b>250</b>																													
2.2	<div>Ogive showing the distribution of marks</div>	<div>✓ Shape</div> <div>✓ anchor on <math>x</math>- axis</div> <div>✓ plotting</div>	(3)																											
2.3	<div><math>Q_2 \approx 162</math></div> <div><math>Q_1 \approx 158</math></div> <div><math>Q_3 \approx 167</math></div> <div><math>\therefore \text{IQR} \approx 162</math></div>	<div>✓</div> <div>✓</div> <div>✓</div> <div>✓</div>	(4)																											
2.4	$165 \leq x < 170$	✓ answer	(1)																											
2.5	$\frac{35}{250} = 14\%$	✓ answer	(1)																											
2.6	$\sigma = 6,43$	✓✓ answer	(2)																											
			[14]																											

## MARKING GUIDELINES

MATHEMATICS  
(Paper 2)

## GRADE 11

## QUESTION 3

3.1	$E\left(\frac{3+12}{2}; \frac{0+3}{2}\right)$ $=\left(7\frac{1}{2}; 1\frac{1}{2}\right)$	✓ substitution into midpoint formula ✓ Answer	(2)
3.2	$M_{BC} = \frac{3-0}{12-3}$ $= \frac{1}{3}$	✓ substitution into gradient formula ✓ Answer	(2)
3.3	$\tan \theta = m_{BC} = \frac{1}{3}$ $\theta \tan^{-1}\left(\frac{1}{3}\right) = 18,43^\circ$	✓ $\tan \theta = m_{BC}$ ✓ answer	(2)
3.4	$M_{AD} = M_{BC} = \frac{1}{3}$ $AD \parallel BC$ , equal gradients $M_{AB} = \frac{6-0}{1-3} = -3$ $\therefore M_{AD} \times M_{AB} = \frac{1}{3} \times -3 = -1$ $\therefore AD \perp AB$	✓ $M_{AD} = \frac{1}{3}$ ✓ $M_{AB} = -3$ ✓ $M_{AD} \times M_{AB} = -1$	(3)
3.5	Inclination of new line $= 45^\circ + 18,43^\circ = 63,43^\circ$ $\therefore \tan 63,43^\circ = 2 = m_{\text{line}}$ $\therefore y - 6 = 2(x - 1)$ $y = 2x + 4$	✓ $18,43^\circ$ ✓ $63,43^\circ$ ✓ $m = 2$ ✓ equation	(4)
			[13]

## MARKING GUIDELINES

MATHEMATICS  
(Paper 2)

## GRADE 11

## QUESTION 4

4.1	$M_{QP} = M_{OS} = 6$ $QP \parallel OS$ , equal gradients $y - 17 = 6(x + 3)$ $y = 6x + 35$	$\checkmark M_{QP} = 6$ $\checkmark$ equation	(2)
4.2	$6x + 35 = -x$ $7x = -35$ $x = -5$ $y = -(-5) - 5$ $\therefore Q(-5;5)$	$\checkmark$ setting up equation $\checkmark x = -5$ $\checkmark$ coordinates of Q	(3)
4.3	$OQ^2 = (-5 - 0)^2 + (5 - 0)^2$ $= 50$ $OQ = \sqrt{50} = 5\sqrt{2}$ units	$\checkmark$ substitution into distance formula $\checkmark 5\sqrt{2}$	(2)
4.4	$M_{OS} = 6$ $\therefore$ inclination of OS is $-\tan^{-1}(4) = 80,54^\circ$ $M_{OQ} = -1$ $\therefore$ inclination of QO is $180^\circ - \tan^{-1}(1) = 135^\circ$ $\alpha = 135^\circ - 80,54^\circ \dots$ $= 54,46^\circ$	$\checkmark 80,54^\circ$ $\checkmark 135^\circ$ $\checkmark 54,46^\circ$	(3)
4.5	$QS^2 = OS^2 + OQ^2 - 2OS \cdot OQ \cdot \cos \alpha$ $= 148 + 50 - 2(\sqrt{148})(\sqrt{50} \cdot \cos 54,46^\circ)$ $QS = 9,90$ units	$\checkmark$ correct use of cosine rule $\checkmark$ substitution into formula $\checkmark 9,90$	(3)
			[13]

## MARKING GUIDELINES

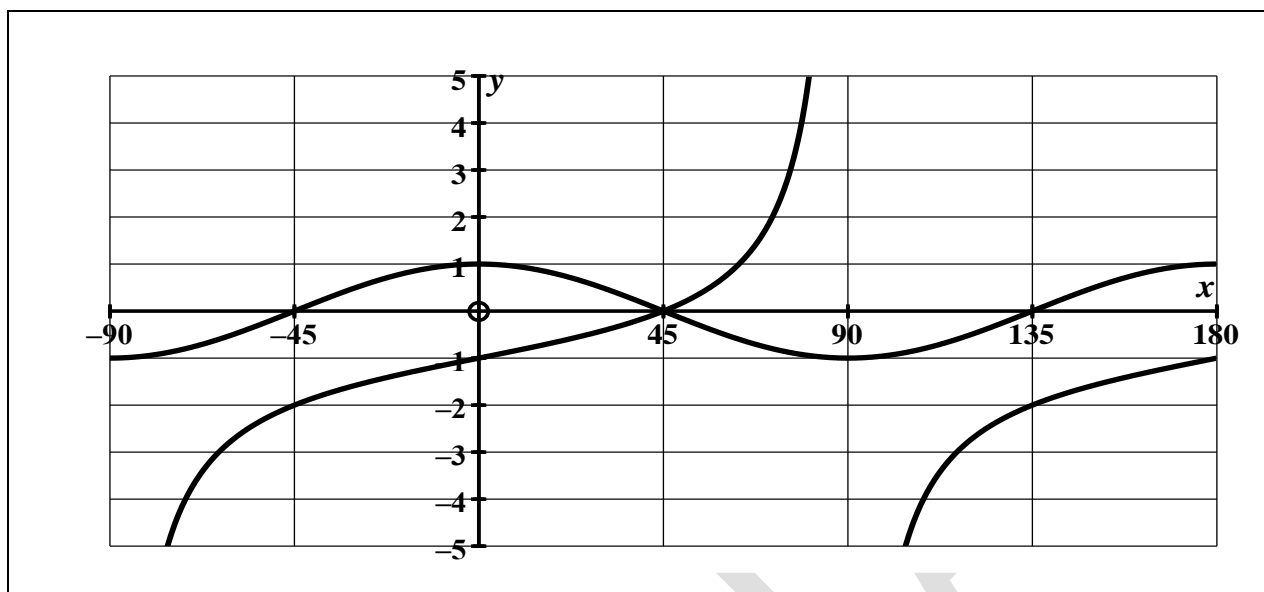
MATHEMATICS  
(Paper 2)

GRADE 11

## QUESTION 5

5.1	5.1.1	$= \cos 62^\circ$ $= k$	✓✓ Answer	(2)
	5.1.2	$= -\cos 62^\circ$ $= -k$	✓✓ Answer	(2)
5.2		$= -\tan x \cdot \cos x = \frac{\sin x}{\cos x} \times \frac{\cos x}{\sin x}$ $= 1$	✓ $\tan x \cdot \cos x$ ✓ $-\sin x$ ✓ $\frac{\sin x}{\cos x}$ ✓ 1	(4)
5.3		$\frac{\cos 50^\circ - \cos \theta}{\sin 40^\circ + \cos \theta} = \frac{-(\cos \theta + \cos 50^\circ)}{\cos \theta + \sin 40^\circ}$ $\cos 50^\circ = \sin 40^\circ$ $\therefore = -1$	✓✓ $\frac{\cos 50^\circ - \cos \theta}{\sin 40^\circ + \cos \theta}$ ✓✓ $-(\cos \theta + \cos 50^\circ)$ ✓ answer	(5)
5.4		$= \sqrt{2^{2\sin 20^\circ} \cdot 2^{3-\tan 45^\circ}}$ $= \sqrt{2^{2 \cdot \frac{1}{2}} \cdot 2^{-3}}$ $= \sqrt{\frac{1}{4}}$ $= \frac{1}{2}$	✓ $\sqrt{2^{2\sin 20^\circ} \cdot 2^{3-\tan 45^\circ}}$ ✓ $\sqrt{2^{2 \cdot \frac{1}{2}} \cdot 2^{-3}}$ ✓ $\frac{1}{2}$	(3)
5.5	5.5.1	$\sin^2 x = \frac{3}{4}$ $\sin x = \pm \frac{\sqrt{3}}{2}$	✓ answer	(1)
	5.5.2	Reference angle = $60^\circ$ $60^\circ; 120^\circ; 240^\circ$	✓✓✓ answers	(3)
				[20]

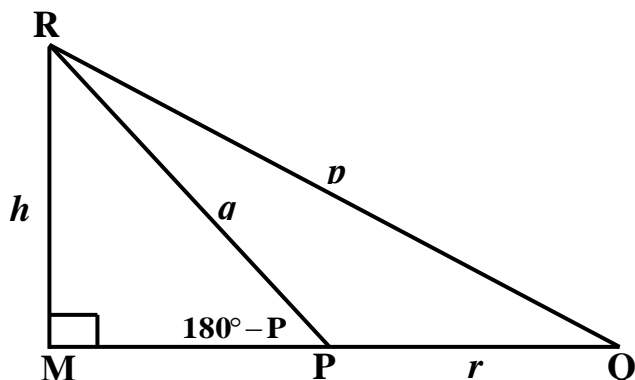
## QUESTION 6



6.1	$f(x) = \tan x - 1$  $g(x) = \cos 2x$		✓ asymptotes ✓ intercepts with axes ✓ shape ✓ period ✓ shape with open dot at $x = -90^\circ$	(5)
6.2	6.2.1	$\cos 2x + 1 = \tan x$ $\therefore \cos 2x = \tan x - 1$ $\therefore x = 45^\circ$	✓ $x = 45^\circ$	(1)
	6.2.2	$2\cos^2 x = 1$ $\therefore 2\cos^2 x - 1 = 0$ $\therefore \cos 2x = 0$ $\therefore x = \pm 45^\circ$ or $x = 135^\circ$	✓ $\cos 2x = 0$ ✓ solutions	(2)
	6.2.3	$-90^\circ < x < -45^\circ$ or $90^\circ < x < 135^\circ$	✓ $-90^\circ < x < -45^\circ$ ✓ $90^\circ < x < 135^\circ$	(2)
6.3	$h(x) = \cos 2(x - 30^\circ)$		✓ $h(x) = \cos 2(x - 30^\circ)$	(1)
				[11]

## QUESTION 7

7.1



Construction: Draw  $RM \perp PQ$  or  $QP$  produced

$$RM = p \sin Q$$

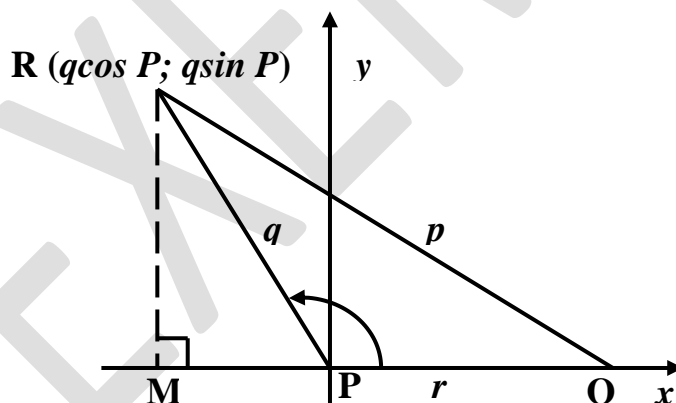
$$\begin{aligned} CM &= q \sin (180^\circ - P) \\ &= q \sin P \end{aligned}$$

$$\therefore p \sin Q = q \sin P$$

Dividing by  $\sin P \cdot \sin Q$  gives

$$\frac{p}{\sin P} = \frac{q}{\sin Q}$$

OR



P is in standard position as shown in the figure.

Construction: Draw  $RM \perp PQ$  or  $QP$  produced

$$RM = q \sin P = p \sin Q$$

Dividing by  $\sin P \cdot \sin Q$  we then have

$$\frac{p}{\sin P} = \frac{q}{\sin Q}$$

✓ construction

✓  $RM = p \sin Q$

✓  $RM = q \sin (180^\circ - P)$

✓  $\therefore p \sin Q = q \sin P$

✓ Dividing by  $\sin P \cdot \sin Q$

OR

✓  $R(q \cos P; q \sin P)$

✓ construction

✓  $RM = q \sin P$

✓  $RM = p \sin Q$

✓ Dividing by  $\sin P \cdot \sin Q$

(5)

## MARKING GUIDELINES

MATHEMATICS  
(Paper 2)

GRADE 11

7.2			
7.2	7.2.1	<p>In <math>\Delta LKJ</math>:</p> $\frac{h}{LJ} = \sin y$ $\therefore h = LJ \sin y \quad \text{--- (1)}$ <p>In <math>\Delta LJI</math>:</p> $\frac{LJ}{\sin x} = \frac{d}{\sin \hat{L}_1}$ <p>but <math>\hat{L}_1 = y - x</math></p> $\therefore LJ = \frac{d \sin x}{\sin (y - x)} \quad \text{--- (2)}$ <p>subst.(2) into (1):</p> $h = \frac{d \sin x \sin y}{\sin (y - x)}$	<p>✓ <math>h</math> sub of formula</p> <p>✓ sine rule</p> <p>✓ <math>\hat{L}_1 = y - x</math></p> <p>✓ LJ</p> <p>✓ substitution</p>
			(5)
	7.2.2	$h = \frac{70 \cdot \sin 12^\circ \cdot \sin 30^\circ}{\sin 18^\circ}$ $= 23,55 \text{ m}$	<p>✓ substitution</p> <p>✓ answer</p>
			(2)
			[12]

## QUESTION 8

8.1	<p>Volume of cylinder:</p> $V_1 = \pi r^2 h$ $= \pi(12)^2 \cdot (19)$ $= 2736 \pi m^3$ <p>Volume of hemisphere (half sphere):</p> $V_2 = \frac{1}{2} \left( \frac{4}{3} \pi r^3 \right)$ $\text{Total Volume} = V_1 + V_2$ $= \frac{1}{2} \left[ \left( \frac{4}{3} \pi r^3 \right) (12)^3 \right]$ $= 1152 \pi m^3$ $\text{Total Volume} = V_1 + V_2$ $= 2736\pi + 1152\pi$ $= 3888\pi m^3$	<p>✓ Substitution</p> <p>✓ answer</p> <p>✓ <math>\frac{1}{2}</math></p> <p>✓ Substitution</p> <p>✓ answer</p> <p>✓ Total Volume in <math>\pi</math></p> <p><i>(5 out of 6 if numerical value of <math>\pi</math> is used )</i></p>	(6)
-----	--	--	-----

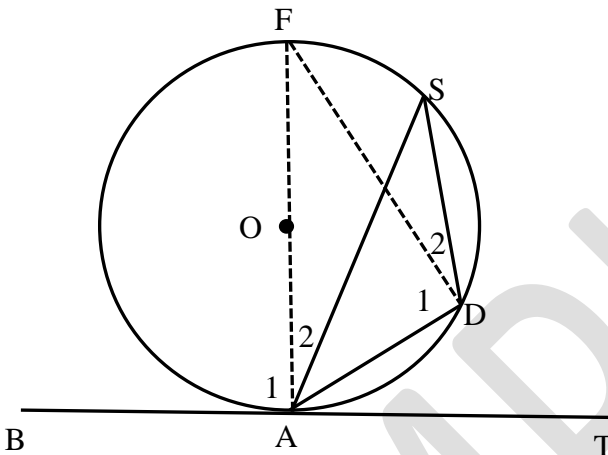
## MARKING GUIDELINES

MATHEMATICS  
(Paper 2)

GRADE 11

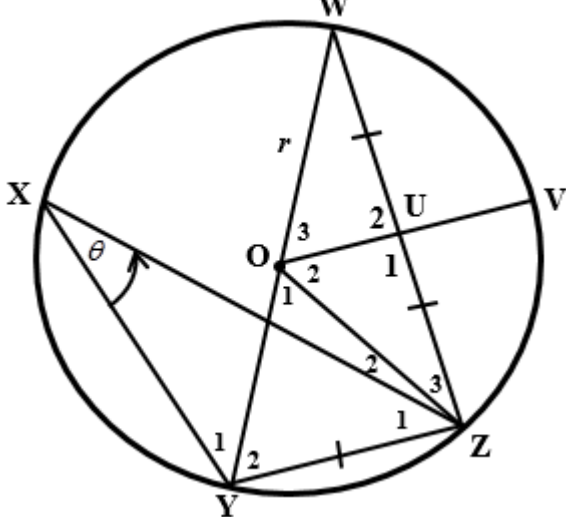
8.2	<p>Surface area of cylinder:</p> $A_1 = 2\pi r^2 + 2\pi rh$ $= 2\pi(12)^2 + 2\pi(12)(19)$ $= 2337,34 \text{ m}^2$ <p>Surface area of hemisphere (half sphere):</p> $A_2 = \frac{1}{2}(4\pi r^2)$ $= \frac{1}{2}[(4\pi r^2)(12)^2]$ $= 452.39 \text{ m}^2$ <p><i>Total Surface Area</i> <math>= A_1 + A_2</math></p> $= 2337,34 + 452.39$ $= 685.73 \text{ m}^2 \approx 686 \text{ m}^2$	<p>✓ Substitution</p> <p>✓ answer</p> <p>✓ Substitution</p> <p>✓ answer</p> <p>✓ Total area</p> <p>✓ correct rounded off answer</p>	<p>(6)</p> <p><b>[12]</b></p>
-----	---	---	-------------------------------

## QUESTION 9

9.1	9.1.1	The opposite angles of a cyclic quadrilateral are <i>supplementary</i> .	✓ answer	(1)
	9.1.2	The angle between a tangent and a chord is <i>equal to an angle in an alternate segment</i> .	✓ answer	(1)
9.2	<div></div> <p>Construct AF(diameter) and FD (chord)</p> $\hat{D}_1 = 90^\circ \quad ( \angle \text{ in a semi-circle } )$ $\hat{A}_1 = 90^\circ \quad ( \text{ rad } \angle \text{ tan } )$ $\hat{A}_2 = \hat{D}_2 \quad ( \angle s \text{ in same seg. } )$ $\hat{A}_1 + \hat{A}_2 = \hat{D}_1 + \hat{D}_2$ $\therefore \hat{BAS} = \hat{D}$		<p>✓ Construction</p> <p>✓ Statement ✓ Reason</p> <p>✓ Statement and Reason</p> <p>✓ Statement and Reason</p> <p>✓ Conclusion</p>	(6)

14

## QUESTION 10

10.1				
10.1.1	$\angle O_1 = 2\theta$ ( $\angle$ at the centre = $2 \times$ the $\angle$ at the circumference)	✓ statement ✓ reason		(2)
10.1.2	$\angle W = \theta$ ( $\angle$ s in the same seg)	✓ statement ✓ reason		(2)
10.1.3	$\angle Z = 90^\circ$ ( $\angle$ in a semi-circle) $\angle Y_2 = 90 - \theta$ (sum of $\angle$ s in a $\Delta$ )	✓ statement & reason ✓ statement & reason		(2)
10.2	$WY^2 = WZ^2 + YZ^2$ (Pythagoras theorem) $YZ = WU = UZ$ (given) $\therefore OW = OY = r$ (given) $\Rightarrow YZ = \sqrt{\frac{4r^2}{5}} = \frac{4r}{\sqrt{5}}$	✓ statement & reason ✓ statement & reason ✓ statement & reason $YZ = \sqrt{\frac{4r^2}{5}}$ <b>OR</b> $YZ = \frac{4r}{\sqrt{5}}$ ✓		(4)
10.3	$OW = OV = r = OU + OV$ (radii) $r^2 = 3^2 + 4^2$ (Pythagoras theorem) $r = 5 \text{ cm}$ $\therefore UV = 5 - 2 = 3 \text{ cm}$	✓ statement ✓ reason $r^2 = 3^2 + 4^2$ ✓ $r = 5 \text{ cm}$ ✓ ✓ answer		(2)
				[12]



## MARKING GUIDELINES

MATHEMATICS  
(Paper 2)

GRADE 11

11.2.2	$\hat{W}_1 = \hat{Q}_4$ (tan-chord theorem) since $\hat{Q}_1 = \hat{Q}_3$ (= chord subt. = $\angle$ ) $\Rightarrow \hat{Q}_4 + \hat{Q}_3 = \hat{W}_1 + \hat{Q}_1$ also $\hat{L}_3 = \hat{W}_1 + \hat{Q}_1$ (ext. $\angle$ of $\Delta$ ) $= \hat{R}_3$ (corr. $\angle$ s, $QT \parallel RS$ ) $\therefore \hat{R}_3 = \hat{Q}_4 + \hat{Q}_3$ PRKQ is a cyclic quad (ext. $\angle$ = int. opp. $\angle$ ) <b>Alternatively:</b> $\hat{Q}_1 + \hat{Q}_3 = \hat{Q}_3 + \hat{S}$ <b>OR</b> $\hat{W}_1 = \hat{Q}_4$ (tan-chord theorem) $\hat{Q}_1 = \hat{Q}_3$ (= chord subt. = $\angle$ ) $\therefore \hat{Q}_4 + \hat{Q}_3 = \hat{W}_1 + \hat{Q}_1$	✓ statement ✓ reason ✓ statement & reason ✓ statement & reason ✓ statement & reason	(5)
			[12]
<b>TOTAL:</b>			<b>150</b>