



**PROVINSIALE EKSAMEN**  
**NOVEMBER 2022**  
**GRAAD 11**  
**NASIENRIGLYNE**

**WISKUNDE (VRAESTEL 2)**

**22 bladsye**

**INSTRUKSIES EN INLIGTING**

- **A** – AKKURAATHEID
- **CA** – KONTINUE AKKURAATHEID
- **S** – BEWERING
- **R** – REDE
- **S/R** – BEWERING met REDE

**NOTAS:**

- Indien 'n kandidaat 'n vraag TWEE keer antwoord, merk slegs die EERSTE poging.
- Indien 'n kandidaat 'n antwoord doodgetrek het, en nie die vraag weer gedoen het nie, merk die doodgetrekte poging.
- Deurlopende akkuraatheid moet op ALLE aspekte van die nasienriglyn toegepas word.
- Dit is ONAANVAARBAAR vir kandidate om waardes/antwoorde aan te neem in 'n poging om probleme op te los.

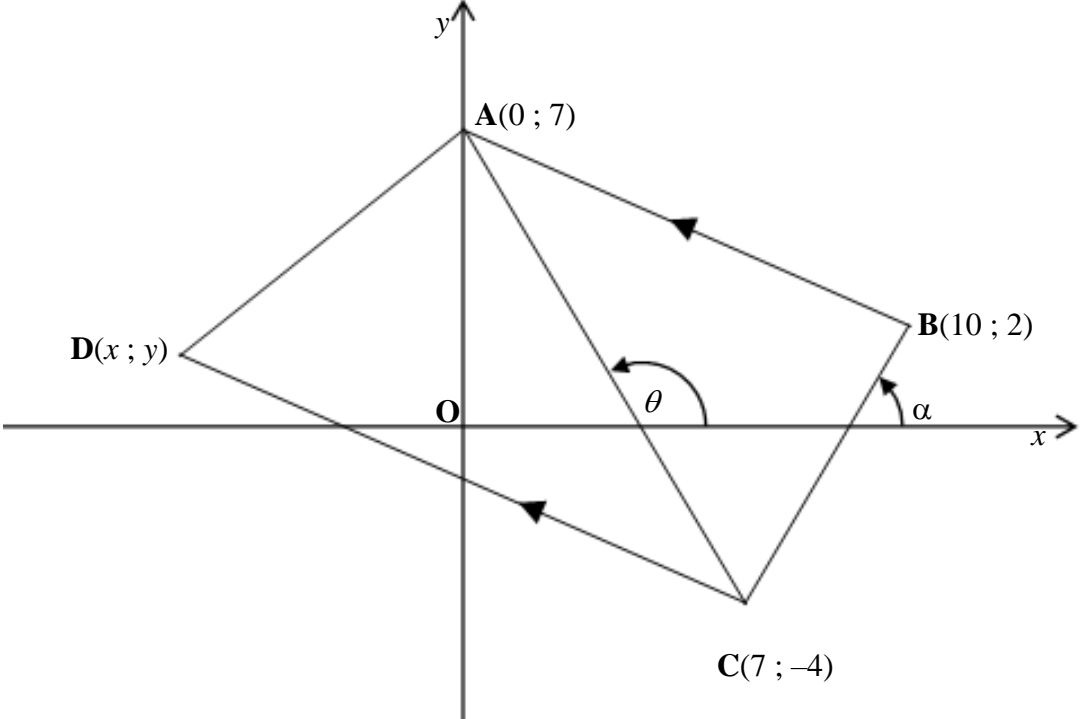
## VRAAG 1

1.1	<p>250      266      287      306      329</p> <p>245 250 255 260 265 270 275 280 285 290 295 300 305 310 315 320 325 330 335 340</p>	✓ Mond (Q1 en Q3) ✓ Snor (Min. en Maks.) ✓ Mediaan	(3)
1.2	Die data is simmetries versprei, 'n normale verspreiding.  <b>OF</b>  Die data is effens skeef na regs.	✓ Verduideliking	(1)
1.3	$\sigma = 23,77$	✓✓ $\sigma$	(2)
1.4	$(\bar{x} - \sigma; \bar{x} + \sigma)$ $= (287 - 23,77; 287 + 23,77)$ $= (263,23; 310,77)$ $\therefore$ 6 munisipaliteite is binne een standaard afwyking.	✓ $\bar{x} = 287$ ✓ Interval ✓ 6	(3)
1.5	$S-IQR = \frac{1}{2} (Q_3 - Q_1)$ $= \frac{1}{2} (306 - 266)$ $= 20$	✓ $Q_3 - Q_1$ ✓ $\frac{1}{2} \times$ ✓ Antwoord	(3)
			<b>[12]</b>

## VRAAG 2

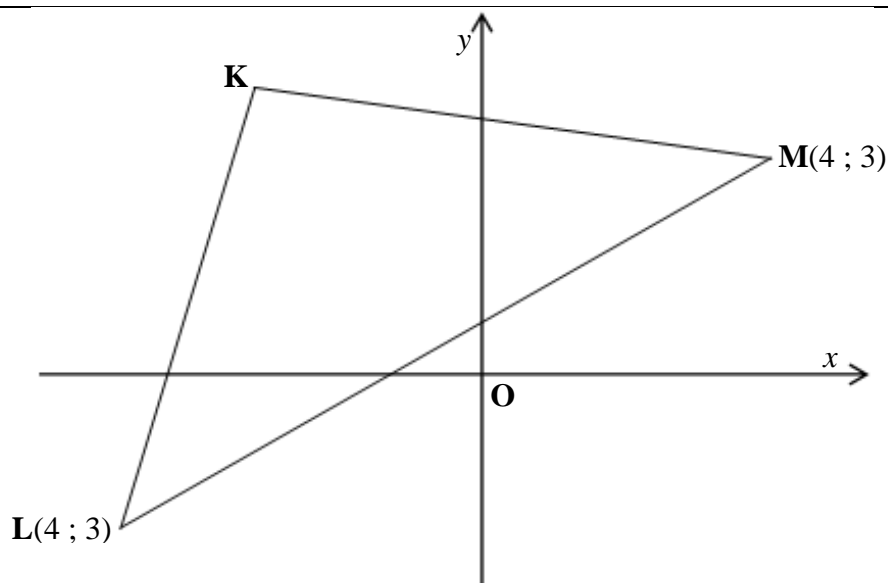
2.1	$p = 5$ $q = 27$ $r = 1$	✓ $p$ ✓ $q$ ✓ $r$	(3)
2.2	<p style="text-align: center;"><b>KUMULATIEWE FREKWENSIEGRAFIEK (OGIEF)</b></p> <p style="text-align: center;">PUNTE</p>		
2.3	$M = 46$	✓✓ $M = 46(\pm 2)$	(2)
2.4	$32 - 24$ $= 8$ $\therefore 8$ leerders behaal 'n punt van meer as 55.	✓ $KF = 24(\pm 1)$ ✓ $32 - KF$	(2)
			<b>[10]</b>

## VRAAG 3

			
3.1	$AB = \sqrt{(0-10)^2 + (7-2)^2}$ $AB = \sqrt{125}$ $AB = 5\sqrt{5}$	✓ Vervang in korrekte formule ✓ AB ✓ Eenvoudigste wortelvorm	(3)
3.2	$m_{AC} = \frac{7-(-4)}{0-7}$ $m_{AC} = -\frac{11}{7}$	✓ Gradiënt substitusie ✓ $m_{AC}$	(2)
3.3	$\tan \theta = -\frac{11}{7}$ $\text{verw.hoek} = 57,528 \dots^\circ$ $\theta = 180^\circ - 57,528 \dots^\circ$ $\theta = 122,47^\circ$	✓ Inklinasie hoek vervang (CA 4.2) ✓ Verw.hoek ✓ $\theta$ (trek verw.hoek af van $180^\circ$ )	(3)

3.4	$\tan \alpha = 2$ $\alpha = 63,434 \dots^\circ$ $\hat{A}CB = 122,47^\circ - 63,434 \dots^\circ$ $\hat{A}CB = 59,04^\circ$ $m_{AB} = \frac{1}{2}$ $\hat{C}AB = 180^\circ - \tan^{-1} \left( \frac{1}{2} \right) - 122,74^\circ$ $\hat{C}AB = 30,69^\circ$ $\hat{A}CD = 30,69^\circ$ [verw. $\angle$ 'e, $AB \parallel CD$ ] $\hat{B}CD = 30,69^\circ + 59,04^\circ$ $\hat{B}CD = 89,73^\circ$ <p style="text-align: center;"><b>OF</b></p> <p><b>Indien <math>\theta</math> nie afgerond is nie</b></p> $\tan \alpha = 2$ $\alpha = 63,434 \dots^\circ$ $\hat{A}CB = 122,4711 \dots^\circ - 63,434 \dots^\circ$ $\hat{A}CB = 59,036 \dots^\circ$ $m_{AB} = \frac{1}{2}$ $\hat{C}AB = 180^\circ - \tan^{-1} \left( \frac{1}{2} \right) - 122,74 \dots^\circ$ $\hat{C}AB = 30,963^\circ$ $\hat{A}CD = 30,963^\circ$ [verw $\angle$ 'e, $AB \parallel CD$ ] $\hat{B}CD = 30,963^\circ + 59,036 \dots^\circ$ $\hat{B}CD = 90^\circ$	$\alpha$ $\hat{A}CB$ $\hat{C}AB$ $\hat{B}CD$ <p style="text-align: center;"><b>OF</b></p> $\checkmark \alpha$ $\checkmark \hat{A}CB$ $\checkmark \hat{C}AB$ $\checkmark \hat{B}CD$	(4)
			[12]

VRAAG 4



4.1	$y = 5x + 9$ en $5y + x - 19 = 0$ $5(5x + 9) + x = 19$ $26x = -26$ $x = -1$ $y = 5(-1) + 9$ $y = 5$ $\therefore K(-1; 4)$	✓ Vervang $y$ ✓ Los op vir $x$ ✓ Vervang $x$ terug GEEN PUNTE vir koördinate nie (gegee)	(3)
4.2	$m_{KL} = 5$ en $m_{KM} = -\frac{1}{5}$ $\therefore m_{KL} \times m_{KM} = -1$ $KL \perp KM$ $\therefore \hat{LKM} = 90^\circ$	✓ $m_{KL}$ ✓ $m_{KM}$ ✓ Definisie van loodregte gradiënte	(3)

4.3	$KL = \sqrt{((-1 - (-2))^2 + (4 - (-1))^2)}$ $KL = \sqrt{26}$ $KN = \sqrt{(-1 - 4)^2 + (4 - 3)^2}$ $KN = \sqrt{26}$ $\text{Area } \Delta KML = \frac{1}{2} (\sqrt{26})(\sqrt{26})$ $\text{Area } \Delta KML = 13 \text{ eenhede}^2$	✓ KL lengte ✓ KN lengte ✓ Vervang in Oppervlak formule ✓ Opp $\Delta KML$	(4)
4.4	$M \left( \frac{-2 + 4}{2}; \frac{3 + (-1)}{2} \right)$ $M(1; 1)$ $m_{NL} = \frac{3 - (-1)}{4 - (-2)}$ $m_{NL} = \frac{2}{3}$ $y - y_1 = -\frac{2}{3}(x - x_1)$ $y - 1 = -\frac{2}{3}(x - 1)$ $y = -\frac{2}{3}x + \frac{5}{2}$	✓ Middelpunt van NL  ✓ Gradiënt van NL  ✓ Loodregte gradiënt  ✓ Vervang Middelpunt  ✓ Vergelyking	(5)
4.5	$m_{NL} = m_{NP} = m_{LP}$ $\frac{2}{3} = \frac{y - 3}{7 - 4}$ $y = 5$	✓ Definisie van kollineêr  ✓ Vervang P(7 ; y)	(2)

4.6	Q(3 ; -2)	✓ $x_Q$ ✓ $y_Q$	(2)
4.7	$\hat{K} = 90^\circ$ [bewys in 4.1] $KL = KN = \sqrt{26}$ [bewys in 4.2] $\therefore$ KLQN is 'n vierkant [’n parallelogram met ’n binnehoek van $90^\circ$ en aangrensende sye gelyk]	✓ Binnehoek van $90^\circ$ ✓ Aangrensende sye gelyk	(2)
			<b>[21]</b>

**VRAAG 5**

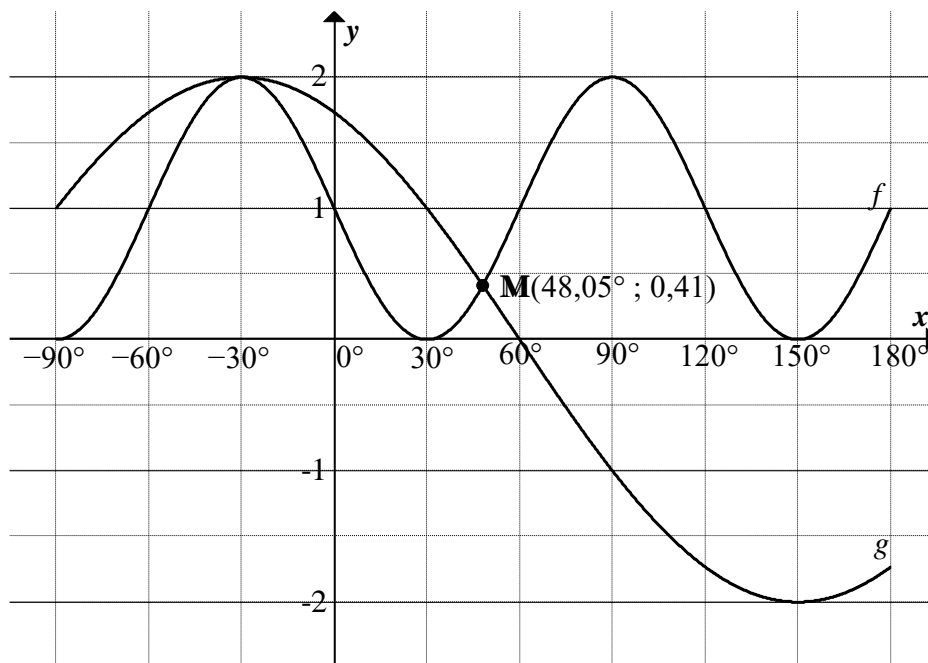
5.1	$\frac{\tan(180^\circ - x) \cos(360^\circ + x)}{2 \cos(90^\circ + x)}$ $= \frac{-\tan x \cdot \cos x}{-2 \sin x}$ $= \frac{-\frac{\sin x}{\cos x} \cdot \cos x}{-2 \sin x}$ $= \frac{-\sin x}{-2 \sin x}$ $= \frac{1}{2}$		✓ $-\tan x$ ✓ $\cos x$ ✓ $-2 \sin x$ ✓ $-\tan x$ identiteit ✓ vereenvoudig ✓ antwoord	(6)
5.2	5.2.1	$LK = \frac{\sin \theta - 2 \sin \theta \cos \theta}{2 \cos^2 \theta + \cos \theta - 1}$ $= \frac{\sin \theta (1 - 2 \cos \theta)}{(2 \cos \theta - 1)(\cos \theta + 1)}$ $= \frac{-\sin \theta (2 \cos \theta - 1)}{(2 \cos \theta - 1)(\cos \theta + 1)}$ $= \frac{-\sin \theta}{\cos \theta + 1}$ $= RK$	✓ $\sin \theta (1 - 2 \cos \theta)$ ✓ $(2 \cos \theta - 1)(\cos \theta + 1)$ ✓ Teken verander/negatiewe faktor ✓ Vereenvoudig <u>met</u> gevolgtrekking	(4)

5.2.2	<p>Ongedefinieerd wanneer</p> $2\cos\theta - 1 = 0$ $\cos\theta = \frac{1}{2}$ <p>Verw. hoek = <math>60^\circ</math></p> <p>QI : <math>\theta = 60^\circ + k.360^\circ ; k \in \mathbb{Z}</math></p> <p>QIV : <math>\theta = 300^\circ + k.360^\circ</math></p> <p><b>OF</b> wanneer</p> $\cos\theta = -1$ $\theta = 180^\circ + k.360^\circ ; k \in \mathbb{Z}$ <p><b>NOTA: Aanvaar ander ekwivalente algemene oplossings</b></p> $\cos\theta - 1 = 0$ $\theta = 180^\circ + k.360^\circ ; k \in \mathbb{Z}$	<p>✓ <math>2\cos\theta - 1 = 0</math></p> <p>✓ QI algemene oplossing</p> <p>✓ QIV algemene oplossing</p> <p>✓ <math>k \in \mathbb{Z}</math></p> <p>✓ <math>\cos\theta = -1</math> algemene oplossing</p>	(5)
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5.3				
5.3.1	$b^2 + 1^2 = 4^2$ $b = \pm \sqrt{15}$ $b = - \sqrt{15} \quad (\text{QII})$		$\checkmark$ Pythagoras substitusie  $\checkmark b = - \sqrt{15}$	(2)
5.3.2	(a)	$\tan \alpha = - \frac{1}{\sqrt{15}}$	$\checkmark \frac{y}{x}$ $\checkmark \text{CA } x$	(2)
	(b)	$\sin(- \alpha - 180^\circ) + \cos(- \alpha)$ $= \sin \alpha + \cos \alpha$ $= \left(\frac{1}{4}\right) + \left(\frac{-\sqrt{15}}{4}\right)$ $= \left(\frac{1 - \sqrt{15}}{4}\right)$	$\checkmark \sin \alpha$  $\checkmark \cos \alpha$  $\checkmark \text{Vervang } x ; y \text{ en } r$	(3)

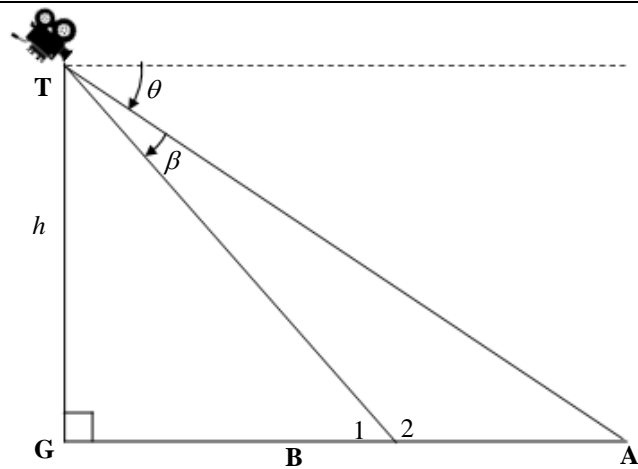
5.3.3	$\cos(\alpha - 90) = \frac{k}{6}$  $\sin \alpha = \frac{k}{6}$  $\therefore \frac{k}{6} = \frac{1}{4}$  $\therefore k = \frac{3}{2}$	$\sin(\alpha - 90) = \frac{m}{6}$  $-\cos \alpha = \frac{m}{6}$  $\cos \alpha = -\frac{m}{6}$  $\therefore \frac{m}{6} = \left( \frac{-\sqrt{15}}{4} \right)$  $\therefore m = \frac{3\sqrt{15}}{2}$	$\checkmark \cos(\alpha - 90) = \frac{k}{6}$  $\checkmark \sin(\alpha - 90) = \frac{m}{6}$  $\checkmark$ Komplementêre herleiding  $\checkmark$ Verhoudings gelyk stel  $\checkmark$ Waardes van $k$ en $m$	
	<p style="text-align: center;"><b>OF</b></p> <p>P lê in QI, daarom is <math>k</math> en <math>m</math> positief</p> <p>'n <math>90^\circ</math> rotasie beteken <math>(x ; y) \rightarrow (y ; x)</math></p> $\frac{r_2}{r_1}$ $\frac{6}{4}$ $\frac{3}{2}$ $\therefore P\left(\frac{3}{2}; \frac{3}{2} \cdot \sqrt{15}\right) = P(k ; m)$ $\therefore k = \frac{3}{2} \text{ en } m = \frac{3\sqrt{15}}{2}$	<p style="text-align: center;"><b>OF</b></p> <p><math>\checkmark</math> P in QI</p> <p><math>\checkmark (x ; y) \rightarrow (y ; x)</math> idee</p> <p><math>\checkmark</math> Verhouding van radii</p> <p><math>\checkmark</math> Pas skaal toe op <math>x</math> en <math>y</math></p> <p><math>\checkmark</math> Waardes van <math>k</math> en <math>m</math></p> <p><b>Antwoord alleen MAG NIE volpunte kry NIE.</b></p>		
				(5)
				[27]

## VRAAG 6



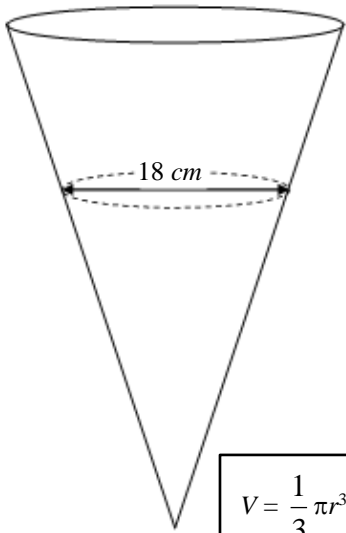
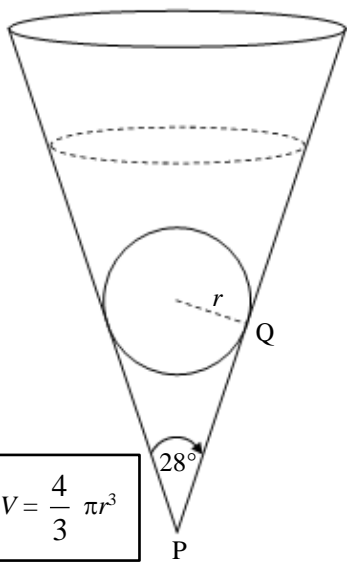
6.1	$A = 2$	✓ Antwoord	(1)
6.2	$120^\circ$	✓ Antwoord	(1)
6.3	$a = -1$ $b = 3$ $p = 30^\circ$	✓ $a$ ✓ $b$ ✓ $p$	(3)
6.4	6.4.1 $-90^\circ \leq x < 60^\circ$	✓ Interval	(1)
	6.4.2 $x = -30^\circ$ of $48,05^\circ \leq x \leq 180^\circ$	✓ Interval ✓ $x = -30^\circ$	(2)
	6.4.3 $60^\circ < x < 150^\circ$ of $150^\circ < x \leq 180^\circ$	✓ $60^\circ$ en $180^\circ$ ✓ Nie ingesl. $150^\circ$ maar $180^\circ$ ingesl.	(2)
6.5	$t = -30^\circ + k \cdot 360^\circ ; k \in \mathbb{Z}$	✓✓ $t = -30^\circ$ ✓ $k \cdot 360^\circ$	(3)
			[13]

VRAAG 7

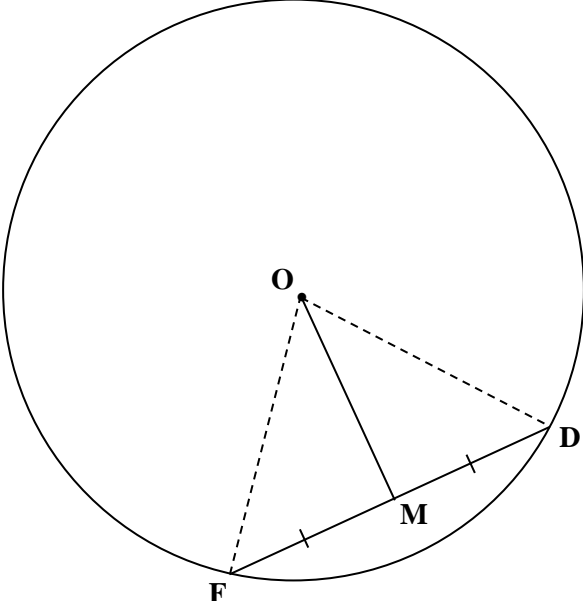


7.1	$\hat{B}_1 = \theta = \beta$	✓ Antwoord	(1)
7.2	<p>In <math>\triangle TBG</math></p> $\sin(\theta + \beta) = \frac{h}{BT}$ $BT = \frac{h}{\sin(\theta + \beta)}$ <p>In <math>\triangle TBA</math></p> $\hat{A} = \theta$ $\frac{x}{\sin \beta} = \frac{BT}{\sin \theta}$ $\frac{x}{\sin \beta} = \frac{h}{\sin(\theta + \beta) \sin \theta}$ $x = \frac{h \sin \beta}{\sin(\theta + \beta) \sin \theta}$	<p>✓ Korrekte verhouding in <math>\triangle TBG</math></p> <p>✓ BT</p> <p>✓ Sin reël substitusie</p> <p>✓ BT substitusie</p> <p>✓ vereenvoudiging</p>	(5)
7.3	$x = \frac{h \sin \beta}{\sin(\theta + \beta) \sin \theta}$ $x = \frac{20 \sin 37^\circ}{\sin(65^\circ + 37^\circ) \sin 65^\circ}$ $x = 13,58 \text{ m}$ <p>Die kat stap 13,58 m</p>	<p>✓ Substitusie</p> <p>✓ Antwoord</p>	(2)
			[8]

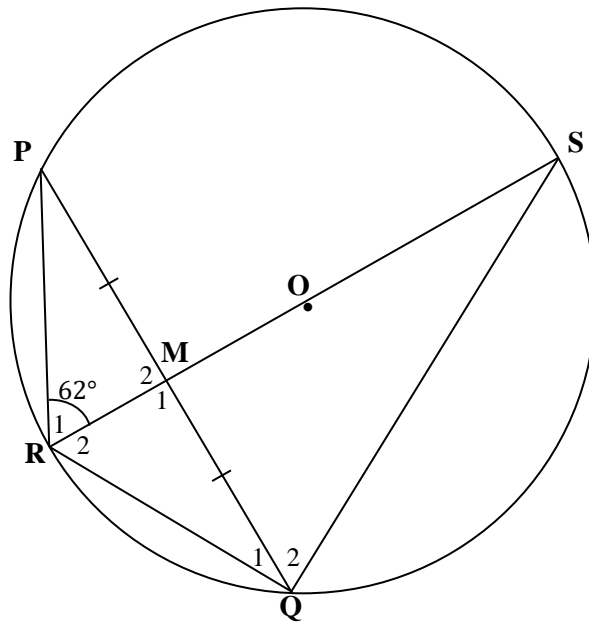
VRAAG 8

<div style="display: flex; justify-content: space-around;"> <div style="text-align: center;"> <b>FIGUUR A</b>   </div> <div style="text-align: center;"> <b>FIGUUR B</b>   </div> </div> <div style="border: 1px solid black; padding: 5px; margin: 10px auto; width: fit-content;"> <math>V = \frac{1}{3} \pi r^3 \quad \text{of} \quad V = \frac{4}{3} \pi r^3</math> </div>			
8.1	$V = \frac{1}{3} \pi (9)^3$  $V = 763,41 \text{ cm}^3$	✓ Substitusie ( $r = 9$ )  ✓ Antwoord	(2)
8.2	$V = \frac{4}{3} \pi r^3$  $268,08 = \frac{4}{3} \pi r^3$  $\sqrt[3]{\frac{3 \times 268,08}{4\pi}} = r$  $r = 4$  $\tan 14^\circ = \frac{4}{PQ}$  $PQ = \frac{4}{\tan 14^\circ}$  $PQ = 16,04 \text{ cm}$	✓ Vervang volume in die korrekte formule  ✓ Onderwerp van die formule  ✓ $r$  ✓ $\tan 14^\circ$  ✓ $PQ$	(5)
			[7]

VRAAG 9

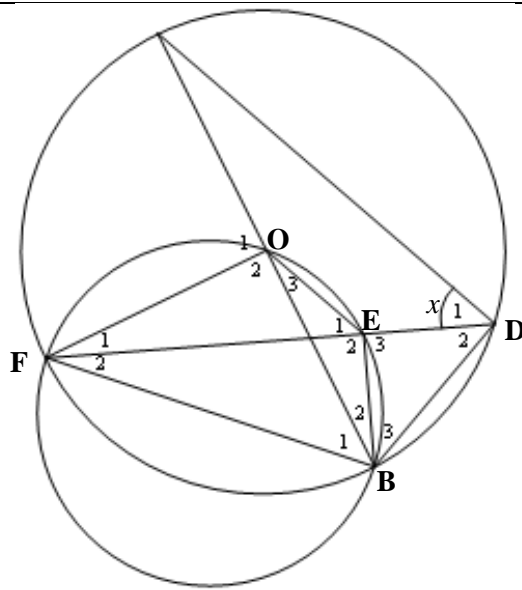
9.1	 <p><i>Konstruksie: Verbind OF en OD</i></p> <p>In <math>\triangle OFM</math> en <math>\triangle ODM</math></p> <p>i. <math>FM = MD</math> [GEGEE]</p> <p>ii. <math>OM</math> is 'n gemeenskaplike sy</p> <p>iii. <math>OF = OD</math> [Radii]</p> <p><math>\therefore \triangle OFM \equiv \triangle ODM</math> [SSS]</p> <p><math>\hat{M}_1 = \hat{M}_2</math> [<math>\triangle OFM \equiv \triangle ODM</math>]</p> <p>Maar <math>\hat{M}_1 + \hat{M}_2 = 180^\circ</math> [<math>\angle</math>'e op 'n reguit lyn]</p> <p><math>\therefore \hat{M}_1 = \hat{M}_2 = 90^\circ</math></p> <p><math>\therefore OM \perp FD</math></p>	<p>✓ Konstruksie</p> <p>✓ R</p> <p>✓ R</p> <p>✓ S</p>	(4)
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9.2



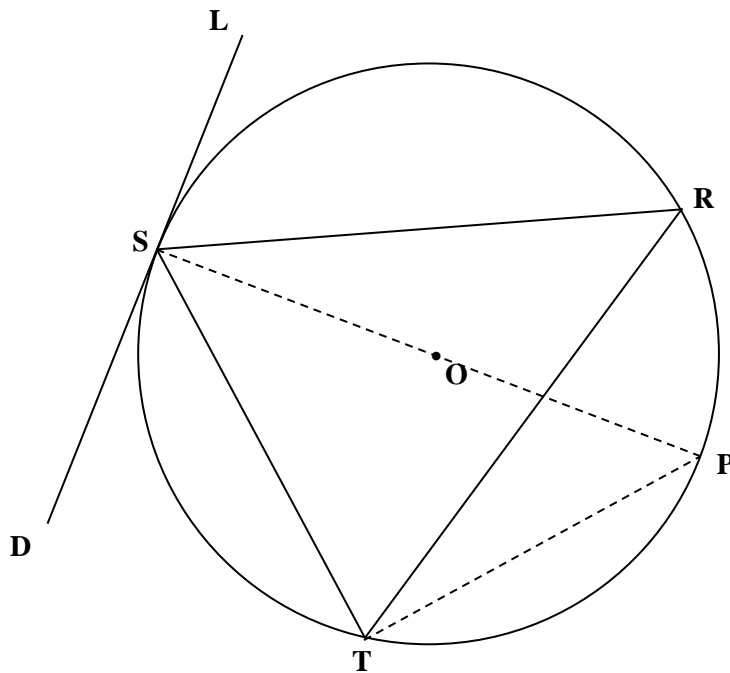
9.2.1	$\hat{M}_2 = 90^\circ$ [lyn van midpunt sirkel, tot midpunt koord]	✓ S ✓ R	(2)	
9.2.2	$\hat{Q}_1 + \hat{Q}_2 = 90^\circ$ [ $\angle$ 'e in 'n halwe sirkel]	✓ S ✓ R	(2)	
9.2.3	$\hat{P} = 28^\circ$ [binne $\angle$ 'e van 'n $\Delta$ ]	✓ S/R		
	$\hat{S} = \hat{P} = 28^\circ$ [ $\angle$ 'e in dieselfde sirkelsegment]	✓ R	(2)	
				<b>[10]</b>

## VRAAG 10



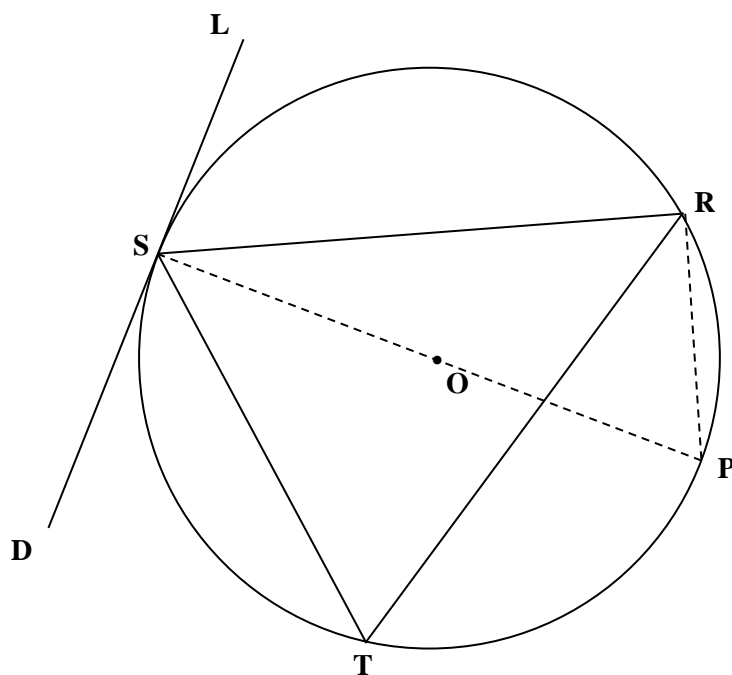
10.1	$\hat{B}_1 = \hat{D}_1 = x$ [∠'e in dieselfde sirkelsegment]	✓ S	
	$\hat{B}_1 = \hat{E}_1 = x$ [∠'e in dieselfde sirkelsegment]	✓ S	(2)
10.2	10.2.1 $\hat{O}_1 = 2x$ [midpunts∠ = 2 × omtreks∠]	✓ S ✓ R	(2)
	10.2.2 $\hat{O}_1 = \hat{O}FB + \hat{B}_1$ [buite ∠ van Δ]	✓ R	
	$2x = \hat{O}FB + x$	✓ $\hat{O}FB = x$	
	$\hat{O}FB = x$		(2)
	10.2.3 $\hat{D}_2 = 90^\circ - x$ [∠'e in 'n halwe sirkel]	✓ S ✓ R	(2)
	10.2.4 $\hat{O}_2 = 180^\circ - 2x$ [∠'e op 'n reguitlyn]	✓ S ✓ R	
	$\hat{O}_2 = \hat{E}_2 = 180^\circ - 2x$ [∠'e in dieselfde sirkelsegment]	✓ S ✓ R	(4)
10.3	$\hat{E}_2 = \hat{B}_3 + \hat{D}_2$ [buite ∠ van Δ]	✓ R	
	$\therefore \hat{B}_3 = 90^\circ - x$	✓ $\hat{B}_3 = 90^\circ - x$	
	$\therefore \hat{B}_3 = \hat{D}_2$ [albei $90^\circ - x$ ]	✓ S	
	$\therefore EB = ED$ [sye teenoor gelyke ∠'e]	✓ R	(4)
			<b>[16]</b>

VRAAG 11



11.1	<p>Konstruksie: Trek middellyn SP en verbind PT</p> <p><math>\hat{L}SR = 90^\circ - \hat{R}SP</math> [rklyn <math>\perp</math> rad]</p> <p><math>\hat{S}TR = 90^\circ - \hat{R}TP</math> [<math>\angle</math>'e in 'n halwe sirkel]</p> <p><math>\hat{R}SP = \hat{R}TP</math> [<math>\angle</math>'e in dieselfde sirkelsegment]</p> <p><math>\therefore \hat{L}SR = \hat{S}TR</math></p>	<p>✓ Konstruksie</p> <p>✓ S ✓ R</p> <p>✓ S ✓ R</p> <p>✓ S ✓ R</p>
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**OF**



Konstruksie: Trek middellyn SP en verbind PR

✓ Konstruksie
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$$\hat{\text{L}}\hat{\text{S}}\text{R} = 90^\circ - \text{R}\hat{\text{S}}\text{P} \quad [\text{rklyn} \perp \text{rad}]$$

✓ S/R

$\widehat{SRP} = 90^\circ$  [ $\angle$  in 'n halwe sirkel]

✓ S ✓ R

$$\hat{S}PR = 90^\circ - \hat{R}SP \quad [\text{binne } \angle\text{'e van } \Delta]$$

[binne  $\angle$ 'e van  $\Delta$ ]

$$\hat{\text{LSR}} = \hat{\text{SPR}} \quad [\text{albei } 90^\circ - \hat{\text{RSP}}]$$

[albei 90° – RŞP]

✓ S

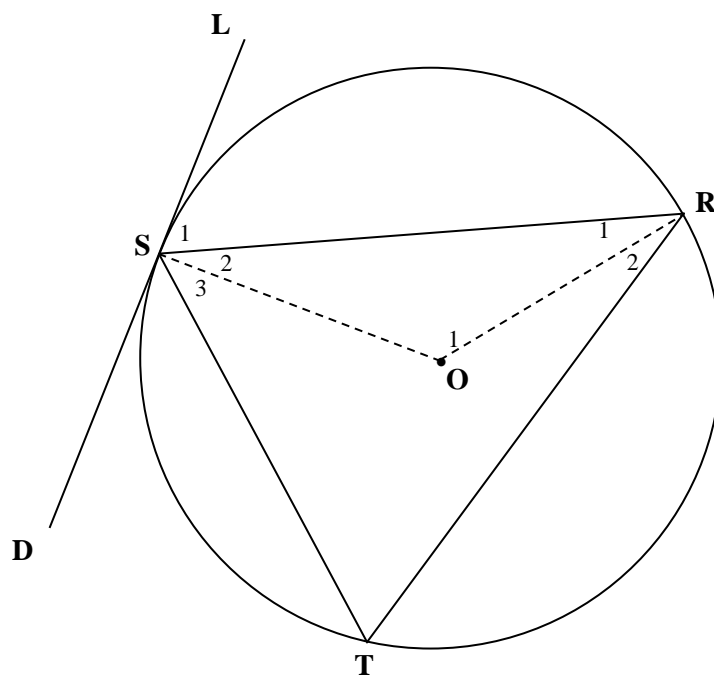
$\hat{S}PR = \hat{S}TR$  [ $\angle$ 'e in dieselde segment]

[∠'e in dieselde segment]

✓ S/R

$$\therefore \mathbf{L}\hat{\mathbf{S}}\mathbf{R} = \mathbf{S}\hat{\mathbf{T}}\mathbf{R}$$

**Of**



Konstruksie: Verbind OS en OR

$$\hat{\mathbf{S}}_1 + \hat{\mathbf{S}}_2 = 90^\circ \quad [\text{rklyn} \perp \text{radius}]$$

$$\hat{S}_2 = 90^\circ - \hat{S}_1$$

$$\hat{R}_1 = \hat{S}_2 = 90^\circ - \hat{S}_1 \quad [\angle\text{'e teenoor gelyke radii}]$$

$$\hat{O}_1 = 180^\circ - 2[90^\circ - \hat{S}_1] = 2\hat{S}_1 \quad [\text{binne } \angle' \text{e van } \Delta]$$

$$\hat{O}_1 = 2\hat{T} \quad [\text{midpunts}\angle = 2 \times \text{omtreks}\angle]$$

$$\therefore \mathbf{L}\hat{\mathbf{S}}\mathbf{R} = \mathbf{S}\hat{\mathbf{T}}\mathbf{R}$$

✓ Konstruksie
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✓ S ✓ R

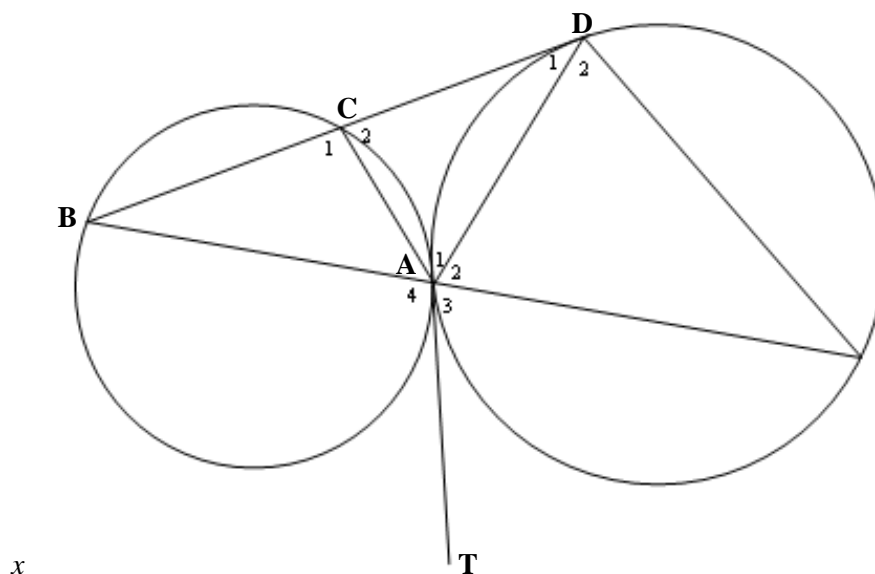
✓ S/R

✓ S

✓ S ✓ R

(6)

11.2



11.2.1  $\angle$  tussen rklyn en koord

✓ R

(1)

11.2.2

$$\hat{A}_4 = 180^\circ - \hat{A}_3 \quad [\angle \text{e op 'n reguitlyn}]$$

✓ SR

$$\hat{C}_1 = 180^\circ - \hat{A}_3 \quad [\angle \text{ tussen rklyn en koord}]$$

✓ S ✓ R

$$\hat{C}_2 = \hat{A}_3 \quad [\angle \text{e op 'n reguitlyn}]$$

✓ SR

$$\hat{D}_2 = \hat{A}_3 \quad [\angle \text{ tussen rklyn en koord}]$$

✓ SR

$$\therefore \hat{C}_2 = \hat{D}_2 \quad [\text{albei gelyk aan } \hat{A}_3]$$

✓ S

$$\hat{D}_1 = \hat{F} \quad [\angle \text{ tussen rklyn en koord /bewys in 11.2.1}]$$

✓ R

$$\hat{A}_1 = \hat{A}_2 \quad [\text{binne } \angle \text{e van } \Delta / 3^{\text{de}} \angle \text{ in } \Delta]$$

(7)

[14]

TOTAAL:

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