KWAZULU-NATAL PROVINCE

EDUCATION
REPUBLIC OF SOUTH AFRICA

NATIONAL SENIOR CERTIFICATE

GRADE 11

MATHEMATICS P2

COMMON TEST

JUNE 2023

MARKING GUIDELINES

MARKS: 100

This marking guideline consists of 9 pages.

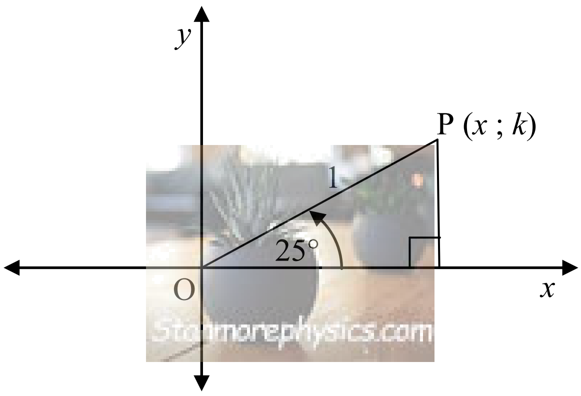
QUESTION 1

1.1.1	$m_{DC}=\frac{y_2-y_1}{x_2-x_1}$ $=\frac{8-0}{1-2}$ $=-8$	✓ substitution ✓ answer (2)
1.1.2	$y=mx+c$ Substitute m and $C(2;0)$: $0=-8(2)+c$ $c=16$ $y=-8x+16$ OR $y-y_1=m(x-x_1)$ $y-0=-8(x-2)$ $y=-8x+16$	✓ substitution ✓ answer (2)
	OR Substitute m and $D(1;8)$: $8=-8(1)+c$ $c=16$ $y=-8x+16$ OR $y-8=-8(x-1)$ $y-8=-8x+8$ $y=-8x+16$	**OR** ✓ substitution ✓ answer (2)
1.1.3	R is the midpoint of AC [diagonals of a parm bisect] $R\left(\frac{-6+2}{2};\frac{4+0}{2}\right)$ $=R(-2;2)$	✓ -2 ✓ 2 (2)
1.1.4	$B(-5;-4)$	✓ -5 ✓ -4 (2)
1.1.5	Substitute $y=-4$ in $y=-8x+16$: $-4=-8x+16$ [BE ‖ to x-axis] $8x=20$ $x=\frac{5}{2}$ Length of BE $=\frac{5}{2}-(-5)$ $=\frac{15}{2}=7\frac{1}{2}$	✓ At E, $y=-4$ ✓ substitution of $y=-4$ into equation of DE ✓ x-value of E ✓ answer (4)

1.1.6	$m_{AC} = \frac{0-4}{2-(-6)} = -\frac{1}{2}$ $m_{BD} = \frac{8-(-4)}{1-(-5)} = 2$ $m_{AC} \times m_{BD} = -\frac{1}{2} \times 2 = -1$ $\therefore$ AC $\perp$ BD [product of gradients = 1] $\therefore$ ABCD is a rhombus [parm with $\perp$ diagonals]	✓ calculating m_{AC} ✓ calculating m_{BD} ✓ lines perpendicular ✓ reason (4)
	OR	**OR**
	$AD = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $= \sqrt{(1-(-6))^2 + (8-4)^2}$ $= \sqrt{7^2 + 4^2}$ $= \sqrt{65}$ $CD = \sqrt{(1-2)^2 + (8-0)^2}$ $= \sqrt{(-1)^2 + 8^2}$ $= \sqrt{65}$ $\therefore$ AD = CD $\therefore$ ABCD is a rhombus [parm with 2 adjacent sides =] (similarly for any other two adjacent sides of ABCD)	✓ substitution in distance formula for AD or CD ✓ length of AD ✓ length of CD ✓ reason (4)
1.1.7 (a)	$\tan\alpha = m_{DC} = -8$ reference $\angle$: $82,87^\circ$ $\alpha = 180^\circ - 82,87^\circ = 97,13^\circ$	✓ $\tan\alpha = m_{DC}$ ✓ $97,13^\circ$ (2)
1.1.7(b)	$\tan A\hat{C}F = m_{AC} = -\frac{1}{2}$ $\therefore \alpha + \beta = 180^\circ - 26,57^\circ$ $= 153,43^\circ$ $\therefore \beta = 153,43^\circ - 97,13^\circ$ $= 56,30^\circ$	✓ $\tan A\hat{C}F = m_{AC} = -\frac{1}{2}$ ✓ size of $\alpha + \beta$ ✓ answer (3)
1.1.8	Height of ΔCEB = 4 units Area of $\Delta CEB = \frac{1}{2} \times \text{base} \times \text{height}$ $= \frac{1}{2} \times \frac{15}{2} \times 4$ $= 15 \text{ units}^2$	✓ height = 4 units ✓ substitution in area formula ✓ answer (3)

1.2.1	$m = \tan 16,70^\circ$ $= 0,3$	✓ answer (1)
1.2.2	Equation of EF: $y = 0,3x + 1,2$ For x-intercept, substitute $y = 0$: $0 = 0,3x + 1,2$ $x = -4$, and therefore: EO = 4 units $G(0\,;-4)$ $m_{GH} = 0,3 = \frac{0-(-4)}{x_H - 0}$ $0,3 = \frac{4}{x_H}$ $x_H = \frac{4}{0,3} = 13,33$ H(13,33 ; 0) **OR** Equation of GH: $y = 0,3x - 4$ For x-intercept of GH, substitute $y = 0$: $0 = 0,3x - 4$ $x = 13,33$ $\therefore H(13,33\,;0)$	✓ substitution of m and y into equation of line ✓ EO = 4 units **or** $x = -4$ at E ✓ $G(0\,;-4)$ ✓ $0,3 = \frac{0-(-4)}{x_H - 0}$ **OR** subst. $y = 0$ in equation of GH. ✓ answer (5)
		[30]

QUESTION 2

2.1.1	 $x^2 = r^2 - y^2$ [Theorem of Pythagoras] $= 1^2 - k^2$ $\therefore x = \sqrt{1-k^2}$ $\cos 25^\circ = \frac{\sqrt{1-k^2}}{1} = \sqrt{1-k^2}$ **OR** $\sin^2 25^\circ + \cos^2 25^\circ = 1$ $\cos^2 25^\circ = 1 - \sin^2 25^\circ$ $= 1 - k^2$ $\therefore \cos 25^\circ = \sqrt{1-k^2}$	✓ correct sketch ✓ $x = \sqrt{1-k^2}$ ✓ answer (3) **OR** ✓ square identity ✓ $\cos^2 25^\circ$ in terms of k ✓ answer (3)

2.1.2	$\sin 205^\circ = -\sin 25^\circ$ $= -k$	✓ $-\sin 25^\circ$ ✓ answer (2)
2.1.3	$\tan 385^\circ = \tan(360^\circ + 25^\circ)$ $= \tan 25^\circ$ $= \dfrac{k}{\sqrt{1-k^2}}$	✓ $\tan 25^\circ$ ✓ answer (2)
2.2	LHS $= \dfrac{-\tan x}{\cos x} + \dfrac{1}{\sin x \cos^2 x}$ $= -\dfrac{\sin x}{\cos x} \times \dfrac{1}{\cos x} + \dfrac{1}{\sin x \cos^2 x}$ $= \dfrac{-\sin x}{\cos^2 x} + \dfrac{1}{\sin x \cos^2 x}$ $= \dfrac{-\sin^2 x + 1}{\sin x \cos^2 x}$ $= \dfrac{\cos^2 x}{\sin x \cos^2 x}$ $= \dfrac{1}{\sin x}$ = RHS	✓ $-\dfrac{\sin x}{\cos x}$ ✓ $\dfrac{-\sin x}{\cos^2 x}$ ✓ adding two fractions ✓ $1 - \sin^2 x = \cos^2 x$ (4)
2.3	$\dfrac{\sqrt{3}\sin x \sin^2 58^\circ - \sqrt{3}\sin^2 212^\circ . \cos(x + 90^\circ)}{\tan 120^\circ . \sin x}$ $= \dfrac{\sqrt{3}\sin x \sin^2 58^\circ - \sqrt{3}(-\sin 32^\circ)^2 . (-\sin x)}{-\tan 60^\circ . \sin x}$ $= \dfrac{\sqrt{3}\sin x \sin^2 58^\circ + \sqrt{3}\sin^2 32^\circ . \sin x}{-\sqrt{3} . \sin x}$ $= \dfrac{\sqrt{3}\sin x(\sin^2 58^\circ + \sin^2 32^\circ)}{-\sqrt{3} . \sin x}$ $= -(\sin^2 58^\circ + \sin^2 32^\circ)$ $= -(\sin^2 58^\circ + \cos^2 58^\circ)$ $= -1$	✓ $-\sin 32^\circ$ ✓ $-\sin x$ ✓ $-\tan 60^\circ$ ✓ $\tan 60^\circ = \sqrt{3}$ ✓ factorisation ✓ $\sin 58^\circ = \cos 32^\circ$ ✓ answer (7)
		[18]

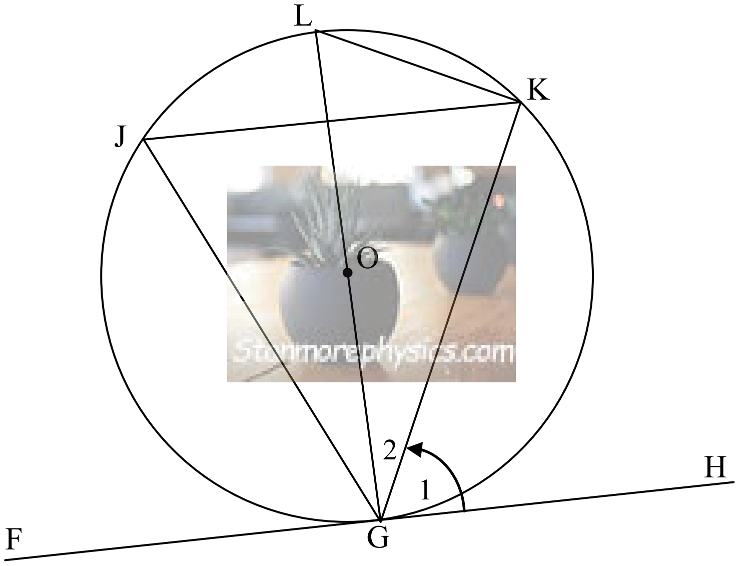
QUESTION 3

3.1	$2\cos^2 x - 7\cos x - 2\sin^2 x = 0$ $2\cos^2 x - 7\cos x - 2(1-\cos^2 x) = 0$ $2\cos^2 x - 7\cos x - 2 + 2\cos^2 x = 0$ $4\cos^2 x - 7\cos x - 2 = 0$ $(4\cos x + 1)(\cos x - 2) = 0$ $4\cos x = -1$ or $\cos x = 2$ $\cos x = -\frac{1}{4}$ no solution Ref $\angle$: $75{,}52^\circ$ Quadrant 2: $x = 180^\circ - 75{,}52^\circ + k.360^\circ$ $x = 104{,}48^\circ + k.360^\circ,\ k \in Z$ **or** Quadrant 3: $x = 180^\circ + 75{,}52^\circ + k.360^\circ$ $x = 255{,}52^\circ + k.360^\circ,\ k \in Z$	✓ using $\sin^2 x = 1 - \cos^2 x$ ✓ standard form ✓ factors ✓ $\cos x = 2$: no solution ✓ $\cos x = -\frac{1}{4}$ ✓ $x = 104{,}48^\circ + k.360^\circ,\ k \in Z$ ✓ $x = 255{,}52^\circ + k.360^\circ,\ k \in Z$ ($k \in Z$: should be written at least once.) (7)
3.2.1	$a = 1$ $b = 3$	✓ value of a ✓ value of b (2)
3.2.2	amplitude = 2	✓ answer (1)
3.2.3	period = 120°	✓ answer (1)
3.2.4	$x \in [-68{,}91^\circ ; -30^\circ]$ **OR** $-68{,}91^\circ \le x \ge -30^\circ$	✓ ✓ answer (2)
3.2.5	$h(x) = -2\sin(x + 90^\circ) - 1$ **OR** $h(x) = -2\cos x - 1$	✓ $-2\sin(x+90^\circ)$ ✓ -1 (2) **OR** ✓ $-2\cos x$ ✓ -1 (2)
3.2.6	$2\sin x = k$ $2\sin x + 1 = k + 1$ $f(x) = k + 1$ will have no real roots if $f(x) > 3$ or $f(x) < -1$ $k + 1 > 3$ or $k + 1 < -1$ $k > 2$ or $k < -2$	✓ $k > 2$ ✓ $k < -2$ (2)
		[17]

QUESTION 4

4.1	$B\hat{C}E = 90^\circ$ [$\angle$ in a semicircle] $\hat{C}_1 = 90^\circ - 52^\circ$ $= 38^\circ$	✓ S ✓R ✓ answer (3)
4.2	$\hat{C}_3 = 20^\circ$ [alternate $\angle$s; BE ‖ CD]	✓S/R (1)
4.3	$B\hat{C}D = 90^\circ + 20^\circ = 110^\circ$ $B\hat{A}D = 180^\circ - B\hat{C}D$ [opp. $\angle$s of cyclic quad.] $= 180^\circ - 110^\circ = 70^\circ$	✓ R ✓ answer (2)
4.4	$\hat{A}_3 = \hat{E}_1 = 20^\circ$ [$\angle$s in the same segment] $\therefore \hat{A}_2 = 70^\circ - 20^\circ = 50^\circ$	✓ S/R ✓ answer (2)
		[8]

QUESTION 5

5.1	Construction: Draw diameter LOG and join L and K.  $\hat{G}_1 + \hat{G}_2 = 90^\circ$ [tangent $\perp$ to radius] $L\hat{K}G = 90^\circ$ [$\angle$ in a semicircle] $\hat{L} + \hat{G}_2 = 90^\circ$ [sum of $\angle$s in a triangle] $\therefore \hat{L} = \hat{G}_1$ But: $\hat{L} = \hat{J}$ [$\angle$s in the same segment] $\therefore \hat{G}_1 = \hat{J}$	✓ construction ✓ S ✓ R ✓ S/R ✓ S/R (5)

5.2.1	$W\hat{P}O = 90^\circ$ [tangent $\perp$ to radius] $W\hat{R}O = 90^\circ$ [tangent $\perp$ to radius] $\therefore W\hat{P}O + W\hat{R}O = 180^\circ$ $\therefore$ PWRO is a cyclic quadrilateral [converse: opp. $\angle$ s of a cyclic quad.]	✓ S/R (one mark for either of the two statements with reason) ✓ S ✓ R (3)
5.2.2	$PO = OR$ [radii] $\hat{W}_1 = \hat{W}_2$ [subtended by = chords in cyclic quad PWRO] **OR** In ΔWPO and ΔWRO: 1. $W\hat{P}O = W\hat{R}O$ [proved above] 2. WP = WR [two tangents from same point] 3. PO = RO [radii] $\therefore \Delta WPO \equiv \Delta WRO$ [s;$\angle$;s] $\therefore \hat{W}_1 = \hat{W}_2$ [$\equiv \Delta$s] **OR** $\therefore \Delta WPO \equiv \Delta WRO$ [s;s;s] $\therefore \hat{W}_1 = \hat{W}_2$ [$\equiv \Delta$s] **OR** $\therefore \Delta WPO \equiv \Delta WRO$ [90°, hyp; s] $\therefore \hat{W}_1 = \hat{W}_2$ [$\equiv \Delta$s]	✓ S/R ✓ R (2) **OR** ✓ congruent Δs ✓ reason for congruency (2) **OR** ✓ congruent Δs ✓ reason for congruency (2) **OR** ✓ congruent Δs ✓ reason for congruency (2)
5.2.3	$P\hat{O}R = P\hat{W}V = 134^\circ$ [ext. $\angle$ of cyclic quad.] $\hat{S} = \frac{1}{2} P\hat{O}R$ [$\angle$ at centre $= 2 \times \angle$ at circum.] $= 67^\circ$ **OR** $\hat{W}_2 = \frac{180^\circ - 134^\circ}{2}$ [$\angle$s on a straight line; $\hat{W}_1 = \hat{W}_2$] $= 23^\circ$ $\hat{O}_1 = 180^\circ - (90^\circ + 23^\circ)$ [sum of $\angle$s of ΔQRO] $= 67^\circ$ Similarly: $\hat{O}_2 = 67^\circ$ $\therefore P\hat{O}R = 134^\circ$ $\hat{S} = \frac{1}{2} P\hat{O}R$ [$\angle$ at centre $= 2 \times \angle$ at circumf.] $= 67^\circ$	✓ S ✓R ✓ R ✓ answer (4) **OR** ✓ $\hat{W}_2 = 23^\circ$ ✓ $P\hat{O}R = 134^\circ$ ✓ R ✓ answer (4)
		[14]

QUESTION 6

6.1	$\angle$s in the same segment	✓R (1)
6.2	$\hat{A}_2 = \hat{C}_2 = x$ [$\angle$s opp. = sides] **OR** [= chords subtend = $\angle$s] $\hat{A}_1 = \hat{C}_2 = x$ [tan-chord-theorem] $\therefore$ DA bisects $\hat{AEF}$	✓S ✓R ✓S ✓R (4)
6.3	$\hat{AOD} = 2 \times \hat{C}_2$ [$\angle$ at centre = 2 × $\angle$ at circum.] $= 2x$ $\therefore \hat{EAF} = \hat{AOD}$ [both = $2x$] $\therefore$ EA is a tangent to the circle through A, O and F [converse tan-chord-theorem]	✓R ✓S ✓S ✓reason (4)
6.4	$\hat{A}_3 = 90^\circ - \angle x$ [tangent $\perp$ to radius] $\hat{AFO} = 180^\circ - (90^\circ - \angle x + 2x)$ [sum of $\angle$s in a triangle] $= 90^\circ$ $\therefore$ AF = FC [line from centre $\perp$ to chord]	✓S ✓R ✓ $\hat{AFO} = 90^\circ$ ✓R (4)
		[13]

TOTAL: 100