Basic Education

KwaZulu-Natal Department of Basic Education
REPUBLIC OF SOUTH AFRICA

MATHEMATICS

COMMON TEST

JUNE 2016

NATIONAL SENIOR CERTIFICATE

GRADE 10

MARKS: 100

TIME: 2 hours

This question paper consists of 7 pages and a 1 page diagram sheet.

INSTRUCTIONS AND INFORMATION

Read the following instructions carefully before answering the questions:

1. This question paper consists of 8 questions.
2. Answer ALL the questions.
3. Clearly show ALL calculations, diagrams, graphs, et cetera, which you have used in determining the answers.
4. Answers only will NOT necessarily be awarded full marks.
5. You may use an approved scientific calculator (non-programmable and non-graphical), unless stated otherwise.
6. If necessary, round off answers to TWO decimal places, unless stated otherwise.
7. ONE diagram sheet for answering QUESTION 7.1, QUESTION 7.2 and QUESTION 8 is attached at the end of this question paper. Write your name on this sheet in the space provided and insert the sheet inside the back cover of your ANSWER BOOK.
8. Diagrams are NOT necessarily drawn to scale.
9. Number the answers correctly according to the numbering system used in this question paper.
10. Write neatly and legibly.

QUESTION 1

1.1 On a number line, indicate between which two integers does $\sqrt{11}$ lies. (2)

1.2 Write $0,\dot{7}$ as a common fraction. Clearly show all your working details. (3)

1.3 Determine the product of the following and simplify fully:

$(x-2)(x^2+5x-1)$ (3)

1.4 Simplify the following expression fully:

$$\frac{x^2-4}{2x^2+5x+2} \div \frac{x^3-8}{6x+3}$$ (5)

1.5 Simplify the following expression:

$$\frac{2^{2x-1}.2.4^{x+1}}{4^{2x}}$$ (4)
[17]

QUESTION 2

2.1 Solve for x: $2x^2-7x+3=0$ (3)

2.2 Solve the following inequality: $-7(3x+2)+5(2x-3)>4$

and illustrate your answer on a number line if x is a real number. (4)

2.3 Solve for x: $3^x=\frac{1}{81}$ (3)

2.4 Solve for x and y:

$4x+y=-5$ and $-3x+4y=18$ (5)
[15]

QUESTION 3

Sipho prepares himself for a road race. In the first week he will run 10km. He will then increase the distance by 4km every week.

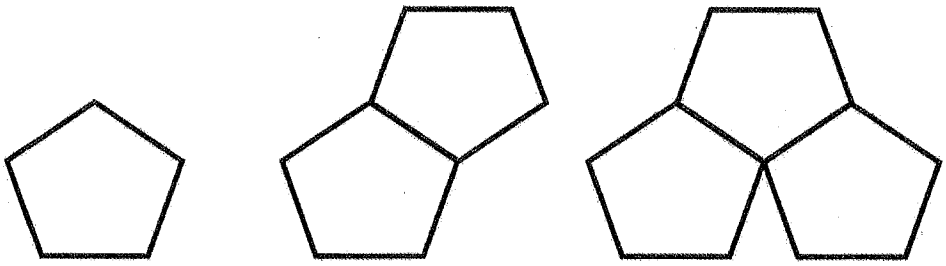
3.1 Write down a number pattern for the distance he will run every week. (1)

3.2 Calculate the total distance that he will run in the first 6 weeks. (2)

3.3 By using a formula, determine the distance that he will run in week 12. (2)
[5]

QUESTION 4

Matches are used to form the regular pentagon patterns below:



Pattern 1 Pattern 2 Pattern 3

4.1 How many matches are needed to make Pattern 4? (2)

4.2 What relationship do you recognise between the number of pentagons and the number of matches used for each pattern? (2)

4.3 Write down a rule or formula to generalise a relationship between the number of pentagons(n) and the number of matches(m). (2)

4.4 Calculate how many matches are required to make a pattern with 25 pentagons. (2)

4.5 Calculate how many regular pentagons can be made from 173 matches? (2)
[10]

QUESTION 5

The following functions are given: $f(x)=2^x+1$ and $g(x)=3x^2-12$.

5.1 Sketch the graphs of f and g on the same set of axes.
Show all intercepts with the axes, turning point(s) and asymptotes where applicable. (8)

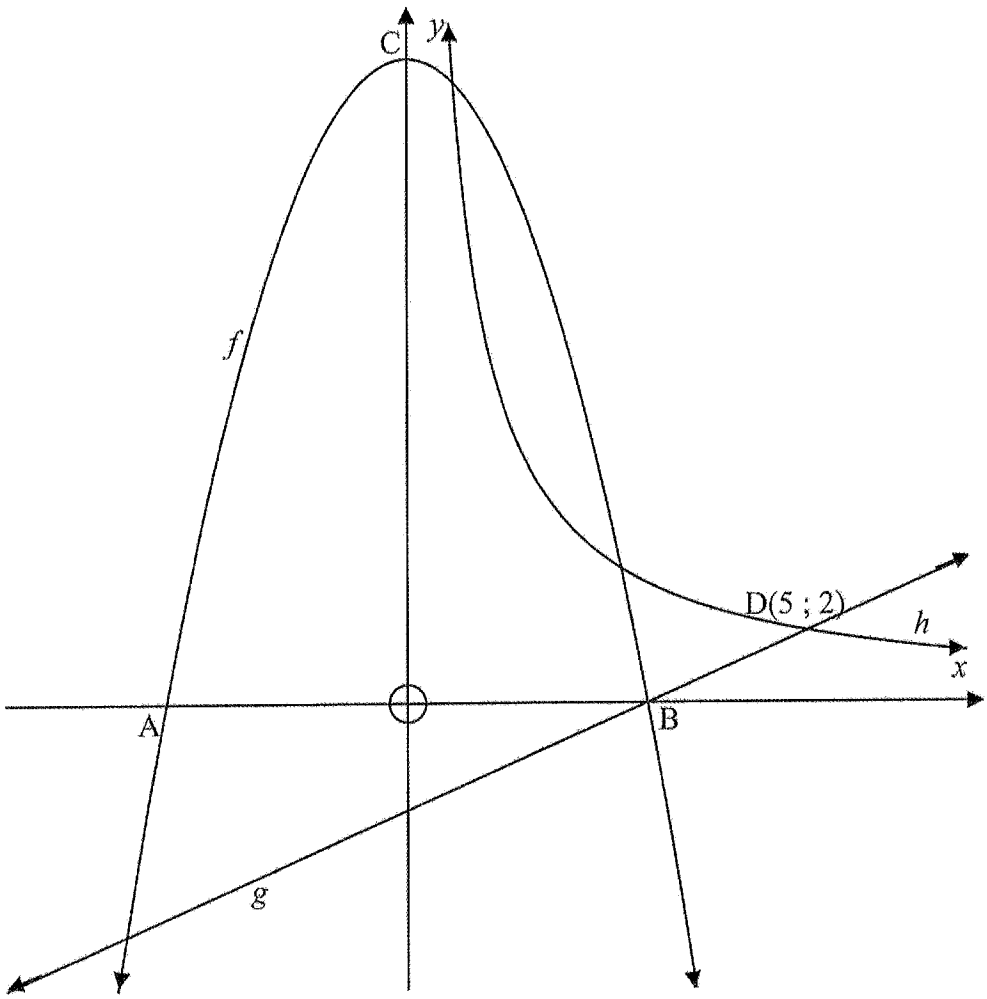
5.2 Use your graphs to solve for x if:

5.2.1 $g(x)>0$ (2)

5.2.2 $f(x)\geq 2$ (1)
[11]

QUESTION 6

Sketched below are the graphs of $f(x) = -2x^2 + 18$, $g(x) = mx + c$ and $h(x) = \frac{k}{x}$ where $x > 0$. The graph of f intersects the x–axis at A and B and the y–axis at C, which is also the turning point of f. The graph of g also intersects the x–axis at B. One of the points of intersection of g and h is D(5 ; 2).



Use the graphs and the information given above to answer the following :

6.1 The coordinates of A and B. (4)

6.2 The length of OC. (1)

6.3 The values of m and c. (3)

6.4 The equation of h. (2)

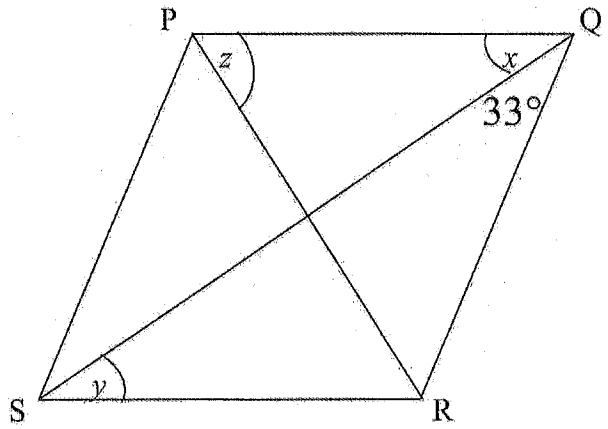
6.5 The range of f. (2)

6.6 The equation of the straight line through B perpendicular to g. (3)
[15]

QUESTION 7

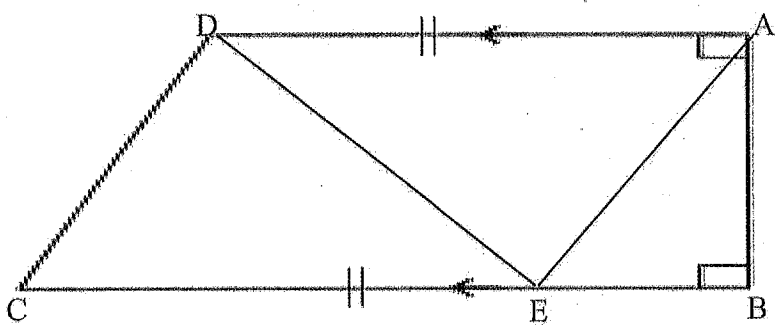
In this question, you must give a reason to justify each of your statements.

7.1 In the diagram below, PQRS is a rhombus. Calculate the size of angles x, y and z. Where $R\hat{P}Q = z$; $P\hat{Q}S = x$ and $Q\hat{S}R = y$.



(6)

7.2 In the diagram below, ABCD is a trapezium with AD // EC and AD = EC. The area of $\Delta ABE = 16cm^2$ and BE = 8cm.



7.2.1 Calculate the perpendicular height of ΔABE. (3)

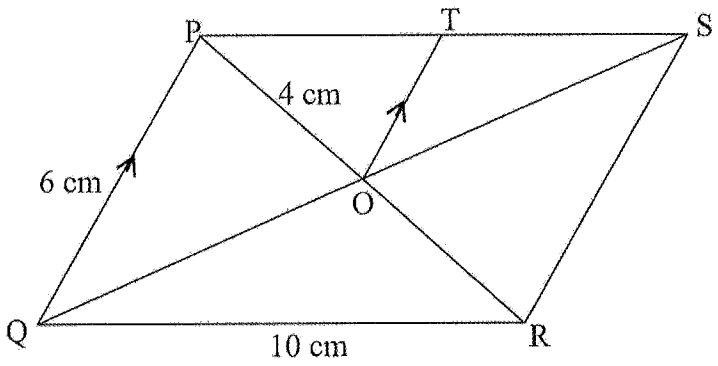
7.2.2 If AD = 10cm, calculate the area of ΔADE. (2)

7.2.3 Calculate the area of trapezium ABCD. (3)
[14]

QUESTION 8

In this question, you must give a reason to justify each of your statements.

In the figure below, PQRS is a parallelogram with diagonals PR and QS intersecting at O. PO = 4 cm, PQ = 6 cm, QR = 10 cm and TO // PQ.



8.1 Calculate the length of PR. (3)

8.2 Calculate the length of ST. (5)

8.3 Calculate the length of OT. (2)

8.4 Prove by calculation that $\hat{QPR} = 90°$. (3)

[13]

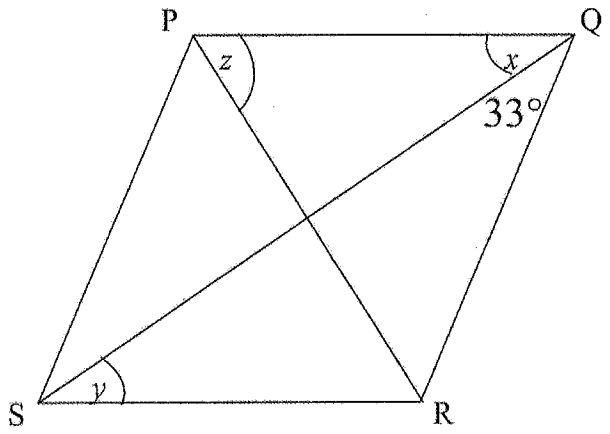
TOTAL MARKS: 100

PLEASE TEAR ON DOTTED LINES

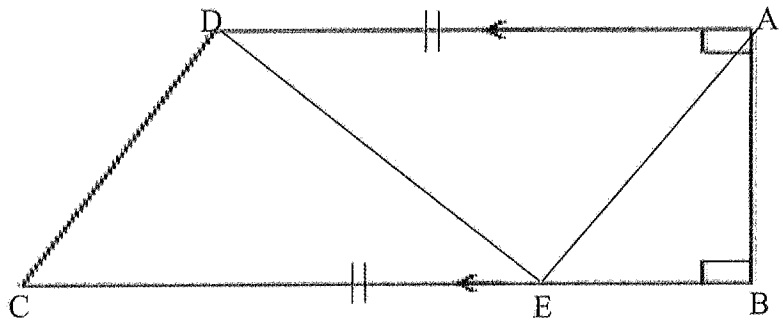
NAME: _____

DIAGRAM SHEET

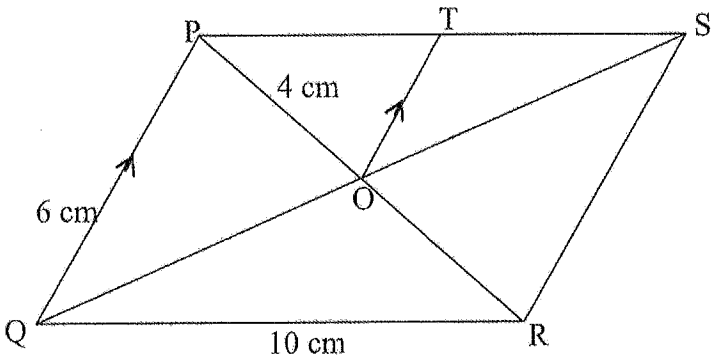
QUESTION 7.1



QUESTION 7.2



QUESTION 8



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MEMORANDUM

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GRADE 10

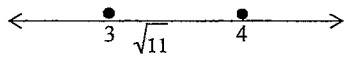
MARKS: 100

TIME: 2 hours

SYMBOL	DESCRIPTION
✓S	Mark for correct statement, no penalty for reason omitted
✓R	Mark for reason
✓S/R	Mark for correct statement with correct reason

This memorandum consists of 8 pages.

QUESTION 1

1.1		✓3 ✓4 (2)
1.2	Let $x = 0,\dot{7}$ Then $10x = 7,\dot{7}$ i.e. $9x = 7$ $\therefore x = \frac{7}{9}$	✓$10x = 7,\dot{7}$ ✓$9x = 7$ ✓$x = \frac{7}{9}$ (3)
1.3	$(x-2)(x^2+5x-1)$ $= x^3+5x^2-x-2x^2-10x+2$ $= x^3+3x^2-11x+2$	✓x^3+5x^2-x ✓$-2x^2-10x+2$ ✓ answer (3)
1.4	$\frac{x^2-4}{2x^2+5x+2} \div \frac{x^3-8}{6x+3}$ $= \frac{(x+2)(x-2)}{(x+2)(2x+1)} \times \frac{3(2x+1)}{(x-2)(x^2+2x+4)}$ $= \frac{3}{x^2+2x+4}$	✓$(x+2)(x-2)$ ✓$(x+2)(2x+1)$ ✓ invert & multiply by $3(2x+1)$ ✓$(x-2)(x^2+2x+4)$ ✓ answer (5)
1.5	$\frac{2^{2x-1}.2.4^{x+1}}{4^{2x}}$ $= \frac{2^{2x-1}.2.(2^2)^{x+1}}{(2^2)^{2x}}$ $= \frac{2^{2x-1}.2.2^{2x+2}}{2^{4x}}$ $= 2^2$ $=4$	✓ prime base of 2 ✓ application of laws ✓ simplification ✓ answer (4)
		[17]

QUESTION 2

2.1	$2x^2-7x+3=0$ $(2x-1)(x-3)=0$ $x=\frac{1}{2}$ or $x=3$	✓ factors ✓ $x=\frac{1}{2}$ ✓ $x=3$ (3)
2.2	$-7(3x+2)+5(2x-3)>4$ $-21x-14+10x-15>4$ $-11x>33$ $x<-3$  	✓multiplying out ✓ simplification ✓ answer ✓ number line (4)
2.3	$3^x=\frac{1}{81}$ $3^x=\frac{1}{3^4}$ $3^x=3^{-4}$ $x=-4$	✓ $81=3^4$ ✓ $\frac{1}{3^4}=3^{-4}$ ✓ answer (3)
2.4	$4x+y=-5$ $\Rightarrow y=-4x-5$ $-3x+4(-4x-5)=18$ $-3x-16-20=18$ $-19x=38$ $x=-2$ $y=-4(-2)-5$ $=3$	✓ $y=-4x-5$ ✓ substitution ✓ multiplying out ✓ $x=-2$ ✓ $y=3$ (5)
		[15]

QUESTION 3

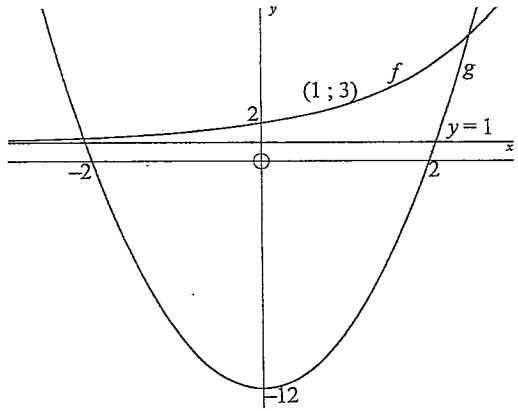
3.1	Distance he will run every week: 10 ; 14 ; 18 ; …	✓ 10 ; 14 ; 18 ; … (1)
3.2	Total distance that he will run in the first 6 weeks: $10+14+18+22+26+30$ $=120$ km	✓10+14+18+22+26+30 ✓ answer (2)
3.3	Distance that he will run in week 12: $Tn=4n+6$ $T_{12}=4(12)+6$ $=54$ km	✓ substitute 12 for n into $Tn=4n+6$ ✓ answer (2) N.B. Answer only: full marks
		[5]

QUESTION 4

4.1	17 matches	✓✓ 17 (2)
4.2	5 matches are needed for the first pentagon, thereafter you only need 4 matches for every additional pentagon.	✓ 5 matches are needed for the first pentagon ✓ 4 matches for every additional pentagon (2)
4.3	$m=4n+1$	✓ $4n$ ✓ $+1$ (2)
4.4	No. of matches required to make a pattern with 25 pentagons: $4(25)+1$ $=101$	✓ substitute 25 for n into rule found in 4.3 ✓ answer (2) N.B. Answer only: full marks
4.5	$4n+1=173$ $4n=172$ $n=43$ 43 pentagons can be made from 173 matches	✓ $4n+1=173$ ✓ $n=43$ (2)
		[10]

QUESTION 5

$f(x)=2^x+1$ and $g(x)=3x^2-12$.

5.1		✓ y–intercept of f: (0;2) ✓ another point on f eg. (1 ; 3) ✓ shape of f i.e. increasing exponential fn. ✓ asymptote of f: $y=1$ x–intercepts of g: ✓(–2 ; 0) and ✓ (2 ; 0) ✓y–intercept of g:(0;–12) ✓ shape of g i.e. minimum value parabola (8)
5.2	$x<-2$ or $x>2$ **OR** $x\in(-\infty\,;-2)\cup(2\,;\infty)$	✓✓ answer (2)
5.3	$x\geq 0$ **OR** $x\in[0\,;\infty)$	✓ answer (1)
		[11]

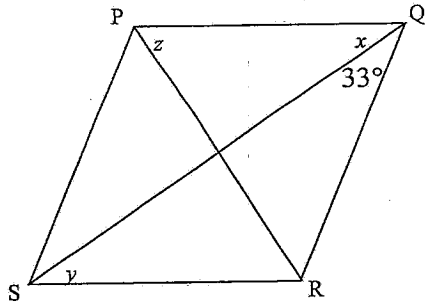
QUESTION 6

$f(x)=-2x^2+18$, $g(x)=mx+c$ and $h(x)=\frac{k}{x}$ where $x>0$.

6.1	The x-intercepts of f are: $-2x^2+18=0$ $-2(x^2-9)=0$ $2(x+3)(x-3)=0$ $x=-3$ or $x=3$ A(–3 ; 0) and B(3 ; 0)	✓ $y=0$ ✓ factors ✓ $x=-3$ or $x=3$ ✓ coordinates of A and B (4)
6.2	OC = 18 units	✓ answer (1)
6.3	$m=\frac{2-0}{5-3}$ $m=1$ $y=x+c$ $2=5+c$ $c=-3$ $y=x-3$	✓ correct substitution into gradient formula ✓ $m=1$ ✓ $c=-3$ (3)
6.4	$2=\frac{k}{5}$ $k=10$ $h(x)=\frac{10}{x}$; $x>0$	✓ substitute (5 ; 2) into h ✓ answer (2)
6.5	$y\leq 18$ **OR** $x\in(-\infty\,;18]$	✓✓ answer (2)
6.6	$y=-x+c$ $0=-3+c$ $c=3$ $y=-x+3$	✓ $m=1$ ✓substitute (3;0) into line ✓ answer (3)
		[15]

QUESTION 7

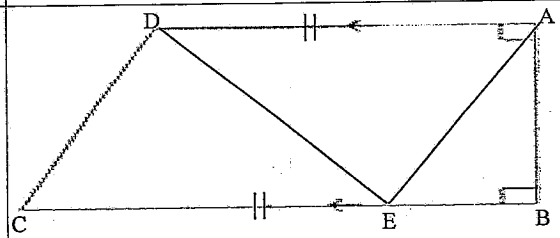
7.1



$x = 33°$	(diagonals of rhombus bisect angles)	✓ S ✓R
$y = 33°$	(angles opp. = sides of rhombus)	✓ S ✓R
$z = 57°$	(sum of angles of $\Delta = 180°$)	✓ S ✓R

(6)

7.2.1



Area of $\Delta ABE = \frac{1}{2} BE \times AB$	✓ formula for area of Δ
$16 = \frac{1}{2}(8) \times AB$	✓ substitution
$\perp$ height $= 4$ cm	✓ answer

(3)

7.2.2

Area of $\Delta ADE = \frac{1}{2} AD \times AB$	
$= \frac{1}{2}(10 \times 4)$	✓ substitution
$= 20\ cm^2$	✓ answer

(2)

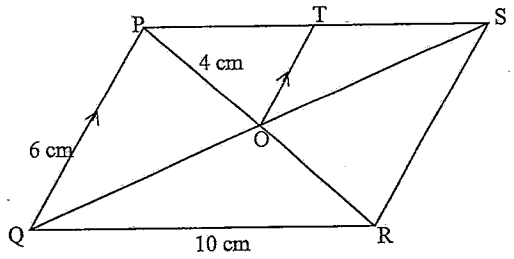
7.2.3

Area of $\Delta ABCD = \frac{1}{2}$(sum of parallel sides)$\times \perp$ height	✓ formula for area of trapezium
$= \frac{1}{2}(10 + 18)(4)$	✓ substitution
$= 56\ cm^2$	✓ answer

(3)

[14]

QUESTION 8



8.1

PO = OR = 4 cm (diagonals of a parm bisect each other)	✓S ✓ R
PR = 8 cm	✓answer

(3)

8.2

OS = OQ (diagonals of a parm bisect each other)	
TO // PQ (given)	
$\therefore$TS = PT (line drawn from the midpoint of one side, parallel to the 2nd side, bisects the 3rd side)	✓ S ✓ R
PS = QR = 10 cm (opp. sides of a parm are equal)	✓S ✓ R
$\therefore$TS = 5 cm	✓answer

(5)

8.3

TS = PT (proved above)	
OS = OQ (proved above)	
$\therefore OT = \frac{1}{2} PQ$ (midpoint theorem)	✓ S/R
$= 3$ cm	✓ answer

(2)

8.4

$QR^2 = 10^2 = 100$	✓ $QR^2 = 100$
$PQ^2 + PR^2 = 6^2 + 8^2$	✓ $PQ^2 + PR^2 = 100$
$= 100$	
$\therefore Q\hat{P}R = 90°$ (converse: theorem of Pythagoras)	✓ R

(3)

[13]

TOTAL MARKS: 100